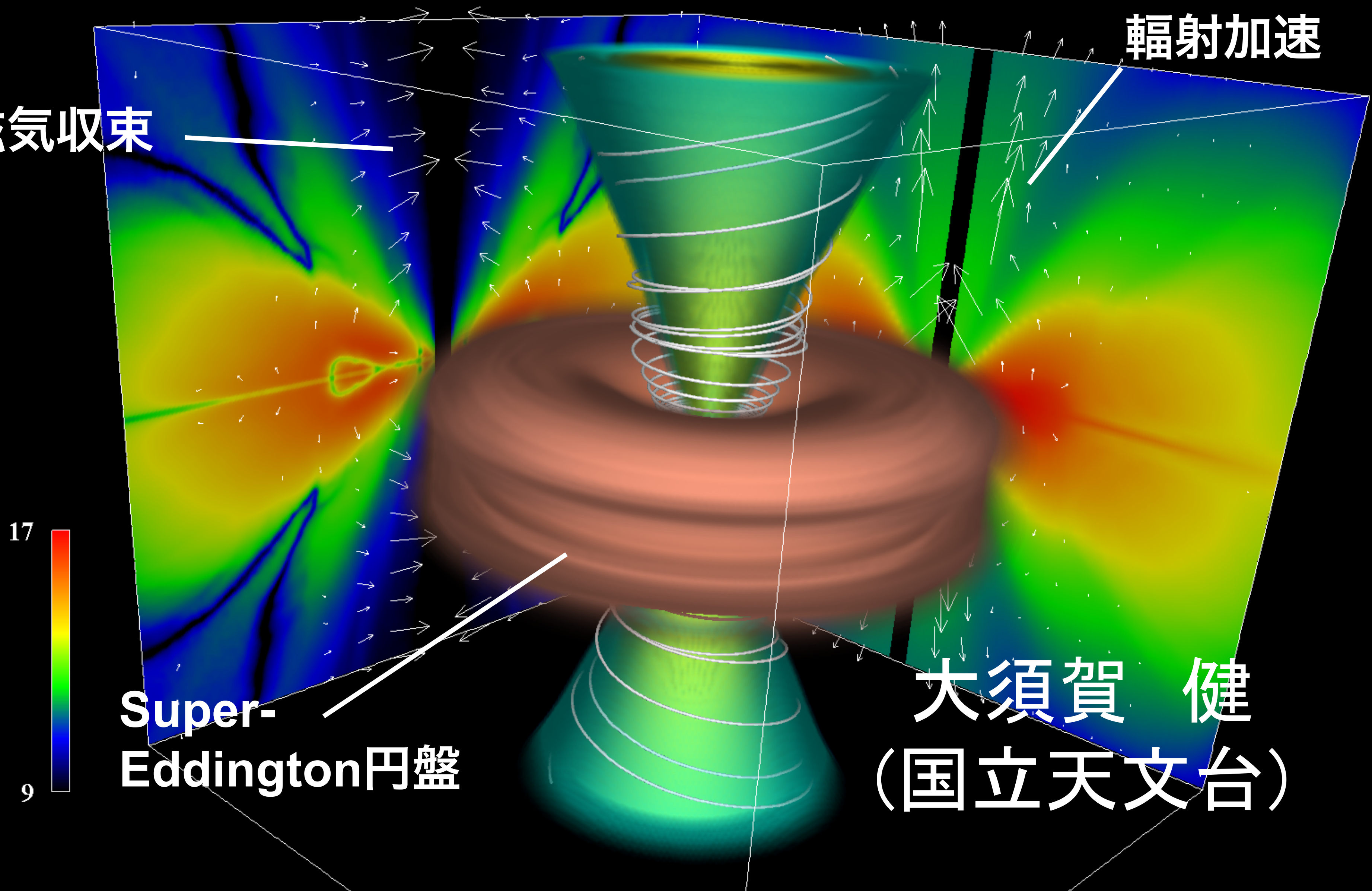
輻射MHDの講義

–輻射流体力学の基礎–



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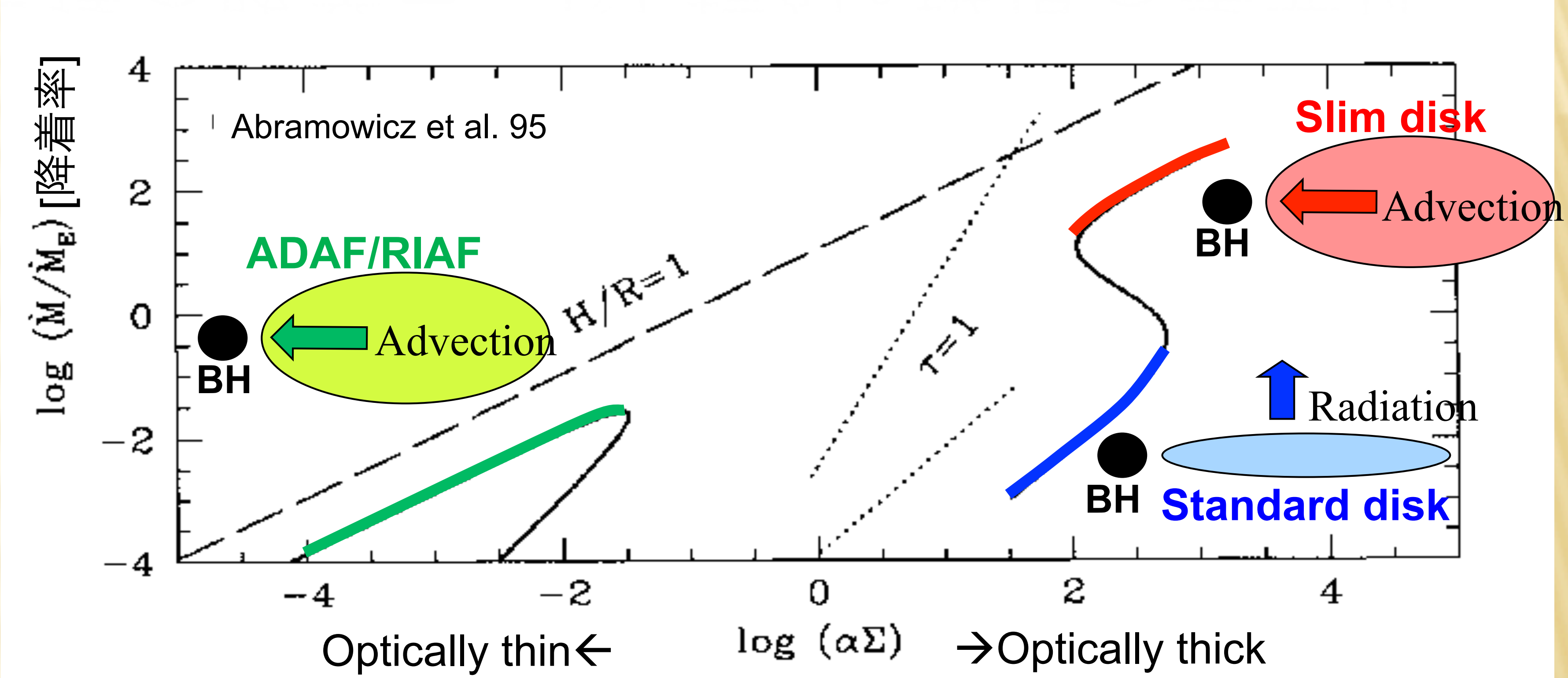
三種の降着モードと輻射&磁場の重要性



ADAF/RIAF	
磁場	**YES**
輻射(冷却)	No
輻射(力)	No

Standard	
磁場	**YES**
輻射(冷却)	**YES**
輻射(力)	No

Slim disk	
磁場	**YES**
輻射(冷却)	**YES**
輻射(力)	**YES**

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輻射流体方程式(1)

Radiation Transfer Eq.

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \mathbf{n}\cdot\nabla I_\nu = (\kappa_\nu + \sigma_\nu)(S_\nu - I_\nu)$$

κ_ν : absorption − opacity

σ_ν : scattering − opacity

S_ν : source − function [emission + scattering]

Radiation Energy, Flux, Stress Tensor

$$E \equiv \int E_\nu d\nu = \frac{1}{c}\int I_\nu d\nu d\Omega$$

$$F^i \equiv \int F_\nu^i d\nu = \int I_\nu n^i d\nu d\Omega$$

$$P^{ij} \equiv \int P_\nu^{ij} d\nu = \frac{1}{c}\int I_\nu n^i n^j d\nu d\Omega$$

輻射流体方程式(2)

Energy-Momentum Tensor $\quad T^{\mu\nu} + R^{\mu\nu}$

(Hydro) $\quad T^{\mu\nu} = \rho_m u^\mu u^\nu - \eta^{\mu\nu} p$

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu = c^2 dt^2 - \left(dx^2 + dy^2 + dz^2\right)$$

$$\rho_m = \rho + \frac{e}{c^2} + \frac{p}{c^2}$$

$$u^\mu = \gamma\left(c, \mathbf{v}\right)$$

(Radiation)

$$R^{\mu\nu} = \begin{pmatrix} E & \mathbf{F}/c \\ \mathbf{F}/c & \mathbf{P} \end{pmatrix}$$

輻射流体方程式(3)

Eq. of Motion

$$\left(T^{\mu\nu} + R^{\mu\nu}\right)_{,\nu} = 0$$

$$\begin{cases} R^{0\nu}{}_{,\nu} = \left(E_{,t} + F^{i}{}_{,i}\right)/c \\ R^{i\nu}{}_{,\nu} = F^{i}{}_{,t} + R^{ij}{}_{,j}/c^2 \end{cases}$$

⬇

$$\begin{cases} \mu = 0; & \left(\gamma^2 \rho_m c^2 - p\right)_{,t} + \left(\gamma^2 \rho_m c^2 v_i\right)_{,i} + \left(E_{,t} + F^{i}{}_{,i}\right) = 0 \\ \mu = i; & \left(\gamma^2 \rho_m v_i\right)_{,t} + \left(\gamma^2 \rho_m c^2 v_i v_j\right)_{,j} + \left(\dfrac{1}{c^2} F^{i}{}_{,t} + R^{ij}{}_{,j}\right) = 0 \end{cases}$$

⬇ (μ=iの式)-(μ=0の式)xv_i/c^2

$$\gamma^2 \rho_m \left(v_{j,t} - v_i v_{j,i}\right) = -p_{,i} - \frac{v_j}{c^2} p_{,t} + \frac{v_j}{c^2}\left(E_{,t} + F^{i}{}_{,i}\right) - \left(\frac{1}{c^2} F^{i}{}_{,t} + R^{ij}{}_{,j}\right)$$

$$\rightarrow \gamma^2 \rho_m \frac{D\mathbf{v}}{Dt} = -\nabla p - \frac{\mathbf{v}}{c^2}\frac{\partial p}{\partial t} + \frac{\mathbf{v}}{c^2}\left(\frac{\partial E}{\partial t} + \nabla\cdot\mathbf{F}\right) - \left(\frac{1}{c^2}\frac{\partial \mathbf{F}}{\partial t} + \nabla\cdot\mathbf{P}\right)$$

輻射流体方程式(4)

Eq. of Gas-Energy

$$u_\mu \left(T^{\mu\nu} + R^{\mu\nu}\right)_{,\nu} = 0$$

$$u_\mu R^{\mu\nu}{}_{,\nu} = \gamma\left(E_{,t} + F^i{}_{,i}\right) - \gamma v_i \left(\frac{1}{c^2} F^i{}_{,t} + R^{ij}{}_{,j}\right)$$

$$\left(e u^\mu\right)_{,\mu} + p u^\mu{}_{,\mu} + \gamma\left(E_{,t} + F^i{}_{,i}\right) - \gamma v_i \left(\frac{1}{c^2} F^i{}_{,t} + R^{ij}{}_{,j}\right) = 0$$

$$\rightarrow \frac{\partial}{\partial t}(\gamma e) + \nabla\cdot(\gamma e \mathbf{v}) + p\left\{\frac{\partial \gamma}{\partial t} + \nabla\cdot(\gamma \mathbf{v})\right\} = -\gamma\left(\frac{\partial E}{\partial t} + \nabla\cdot\mathbf{F}\right) + \gamma \mathbf{v}\cdot\left(\frac{1}{c^2}\frac{\partial \mathbf{F}}{\partial t} + \nabla\cdot\mathbf{P}\right)$$

輻射流体方程式(5)

ここまでをまとめると２つの項(0th & 1st moment eqs.)を追加するだけで流体力学から輻射流体力学へ発展させることができる！

$$
\begin{cases}
\gamma^2 \rho_m \dfrac{D\mathbf{v}}{Dt} = (HD-terms) + \dfrac{\mathbf{v}}{c^2}\left(\dfrac{\partial E}{\partial t} + \nabla\cdot\mathbf{F}\right) - \left(\dfrac{1}{c^2}\dfrac{\partial \mathbf{F}}{\partial t} + \nabla\cdot\mathbf{P}\right) \\[2ex]
\dfrac{\partial}{\partial t}(\gamma e) = (HD-terms) - \gamma\left(\dfrac{\partial E}{\partial t} + \nabla\cdot\mathbf{F}\right) + \gamma\mathbf{v}\cdot\left(\dfrac{1}{c^2}\dfrac{\partial \mathbf{F}}{\partial t} + \nabla\cdot\mathbf{P}\right)
\end{cases}
$$

２つの輻射項は輻射輸送方程式から計算する！

$$
\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \mathbf{n}\cdot\nabla I_\nu = (\kappa_\nu + \sigma_\nu)(S_\nu - I_\nu)
$$

$$
\begin{cases}
\dfrac{\partial E}{\partial t} + \nabla\cdot\mathbf{F} \leftarrow \displaystyle\int\left(\dfrac{1}{c}\dfrac{\partial I_\nu}{\partial t} + \mathbf{n}\cdot\nabla I \right. \\[2ex]
\dfrac{1}{c^2}\dfrac{\partial \mathbf{F}}{\partial t} + \nabla\cdot\mathbf{P} \leftarrow \displaystyle\int\dfrac{\mathbf{n}}{c}\left(\dfrac{1}{c}\dfrac{\partial I_\nu}{\partial t} + \right.
\end{cases}
$$

しかし、物質の性質(吸収・散乱係数)はcomoving-frameで定義されているのでこのままでは使えない！

輻射流体方程式(6)

<u>Lab-frame → Comoving-frame(右辺)</u>

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \mathbf{n}\cdot\nabla I_\nu = (\kappa_\nu + \sigma_\nu)(S_\nu - I_\nu)$$

$$\begin{cases} \dfrac{\partial E}{\partial t} + \nabla\cdot\mathbf{F} \leftarrow \displaystyle\int\left(\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \mathbf{n}\cdot\nabla I_\nu\right) d\nu d\Omega \\ \dfrac{1}{c^2}\dfrac{\partial \mathbf{F}}{\partial t} + \nabla\cdot\mathbf{P} \leftarrow \displaystyle\int\frac{\mathbf{n}}{c}\left(\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \mathbf{n}\cdot\nabla I_\nu\right) d\nu d\Omega \end{cases}$$

輻射輸送方程式の右辺を書き換えて積分

$$\nu\kappa_\nu = \nu_0\kappa_{0\nu},\ \nu\sigma_\nu = \nu_0\sigma_{0\nu}$$

$$\frac{I_\nu}{\nu^3} = \frac{I_{0\nu}}{\nu_0^3},\ \frac{S_\nu}{\nu^3} = \frac{S_{0\nu}}{\nu_0^3}$$

$$\frac{\nu}{\nu_0} = \frac{d\nu}{d\nu_0} = \gamma\left(1 + \frac{\mathbf{n}_0\cdot\mathbf{v}}{c}\right)$$

$$\nu d\nu d\Omega = \nu_0 d\nu_0 d\Omega_0$$

$$\mathbf{n} = \frac{\nu_0}{\nu}\left\{\mathbf{n}_0 + \gamma\frac{\mathbf{v}_0}{c} - \frac{\gamma - 1}{v^2}\mathbf{v}(\mathbf{n}_0\cdot\mathbf{v})\right\}$$

輻射流体方程式(7)

$\kappa_0 : isotropic$

$\sigma_0 : isotropic$

$S_0 = \kappa_0 B : isotropic$

Oth & 1st moment eqs (左辺:lab-frame/右辺:comoving frame)

$$
\begin{cases}
\dfrac{\partial E}{\partial t} + \nabla \cdot \mathbf{F} = \gamma\left(4\pi\kappa_0 B - c\kappa_0 E_0\right) - \gamma \dfrac{\kappa_0 + \sigma_0}{c} \mathbf{v} \cdot \mathbf{F}_0 \\
\dfrac{1}{c^2}\dfrac{\partial \mathbf{F}}{\partial t} + \nabla \cdot \mathbf{P} = \gamma \dfrac{\mathbf{v}}{c^2}\left(4\pi\kappa_0 B - c\kappa_0 E_0\right) - \dfrac{\kappa_0 + \sigma_0}{c}\mathbf{F}_0 - \dfrac{\gamma - 1}{v^2}\dfrac{\mathbf{v}}{c}\left(\kappa_0 + \sigma_0\right)\mathbf{v} \cdot \mathbf{F}_0
\end{cases}
$$

散乱・吸収係数をcomoving-frameで記述できたが、輻射の諸量がlab-frameとcomoving-frameで定義されることになってしまった。どちらかのframeに統一する必要あり！

輻射流体方程式(8)

Mixed-frame Moment Eqs.

ローレンツ変換の式を使って輻射諸量をlab-frameへ

$$R_0^{\mu\nu} = L^{\mu}_{\alpha} L^{\nu}_{\beta} R^{\alpha\beta}$$

$$E_0 = \gamma^2 \left(E - \frac{2}{c^2} v_i \cdot F^i + \frac{v_i v_j}{c^2} P^{ij} \right)$$

$$F_0^i = \gamma \left\{ F^i - \gamma E v^i - v_j P^{ij} + \frac{\gamma - 1}{\gamma} \frac{v^i}{v^2} \left[(2\gamma + 1) v_j \cdot F^j - \gamma v_j v_k P^{jk} \right] \right\}$$

$$P_0^{ij} = P^{ij} - \frac{\gamma}{c^2} \left(v^i F^j + v^j F^i \right) + \frac{\gamma^2}{c^2} v^i v^j E + \frac{\gamma - 1}{v^2} \left(v^i P^{jk} + v^j P^{ik} \right) v_k$$

$$- \frac{2\gamma}{c^2} \frac{\gamma - 1}{v^2} v^i v^j \left(v_k \cdot F^k \right) + \left(\frac{\gamma - 1}{v^2} \right)^2 v^i v^j \left(v_k v_l P^{kl} \right)$$

輻射流体方程式(9)

$$\begin{cases} \dfrac{\partial E}{\partial t} + \nabla\cdot\mathbf{F} = \gamma\left(4\pi\kappa_0 B - c\kappa_0 E_0\right) - \gamma\dfrac{\kappa_0+\sigma_0}{c}\mathbf{v}\cdot\mathbf{F}_0 \\ \dfrac{1}{c^2}\dfrac{\partial\mathbf{F}}{\partial t} + \nabla\cdot\mathbf{P} = \gamma\dfrac{\mathbf{v}}{c^2}\left(4\pi\kappa_0 B - c\kappa_0 E_0\right) - \dfrac{\kappa_0+\sigma_0}{c}\mathbf{F}_0 - \dfrac{\gamma-1}{v^2}\dfrac{\mathbf{v}}{c}\left(\kappa_0+\sigma_0\right)\mathbf{v}\cdot\mathbf{F}_0 \end{cases}$$

⬇

$$\begin{cases} \dfrac{\partial E}{\partial t} + \nabla\cdot\mathbf{F} = \gamma\left(4\pi\kappa_0 B - c\kappa_0 E + \dfrac{\kappa_0}{c}\mathbf{v}\cdot\mathbf{F}\right) + \gamma^3\sigma_0\left[\dfrac{v^2}{c}E + \dfrac{v_i v_j}{c}P^{ij} - \left(1+\dfrac{v^2}{c^2}\right)\dfrac{\mathbf{v}\cdot\mathbf{F}}{c}\right] \\ \dfrac{1}{c^2}\dfrac{\partial\mathbf{F}}{\partial t} + \nabla\cdot\mathbf{P} = \gamma\dfrac{v^i}{c^2}4\pi\kappa_0 B - \gamma\dfrac{\kappa_0+\sigma_0}{c}\left(F^i - v_j P^{ij}\right) \\ \qquad + \gamma^3\sigma_0\left[\dfrac{v^i}{c}E - 2\dfrac{v^i}{c^3}v_j F^j + \dfrac{v^i}{c^3}v_k v_l P^{kl}\right] \end{cases}$$

輻射流体方程式(10)

$$\begin{cases} \gamma^2 \rho_m \dfrac{D\mathbf{v}}{Dt} = (HD-terms) + \dfrac{\mathbf{v}}{c^2}\left(\dfrac{\partial E}{\partial t} + \nabla\cdot\mathbf{F}\right) - \left(\dfrac{1}{c^2}\dfrac{\partial \mathbf{F}}{\partial t} + \nabla\cdot\mathbf{P}\right) \\ \dfrac{\partial}{\partial t}(\gamma e) = (HD-terms) - \gamma\left(\dfrac{\partial E}{\partial t} + \nabla\cdot\mathbf{F}\right) + \gamma\mathbf{v}\cdot\left(\dfrac{1}{c^2}\dfrac{\partial \mathbf{F}}{\partial t} + \nabla\cdot\mathbf{P}\right) \end{cases}$$

⬇

$$\begin{cases} \gamma^2 \rho_m \dfrac{Dv^i}{Dt} = (HD-terms) + \gamma \dfrac{\kappa_0 + \sigma_0}{c}\left\{F^i - \left(v^i E + v_j P^{ij}\right) + \dfrac{v^i}{c^2} v_j F^j\right\} \\ \dfrac{\partial}{\partial t}(\gamma e) = (HD-terms) - 4\pi\kappa_0 B + \gamma^2 c\kappa_0 E - 2\gamma^2 \dfrac{\kappa_0}{c} v_j F^j + \gamma^2 \dfrac{\kappa_0}{c} v_k v_l P^{kl} \end{cases}$$

輻射流体方程式(11)

Mixed-frame Radiation-Hydrodynamic Eqs.

$$
\begin{cases}
\dfrac{\partial}{\partial t}(\gamma\rho) + \nabla\cdot(\gamma\rho\mathbf{v}) = 0 \\
\gamma^2 \rho_m \dfrac{Dv^i}{Dt} = (HD-terms) + \gamma \dfrac{\kappa_0+\sigma_0}{c}\left\{F^i - \left(v^i E + v_j P^{ij}\right) + \dfrac{v^i}{c^2} v_j F^j\right\} \\
\dfrac{\partial}{\partial t}(\gamma e) = (HD-terms) - 4\pi\kappa_0 B + \gamma^2 c\kappa_0 E - 2\gamma^2 \dfrac{\kappa_0}{c} v_j F^j + \gamma^2 \dfrac{\kappa_0}{c} v_k v_l P^{kl} \\
\dfrac{\partial E}{\partial t} + \nabla\cdot\mathbf{F} = \gamma\left(4\pi\kappa_0 B - c\kappa_0 E + \dfrac{\kappa_0}{c}\mathbf{v}\cdot\mathbf{F}\right) + \gamma^3\sigma_0\left[\dfrac{v^2}{c}E + \dfrac{v_i v_j}{c}P^{ij} - \left(1+\dfrac{v^2}{c^2}\right)\dfrac{\mathbf{v}\cdot\mathbf{F}}{c}\right] \\
\dfrac{1}{c^2}\dfrac{\partial\mathbf{F}}{\partial t} + \nabla\cdot\mathbf{P} = \gamma\dfrac{v^i}{c^2}4\pi\kappa_0 B - \gamma\dfrac{\kappa_0+\sigma_0}{c}\left(F^i - v_j P^{ij}\right) \\
\qquad + \gamma^3\sigma_0\left[\dfrac{v^i}{c}E - 2\dfrac{v^i}{c^3}v_j F^j + \dfrac{v^i}{c^3}v_k v_l P^{kl}\right]
\end{cases}
$$

輻射流体方程式(12)

Radiation Hydrodynamic Eqs : O(v/c)

前ページで(v/c)の２次以上のオーダーを落とす

$$
\begin{cases}
\dfrac{\partial}{\partial t}(\rho)+\nabla\cdot(\rho\mathbf{v})=0 \\
\rho\dfrac{Dv^i}{Dt}=\dfrac{\partial p}{\partial x_i}+\dfrac{\kappa_0+\sigma_0}{c}\left\{F^i-\left(v^iE+v_jP^{ij}\right)\right\} \\
\dfrac{\partial e}{\partial t}+\dfrac{\partial}{\partial x_i}\left(ev^i\right)=-p\dfrac{\partial v^i}{\partial x_i}-4\pi\kappa_0B+c\kappa_0E-2\dfrac{\kappa_0}{c}v_jF^j \\
\dfrac{\partial E}{\partial t}+\nabla\cdot\mathbf{F}=4\pi\kappa_0B-c\kappa_0E+\dfrac{\kappa_0-\sigma_0}{c}\mathbf{v}\cdot\mathbf{F} \\
\dfrac{1}{c^2}\dfrac{\partial\mathbf{F}}{\partial t}+\nabla\cdot\mathbf{P}=\dfrac{v^i}{c^2}4\pi\kappa_0B+\sigma_0\dfrac{v^i}{c}E-\dfrac{\kappa_0+\sigma_0}{c}\left(F^i-v_jP^{ij}\right)
\end{cases}
$$

輻射流体方程式(13)

Comoving-frame Radiation Hydrodynamic Eqs : O(v/c)

輻射所領をcomoving-frameに変換

$$
\begin{cases}
\rho \dfrac{Dv^i}{Dt} = \dfrac{\partial p}{\partial x_i} + \dfrac{\kappa_0 + \sigma_0}{c} F_0^i \\
\dfrac{\partial e}{\partial t} + \dfrac{\partial}{\partial x_i}\left(ev^i\right) = -p\dfrac{\partial v^i}{\partial x_i} - 4\pi\kappa_0 B + c\kappa_0 E_0 \\
\dfrac{\partial E_0}{\partial t} + \dfrac{\partial}{\partial x_i}\left(E_0 v^i\right) = -\dfrac{\partial F_0^i}{\partial x_i} + 4\pi\kappa_0 B - c\kappa_0 E_0 - \dfrac{\kappa_0 + \sigma_0}{c} v_i F_0^i - \dfrac{\partial v_j}{\partial x_i} P_0^{ij} - v_j \dfrac{\partial P_0^{ij}}{\partial x_i} \\
\dfrac{1}{c^2}\dfrac{\partial F_0^i}{\partial t} + \dfrac{\partial P_0^{ij}}{\partial x_j} = -\dfrac{\kappa_0 + \sigma_0}{c} F_0^i + 4\pi\kappa_0 B \dfrac{v^i}{c^2} - \dfrac{v^i}{c}\kappa_0 E_0 - \dfrac{1}{c^2}\dfrac{\partial}{\partial x_j}\left(v^i F_0^j + v^j F_0^i\right)
\end{cases}
$$

↓

$$
\frac{\partial E_0}{\partial t} + \frac{\partial}{\partial x_i}\left(E_0 v^i\right) = -\frac{\partial F_0^i}{\partial x_i} + 4\pi\kappa_0 B - c\kappa_0 E_0 - \frac{\partial v_j}{\partial x_i} P_0^{ij}
$$

CLOSURE RELATION(1)

Radiation-Hydrodynamic Eqs. (再掲)

$$\frac{\partial}{\partial t}(\gamma\rho)+\nabla\cdot(\gamma\rho\mathbf{v})=0$$

$$\gamma^2\rho_m\frac{Dv^i}{Dt}=(HD-terms)+\gamma\frac{\kappa_0+\sigma_0}{c}\left\{F^i-\left(v^iE+v_jP^{ij}\right)+\frac{v^i}{c^2}v_jF^j\right\}$$

$$\frac{\partial}{\partial t}(\gamma e)=(HD-terms)-4\pi\kappa_0B+\gamma^2c\kappa_0E-2\gamma^2\frac{\kappa_0}{c}v_jF^j+\gamma^2\frac{\kappa_0}{c}v_l v \cdots$$

$$\frac{\partial E}{\partial t}+\nabla\cdot\mathbf{F}=\gamma\left(4\pi\kappa_0B-c\kappa_0E+\frac{\kappa_0}{c}\mathbf{v}\cdot\mathbf{F}\right)+\gamma^3\sigma_0\left[\frac{v^2}{c}E+\frac{v_iv_j}{\cdots}\cdots\frac{\cdots}{c}\right]$$

$$\frac{1}{c^2}\frac{\partial\mathbf{F}}{\partial t}+\nabla\cdot\mathbf{P}=\gamma\frac{v^i}{c^2}4\pi\kappa_0B-\gamma\frac{\kappa_0+\sigma_0}{c}\left(F^i\cdots\right.$$

$$+\cdots\left[\frac{\cdots}{c}E-2\frac{v^i}{c^3}v_jF^j+\frac{v^i}{c^3}v_kv_lP^{kl}\right]$$

EとFの式しかないのにE, F, Pが変数(式が閉じていない)

CLOSURE RELATION(2)

P_0をE_0の関数として与えることで方程式が閉じさせる

Eddington近似

$$P_0^{ij}(i=j)=\frac{1}{3}E_0$$

Optically thickの極限で正しい

M1-closure (González et al. 07)

$$P_0^{ij}(i=j)=\begin{cases}\frac{1}{3}E_0 & optically-thick\\ E_0(Flux-direction) & optically-thin\end{cases}$$

Optically thickとthinの極限で正しい

CLOSURE RELATION(3)

P_0だけでなくF_0をE_0の関数として与える

<u>FLD近似</u> (Levermore, Pomraning 81)

$$\mathbf{F}_0 = \begin{cases} -\dfrac{c\nabla E_0}{3(\kappa_0+\sigma_0)} & optically-thick \\ -cE_0\dfrac{\nabla E_0}{|\nabla E_0|} & optically-thin \end{cases}$$

Optically thickとthinの極限で正しい

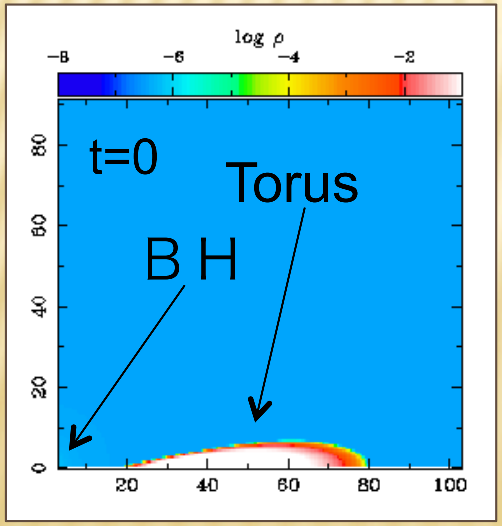
$$P_0^{ij}(i=j) \sim M1-closure\ Method$$

1st moment eq.は解く必要なし
0th moment eq.だけを解けばよい

$$\frac{\partial E_0}{\partial t} + \frac{\partial}{\partial x_i}\left(E_0 v^i\right) = -\frac{\partial F_0^i}{\partial x_i} + 4\pi\kappa_0 B - c\kappa_0 E_0 - \frac{\partial v_j}{\partial x_i} P_0^{ij}$$

FLD輻射(磁気)流体計算の例

- Cylindrical coordinate (r, φ, z)
- BOX SIZE; r=2R_s-105R_s : z=0-103R_s
- Mesh; 512x512, dx=dz=0.2Rs, [1024x512, dx=dz=0.1Rs]
- Axisymmetry & Reflection symmetry (mid-plane)
- Initial condition
  - ✓ Rotating torus with poloidal magnetic fields (β=100)
  - ✓ Non-rotating isothermal corona (No magnetic fileds)



- Closure relation; FLD

基礎方程式

Continuity Equation・・・・・・・ $$\frac{D\rho}{Dt} + \rho\nabla\cdot\mathbf{v} = 0$$

Equation of Motion・・・・・・・・ $$\rho\frac{D\mathbf{v}}{Dt} = -\nabla\left(p + \frac{B^2}{8\pi}\right) + \left(\frac{\mathbf{B}}{4\pi}\cdot\nabla\right)\mathbf{B} - \rho\frac{GM}{(r - r_S)^2} + \frac{\kappa + \sigma}{c}\mathbf{F}_0$$

Gas Energy Equation・・・・・・・ $$\frac{\partial e}{\partial t} + \nabla\cdot(e\mathbf{v}) = -p\nabla\cdot\mathbf{v} - 4\pi\kappa B + c\kappa E_0 + \frac{4\pi}{c^2}\eta J^2$$

Radiation Energy Equation・・ $$\frac{\partial E_0}{\partial t} + \nabla\cdot(E_0\mathbf{v}) = -\nabla\cdot\mathbf{F}_0 + 4\pi\kappa B - c\kappa E_0 - \nabla\mathbf{v}:\mathbf{P}_0$$

Maxwell's Equations・・・・・ $$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times\left(\mathbf{v}\times\mathbf{B} - \frac{4\pi\eta}{c}\mathbf{J}\right) \qquad \mathbf{J} = \frac{c}{4\pi}\nabla\times\mathbf{B}$$

（赤枠）輻射に関する項　（青枠）磁場に関する項

数値計算手順

磁気流体計算(陽解法)

$$\frac{D\rho}{Dt} + \rho\nabla\cdot\mathbf{v} = 0$$

$$\rho\frac{D\mathbf{v}}{Dt} = -\nabla\left(p + \frac{B^2}{8\pi}\right) + \left(\frac{\mathbf{B}}{4\pi}\cdot\nabla\right)\mathbf{B} - \rho\frac{GM}{(r - r_S)^2}$$

$$\frac{\partial e}{\partial t} + \nabla\cdot(e\mathbf{v}) = -p\nabla\cdot\mathbf{v} + \frac{4\pi}{c^2}\eta J^2$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times\left(\mathbf{v}\times\mathbf{B} - \frac{4\pi\eta}{c}\mathbf{J}\right) \qquad \mathbf{J} = \frac{c}{4\pi}\nabla\times\mathbf{B}$$

輻射流束(陰解法)

$$\frac{\partial E_0}{\partial t} = -\nabla\cdot\mathbf{F}_0$$

輻射-ガス相互作用(陰解法)

$$\frac{\partial e}{\partial t} = -4\pi\kappa B + c\kappa E_0$$

$$\frac{\partial E_0}{\partial t} = 4\pi\kappa B - c\kappa E_0 - \nabla\mathbf{v}:\mathbf{P}_0$$

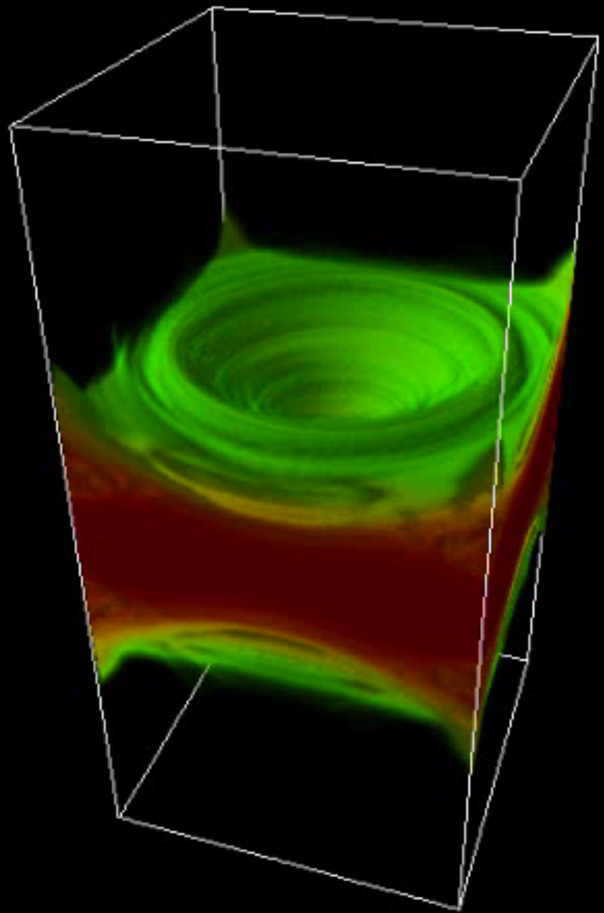
その他の輻射(陽解法)

$$\rho\frac{D\mathbf{v}}{Dt} = \frac{\kappa+\sigma}{c}\mathbf{F}_0$$

$$\frac{\partial E_0}{\partial t} + \nabla\cdot(E_0\mathbf{v}) = 0$$

Ohsuga et al. 2009

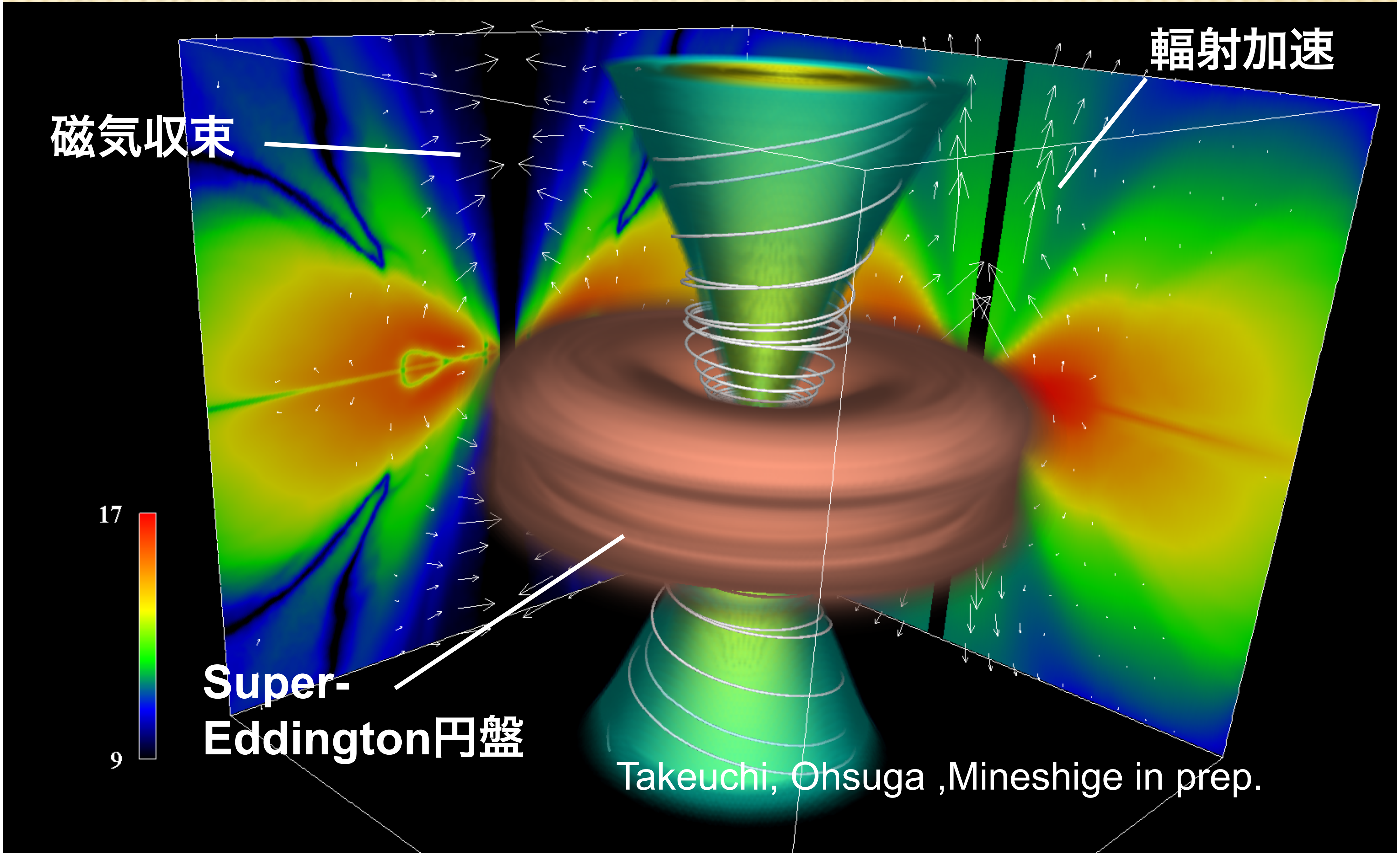
Super-Eddington Accretion+Radiative Jet



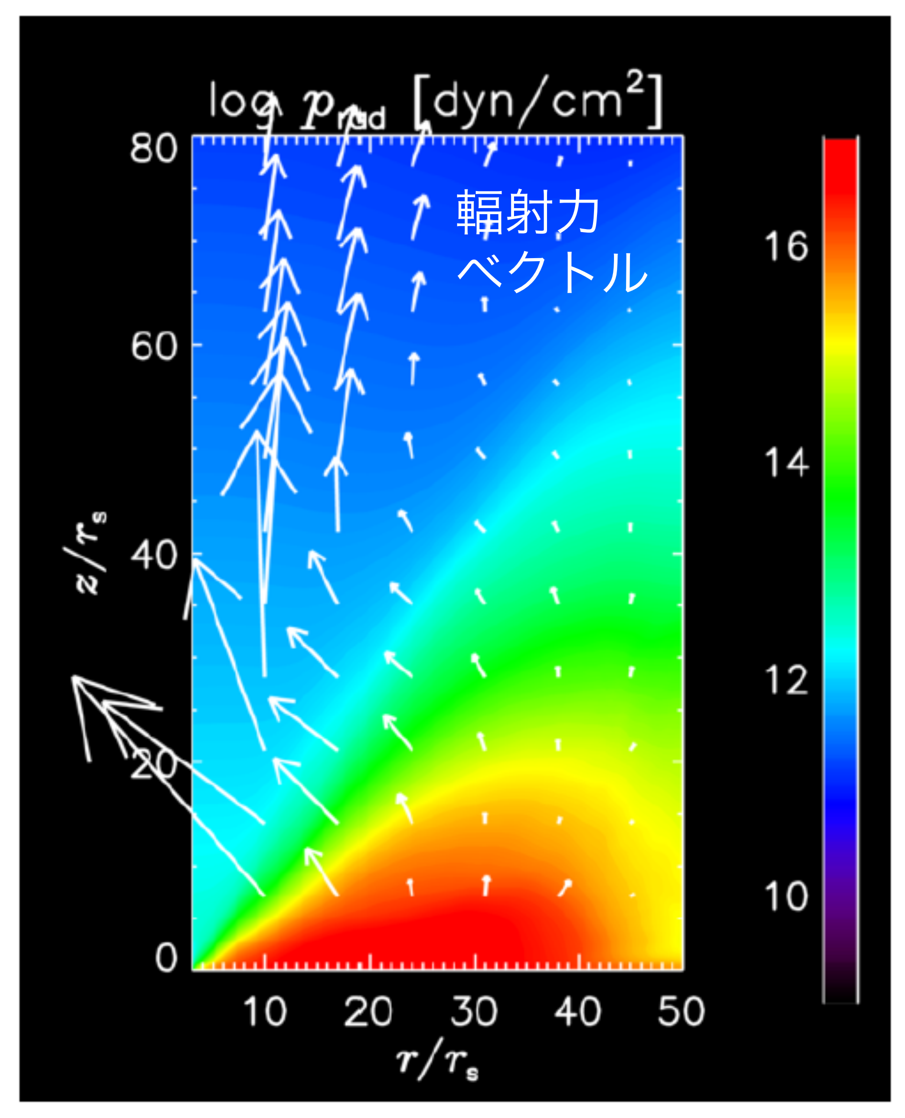
- 幾何学的・光学的に厚い輻射圧優勢円盤が形成
- 輻射圧加速ジェットが噴出

http://th.nao.ac.jp/~ohsugaでダウンロードできます

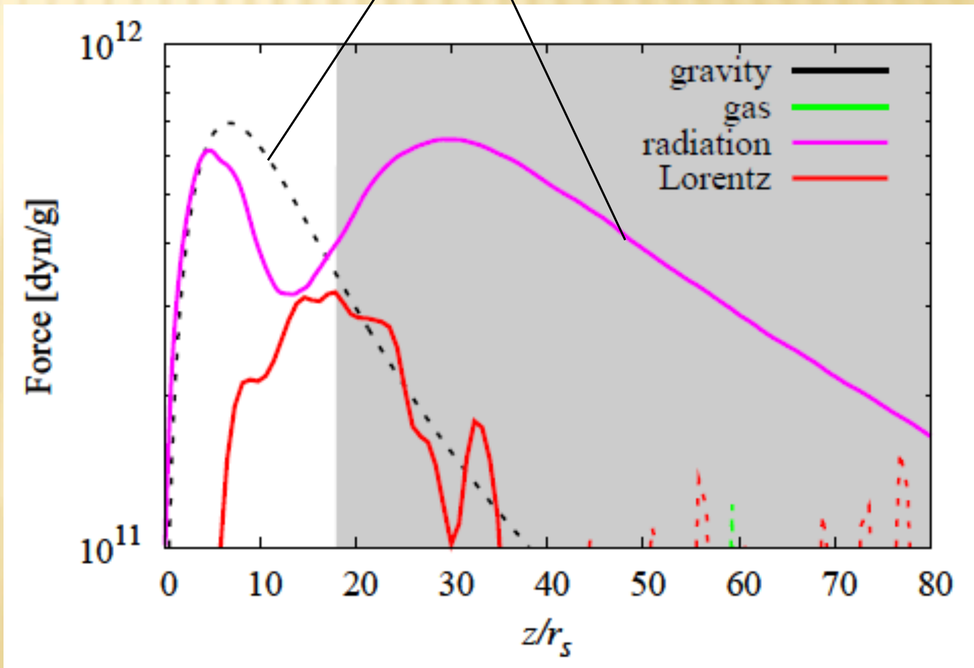
輻射加速＋磁気収束ジェット



加速メカニズム

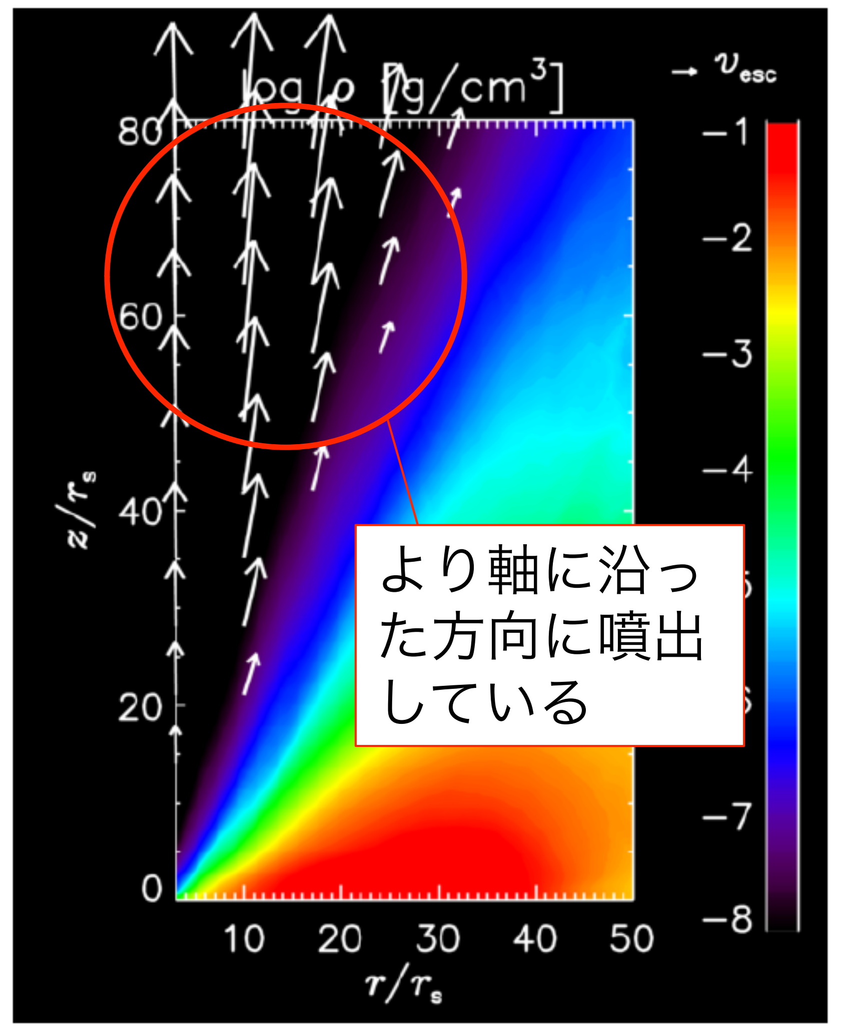


輻射流体計算と同様に、輻射力(>>重力)で加速している！

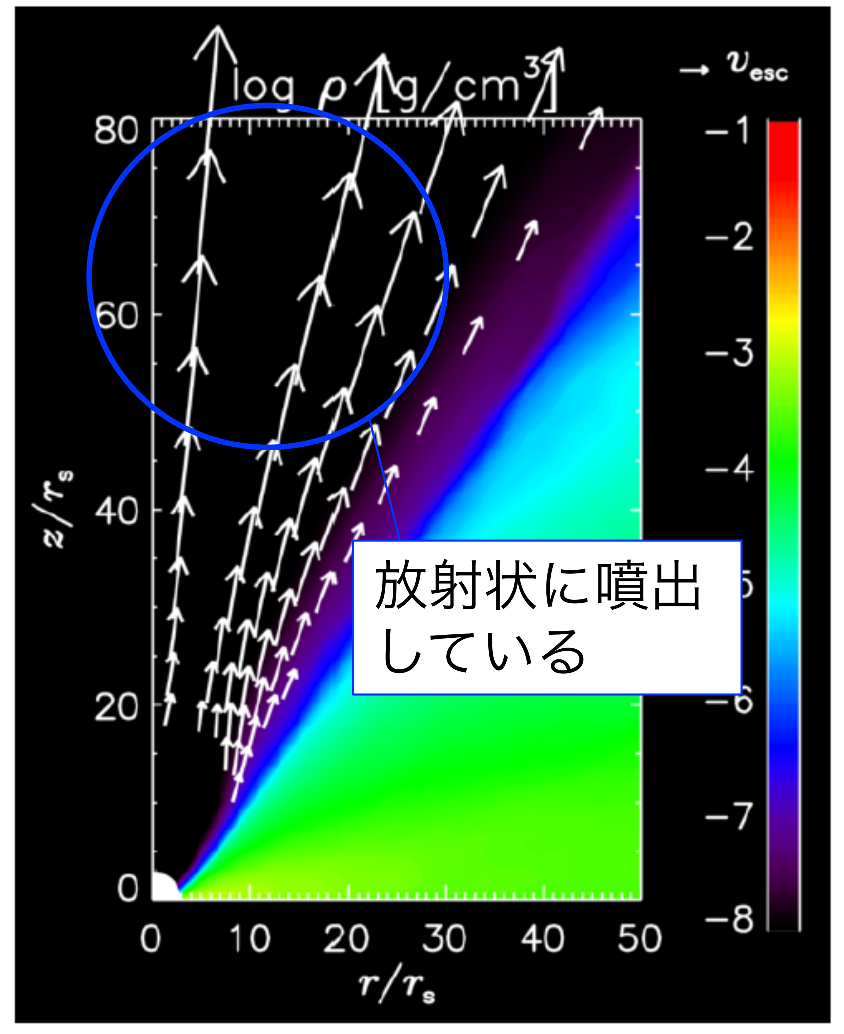


収束性の向上

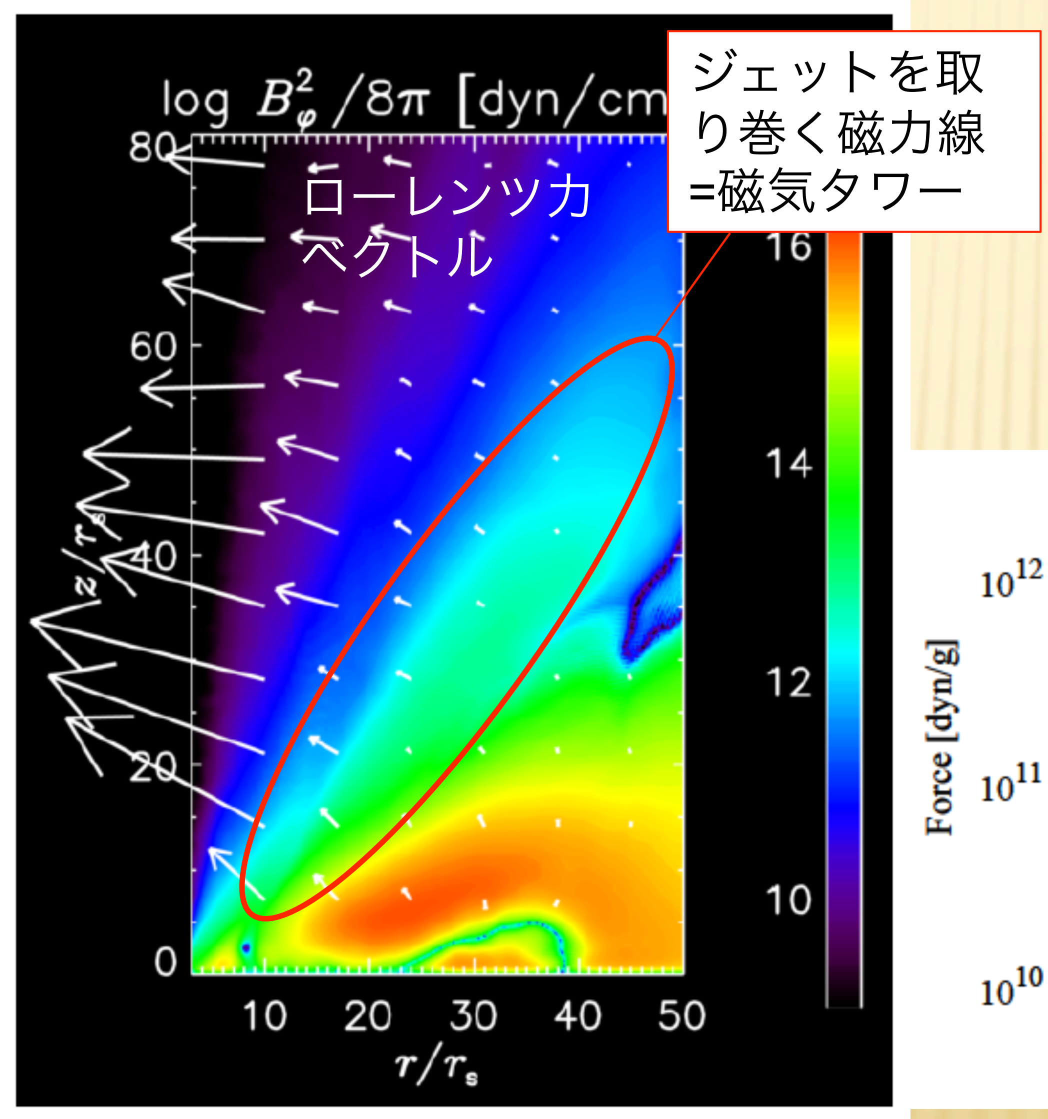
輻射磁気流体計算



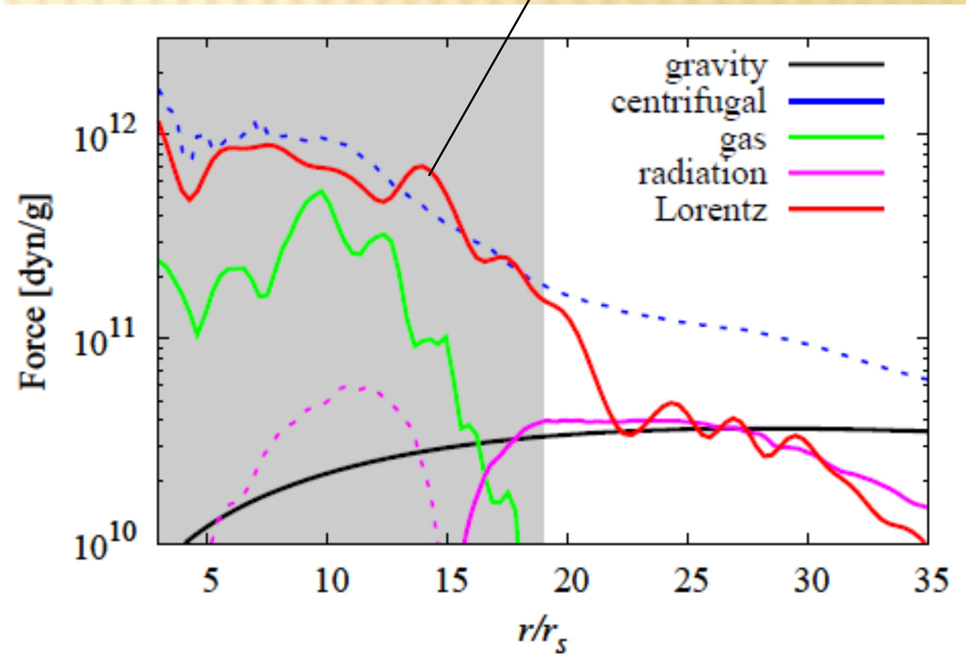
輻射流体計算



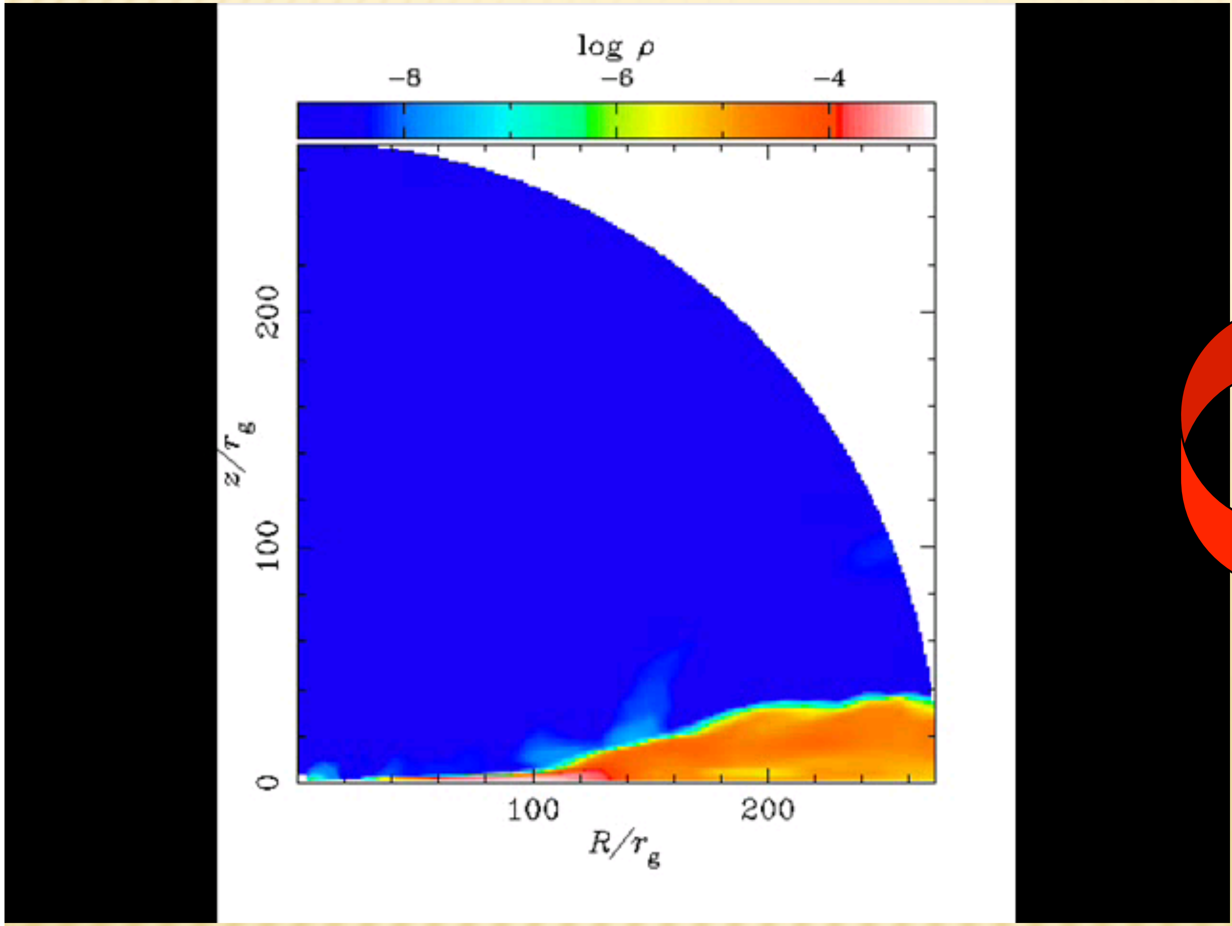
収束メカニズム

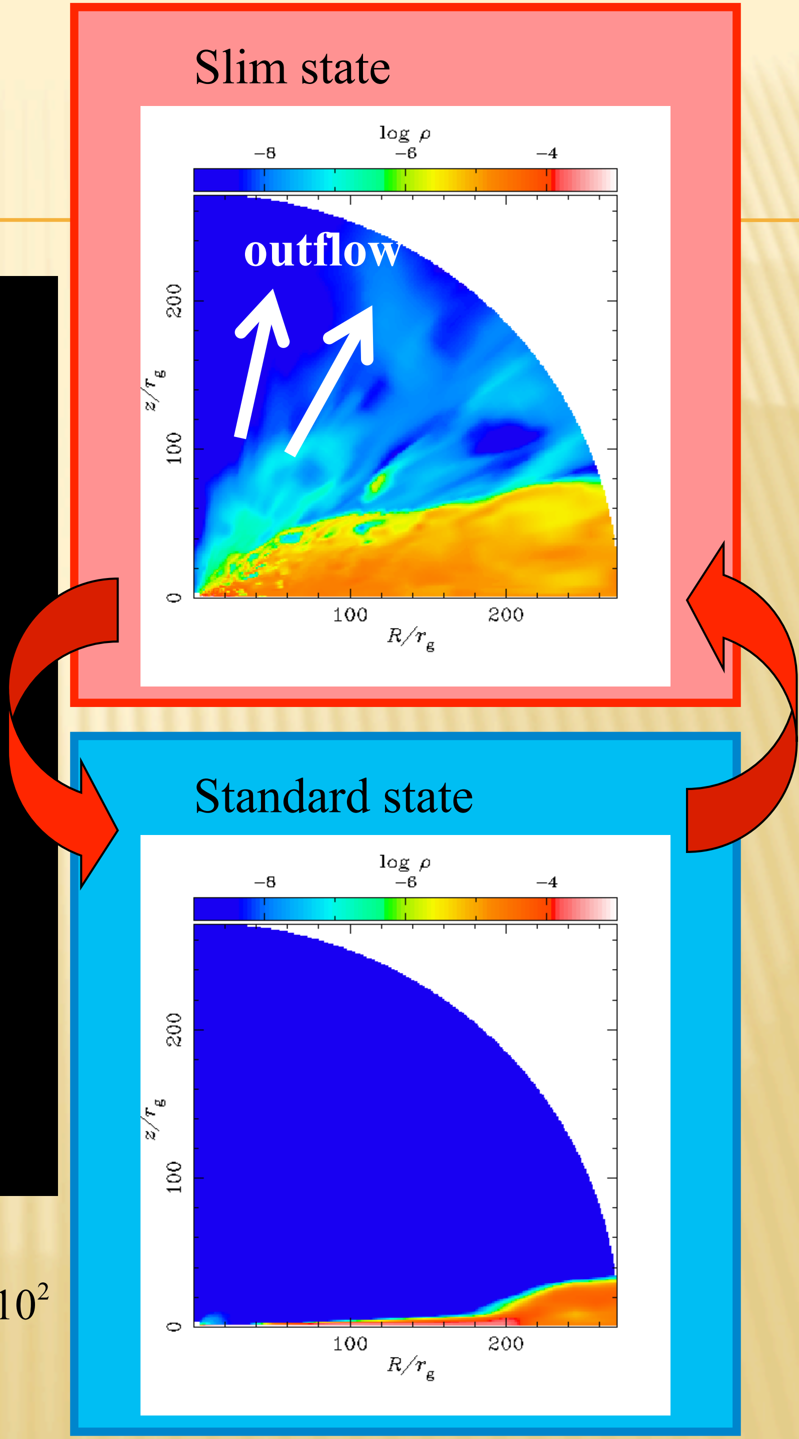


ローレンツ力で収束している
(磁気圧と磁気テンションは同程度)



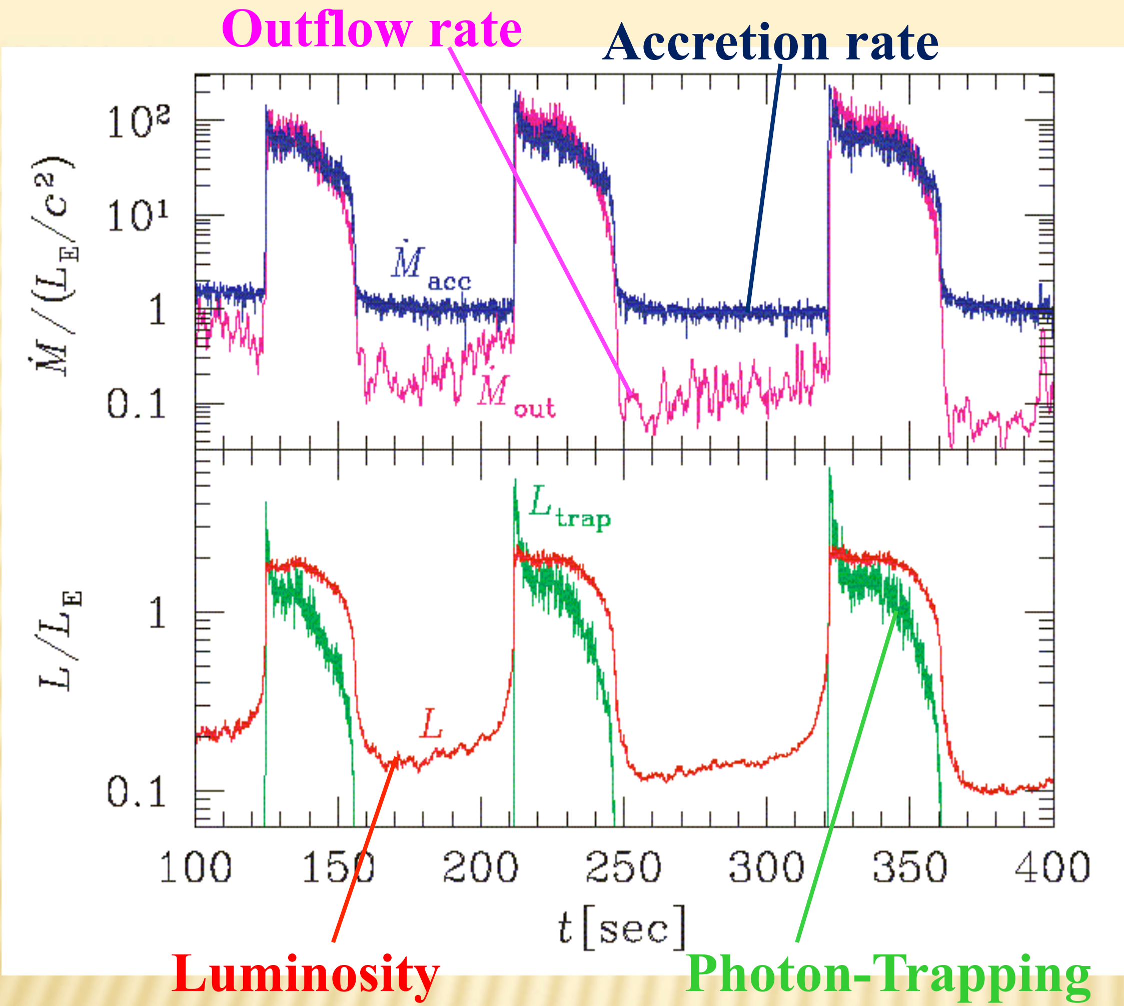
計算例その2:円盤不安定



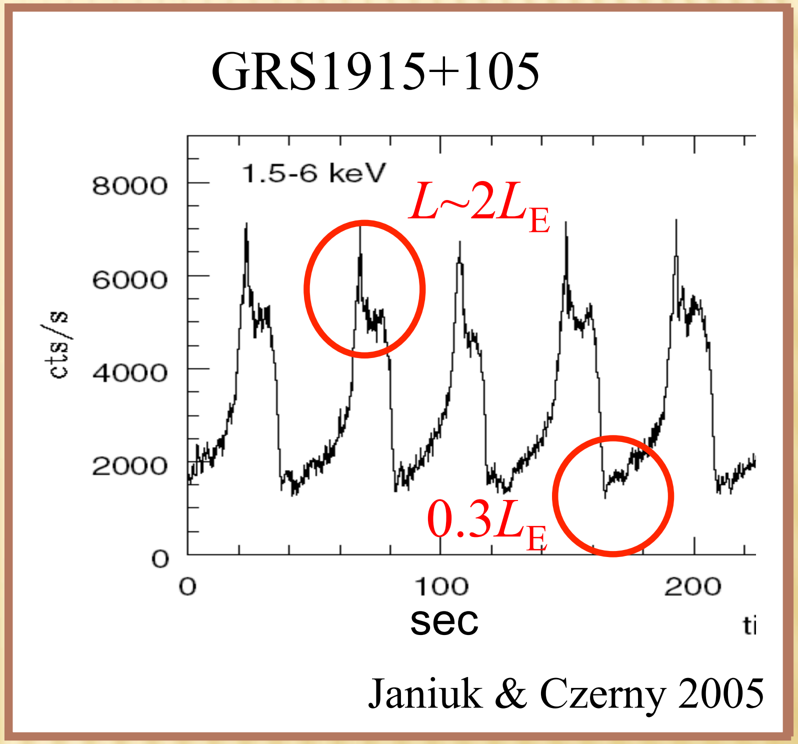


Black hole mass: $M_{BH} = 10 M_{\odot}$

Input mass accretion rate: $\dot{M}_{input} / (L_E / c^2) = 10^2$



Ohsuga 06, ApJ, 640, 923



α粘性入り２次元輻射流体シミュレーションで, 観測を見事に再現した

1. Slim disk状態(~$2L_E$)と標準円盤状態(~$0.3L_E$)を遷移
2. 周期は30-50秒
3. 間欠的にジェットが発生

輻射輸送計算法の改善(M1-CLOSURE)

M1-closure (0次も1次も解く)

0次モーメント $\frac{\partial E_0}{\partial t} + \nabla\cdot(E_0\mathbf{v}) = -\nabla\cdot\mathbf{F}_0 + 4\pi\kappa B - c\kappa E_0 - \nabla\mathbf{v}:\mathbf{P}_0$

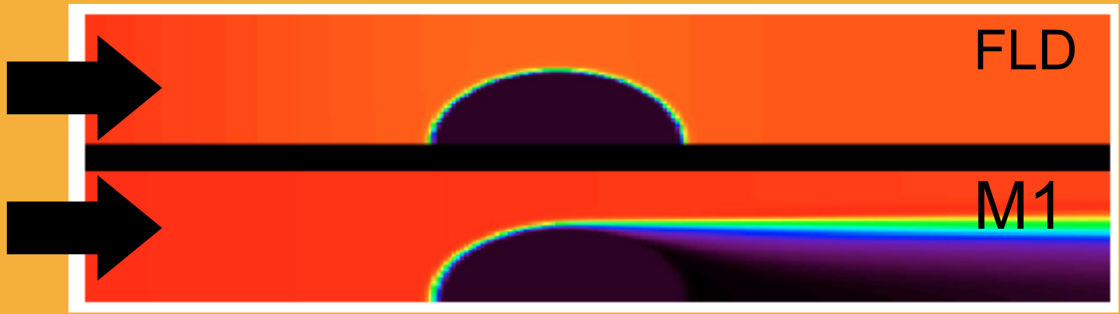
1次モーメント $\frac{\partial \mathbf{F}_0}{\partial t} + \nabla\cdot[\mathbf{F}_0\mathbf{v}] = -c^2\nabla\cdot\mathbf{P}_0 - (\mathbf{F}_0\cdot\nabla)\mathbf{v} - c\chi\mathbf{F}_0$

FL

González et al. 07

照射

照射

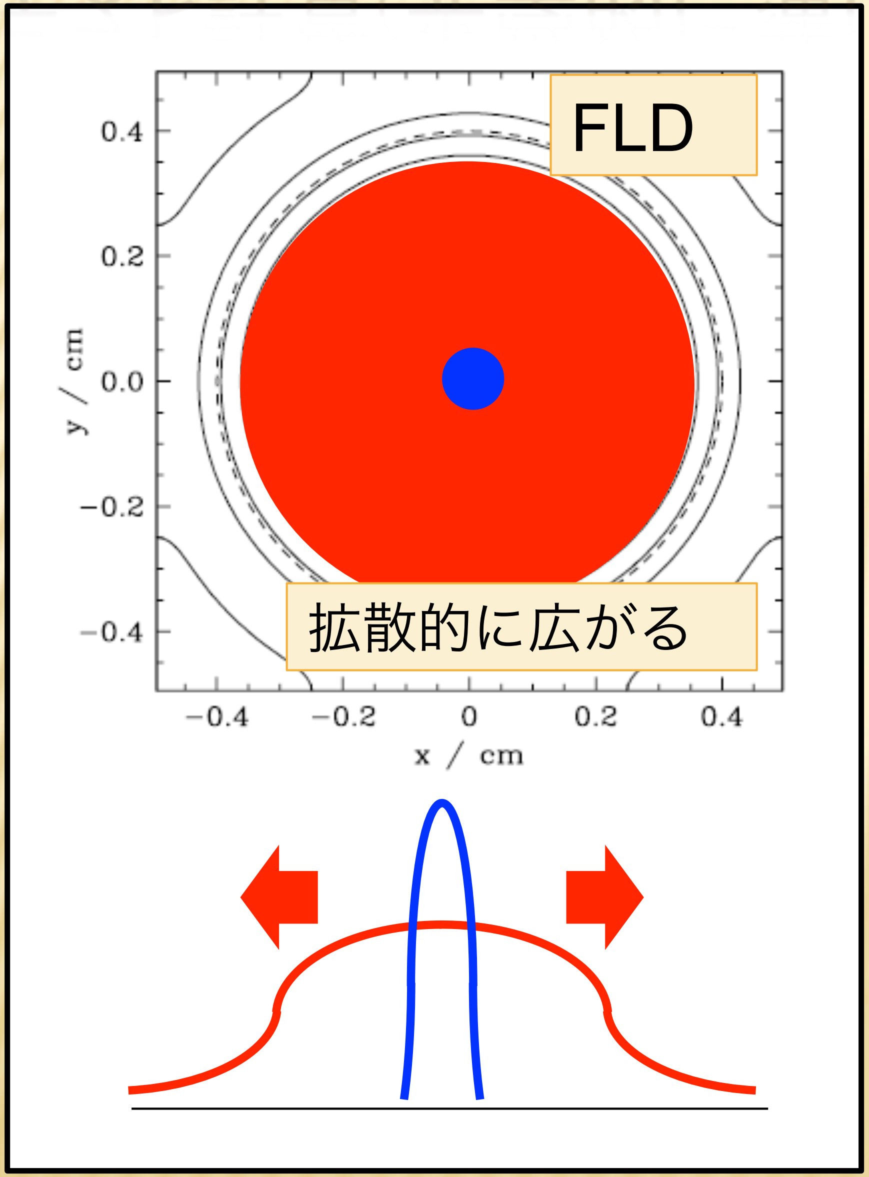


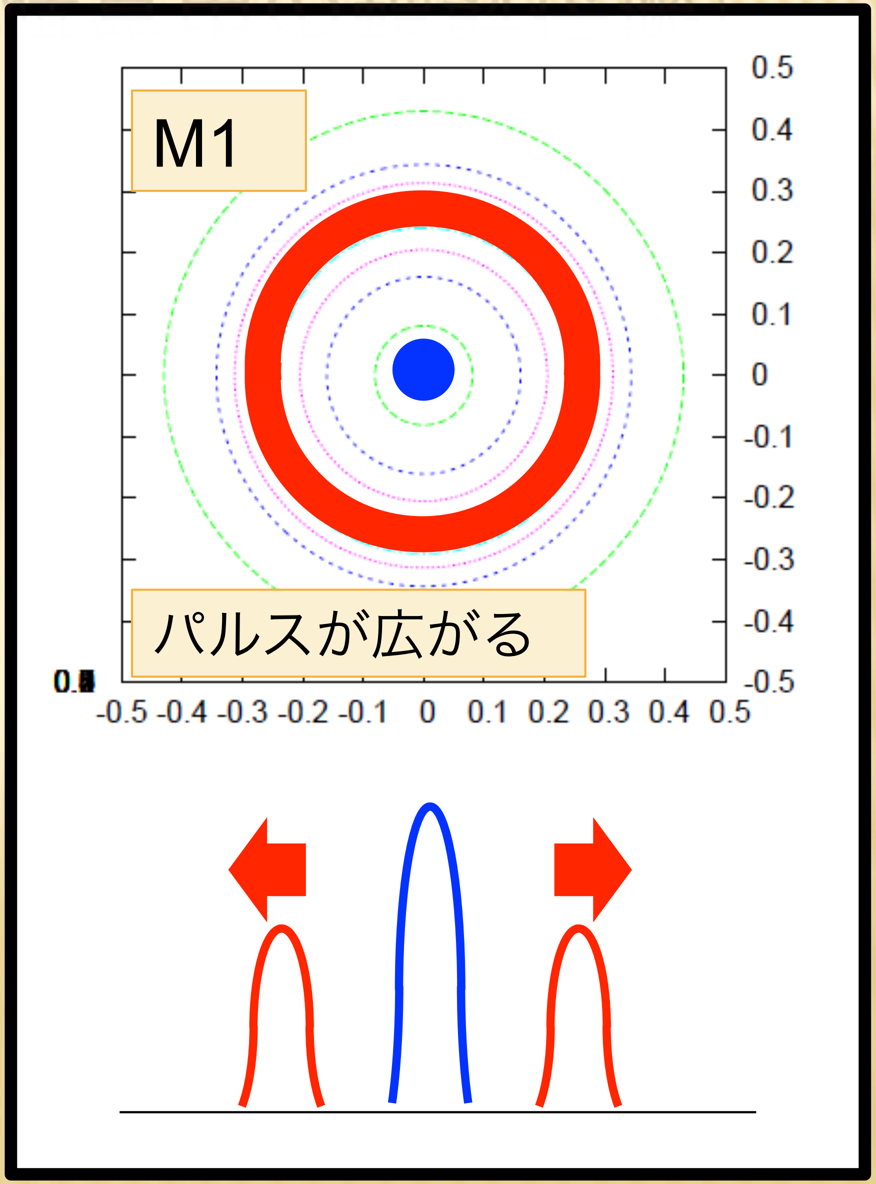
M1は影がちゃんと解ける！

E_0だけが変数

$\mathbf{F}_0$と$\mathbf{P}_0$はE_0の関数

テスト計算(光学的に薄い媒質中の輻射伝搬)



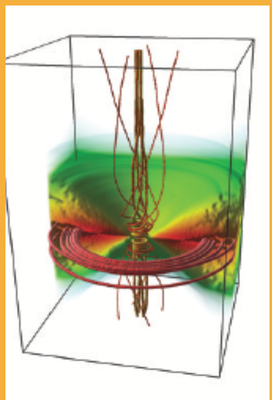


輻射輸送計算法の改善(ART法)

Authentic Radiative Transfer

FLDとARTの違い：輻射Flux

ARTの方が軸に沿っている
→ジェットがよりコリメートされる！



Ohsuga et al. (09)の輻射磁気流体計算のデータを使用

秋月, 梅村, 大須賀 in preparation