ブラックホール磁気圏での磁気リコネクションの数値計算

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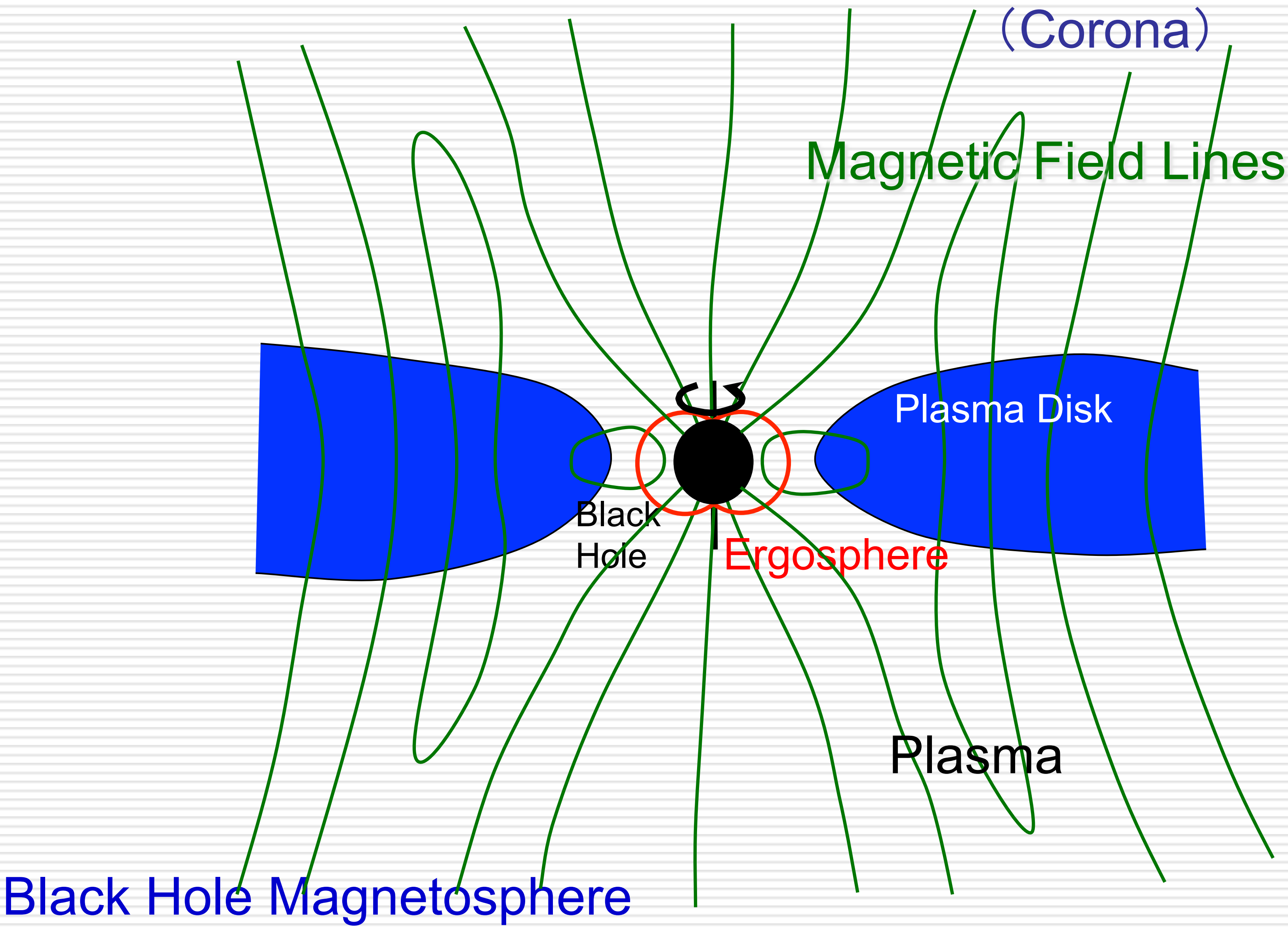
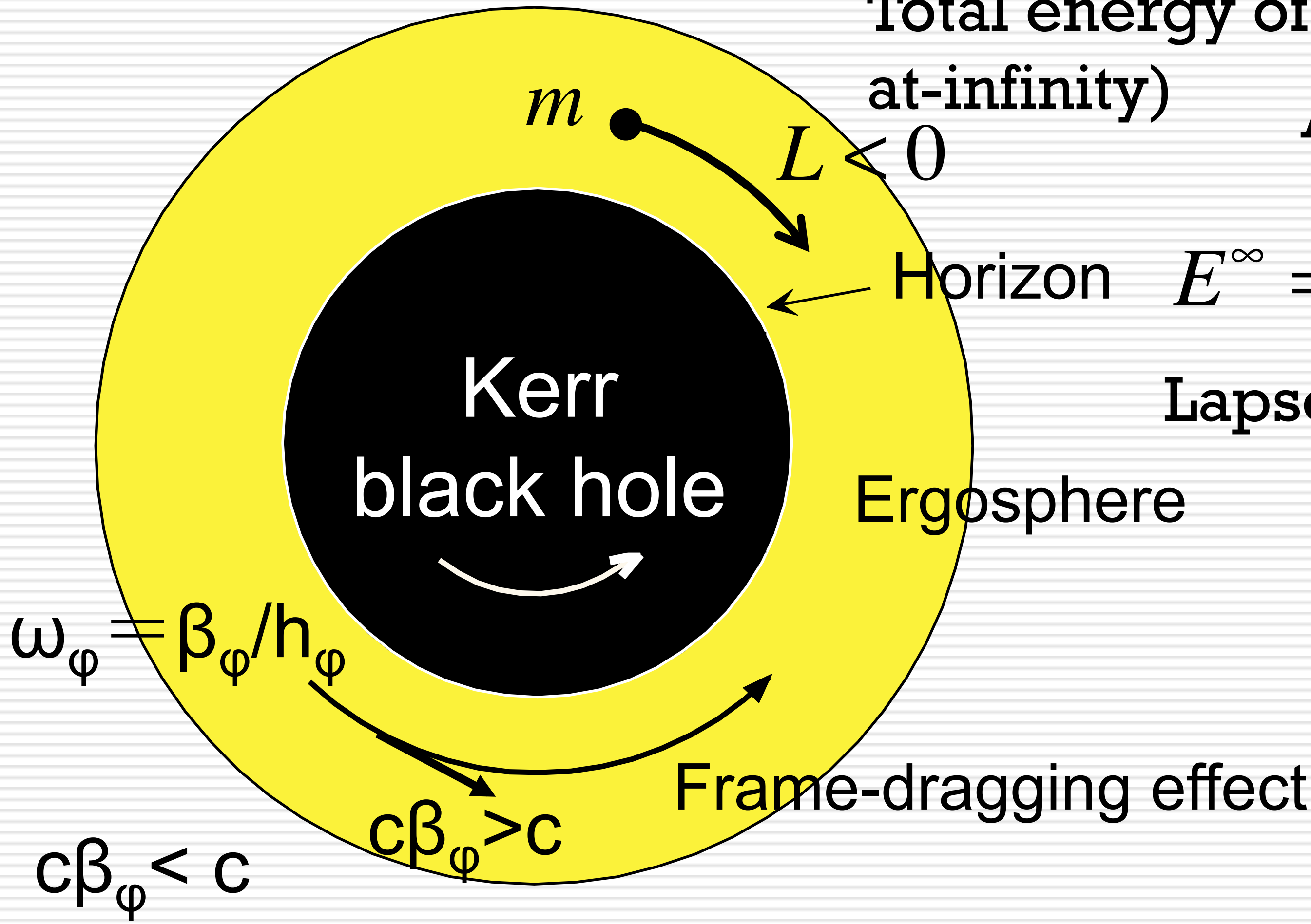
Outline

- Motivation and basis: Magnetic reconnection around astrophysical black holes
- Standard equations of resistive GRMHD
- Test calculations of resistive GRMHD
- A simulation of magnetic reconnection with initially uniform magnetic field around rotating black hole
- Summary and Future plans

Black Hole Magnetosphere



Frame–dragging effect and ergosphere



Total energy of particle m (energy-at-infinity)

Angular momentum

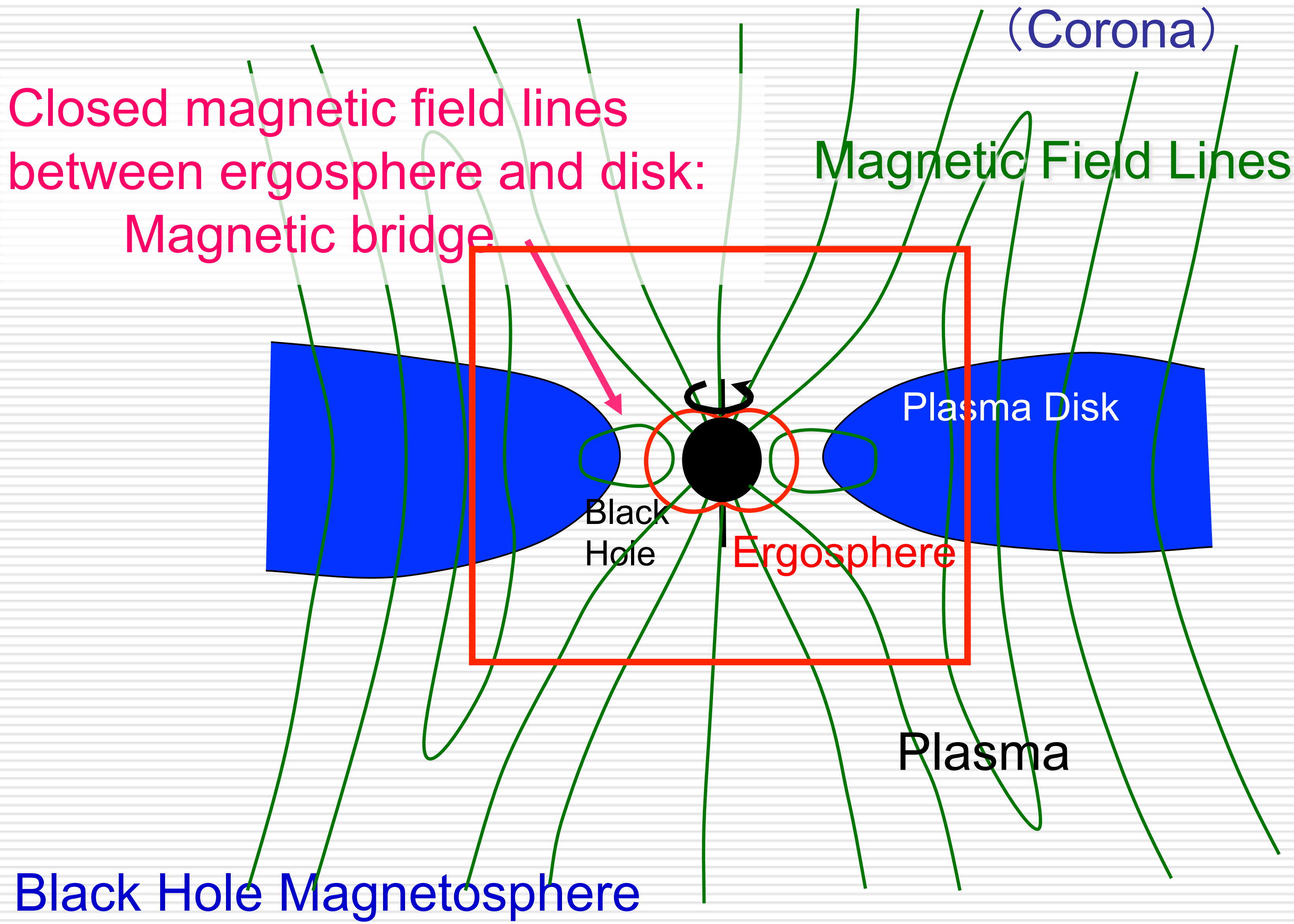
$$E^{\infty} = \alpha m c^2 \gamma + \underline{\omega_{\phi} L}$$

Lapse function

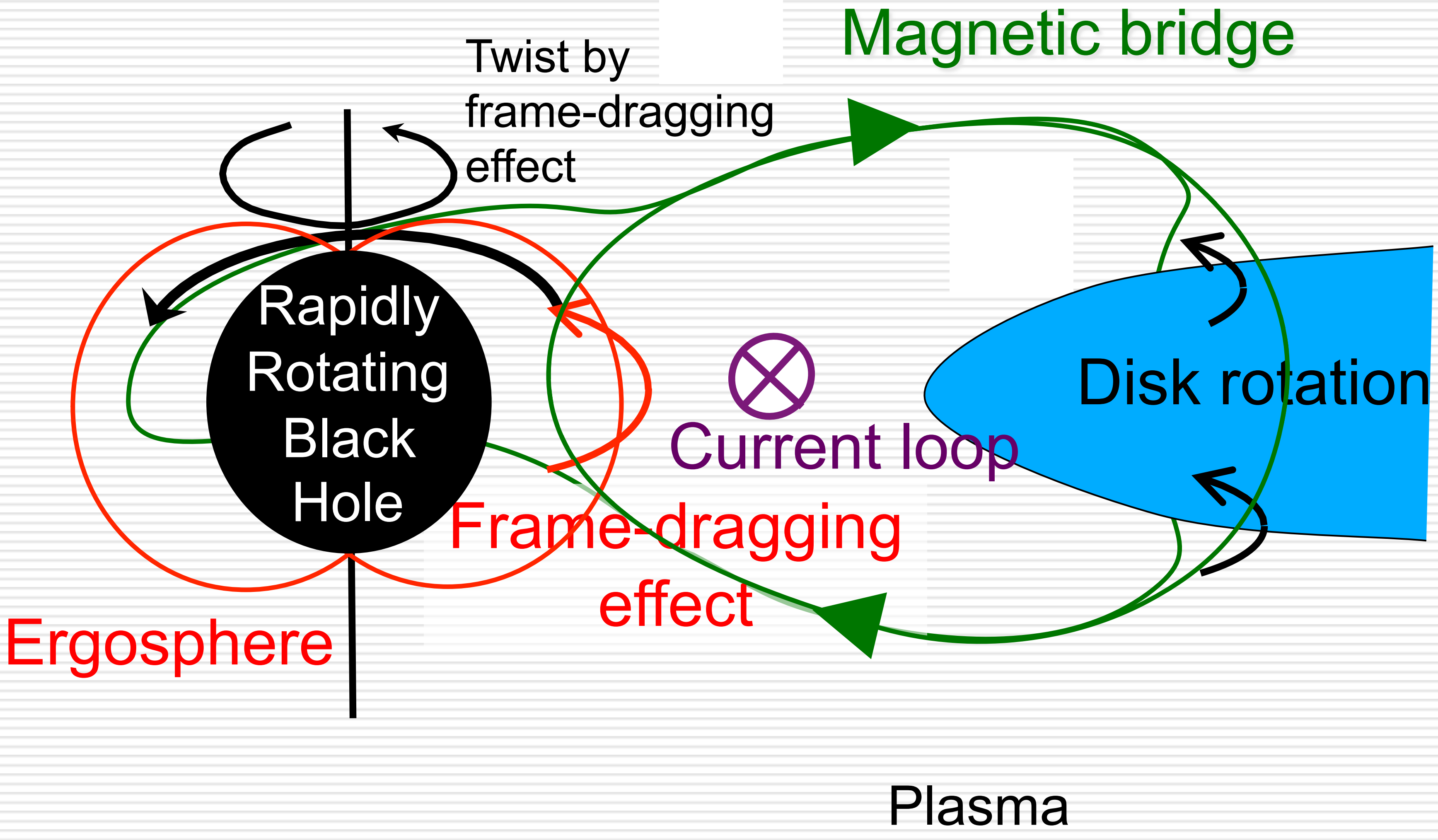
"Frame-dragging potential" (Extremely deep potential without limitation)

In the ergosphere, any matter, energy, and information can't move or propagate in the opposite direction of the black hole rotation.
Plasma in the ergosphere behaves like very heavy plasma disk!

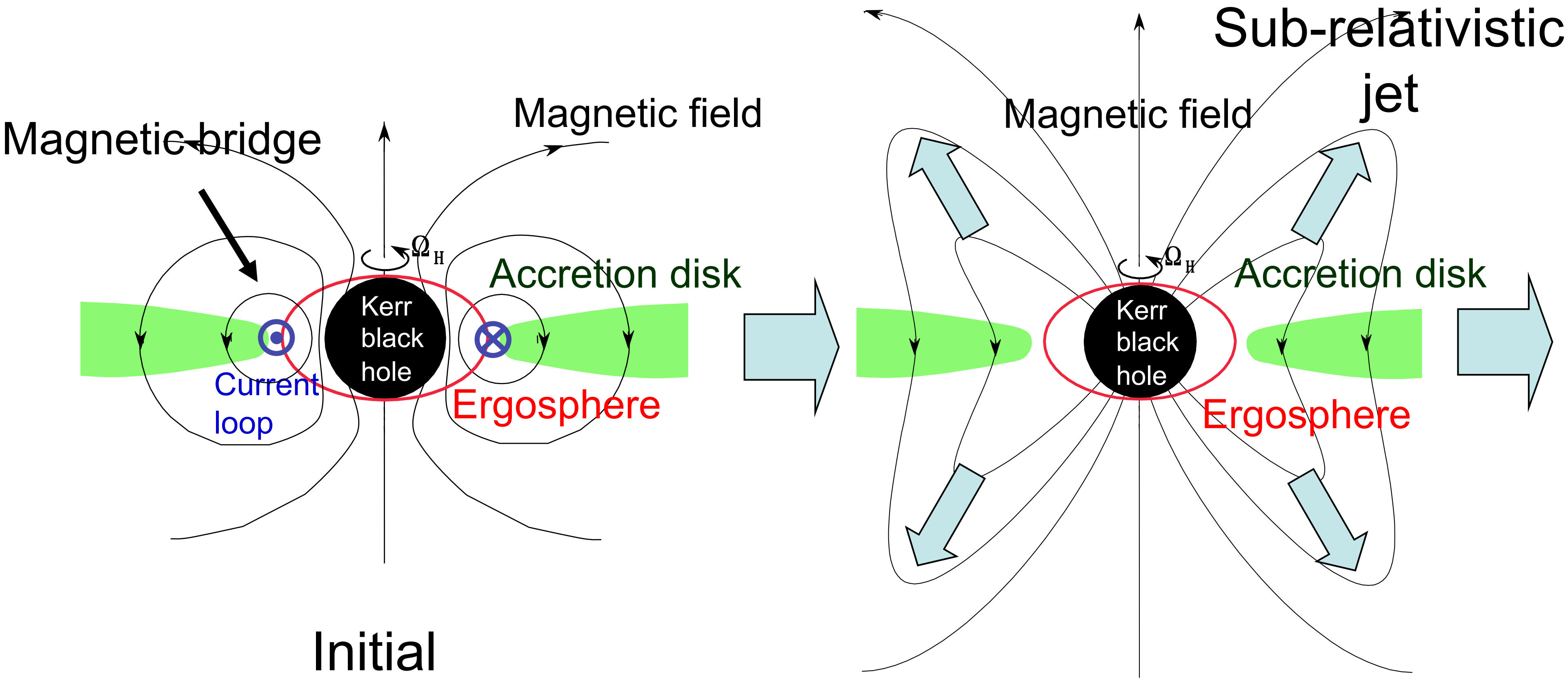
Black Hole Magnetosphere



Twist of magnetic bridge by ergosphere

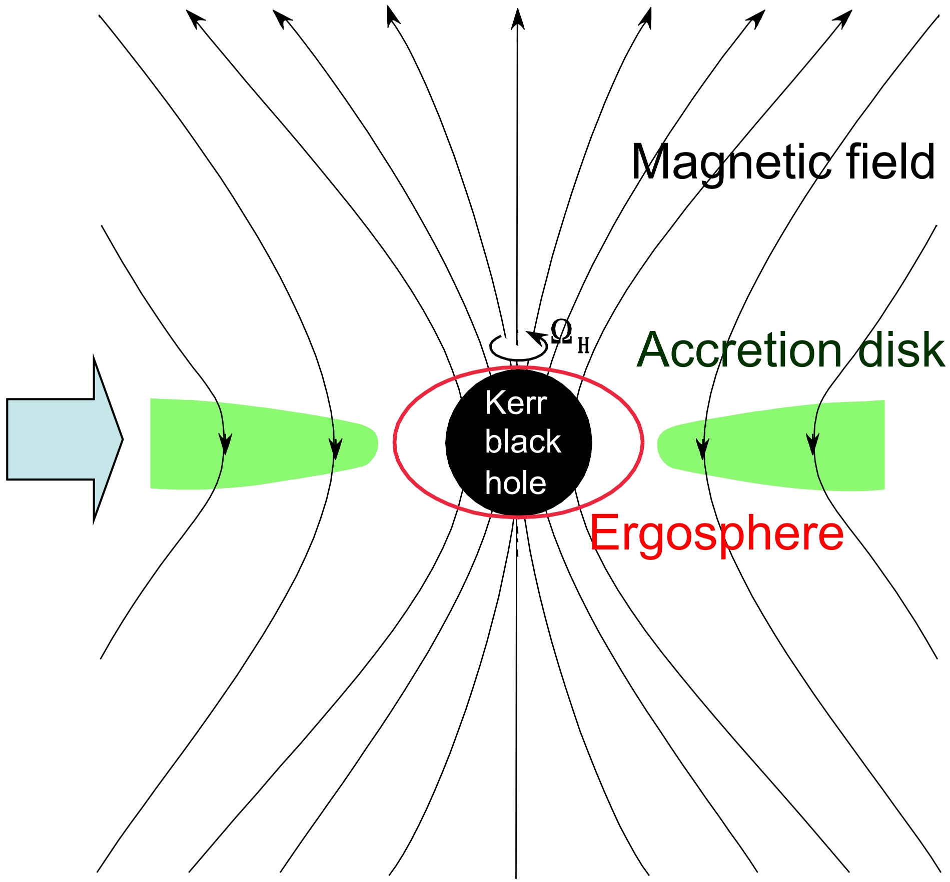


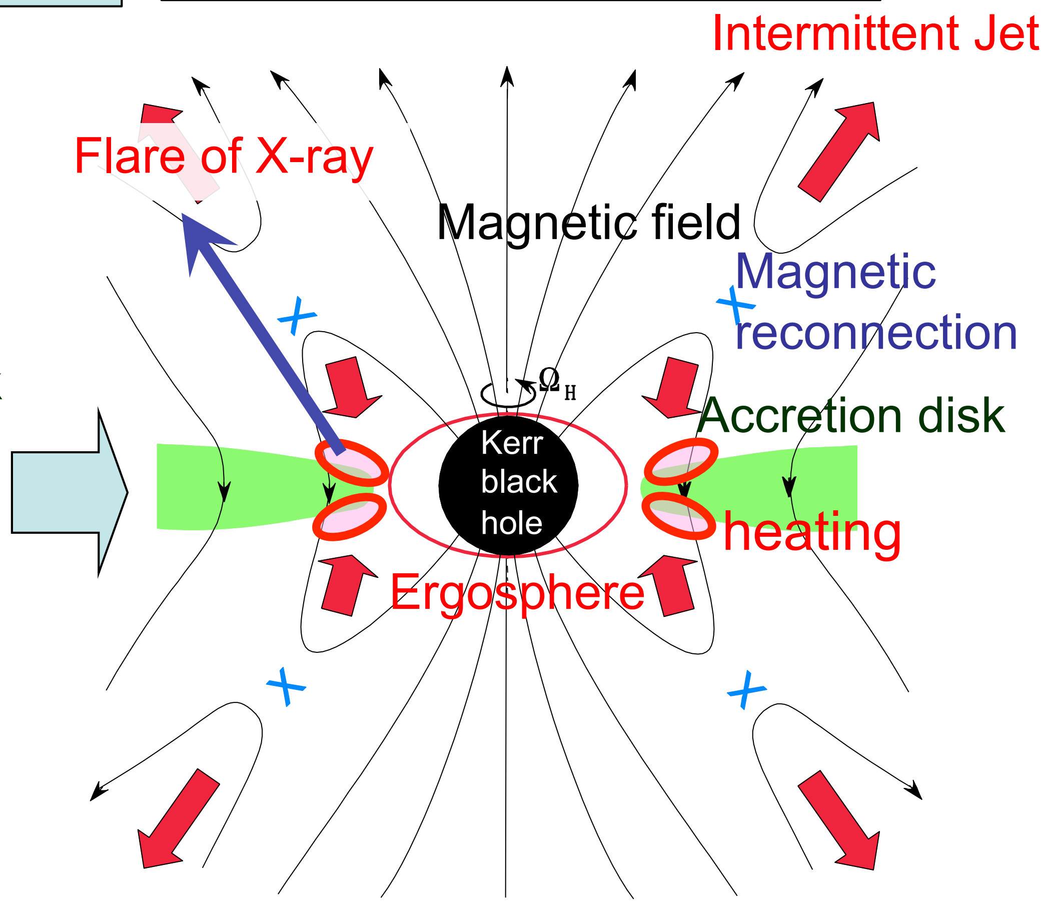
Ideal GRMHD simulations: A phenomena caused by the magnetic bridge near the black hole



Ideal GRMHD result

With finite resistivity





Anti-parallel magnetic field is formed

Magnetic reconnection must be important near black hole horizon!

GRMHD with finite resistivity

Near future important subject

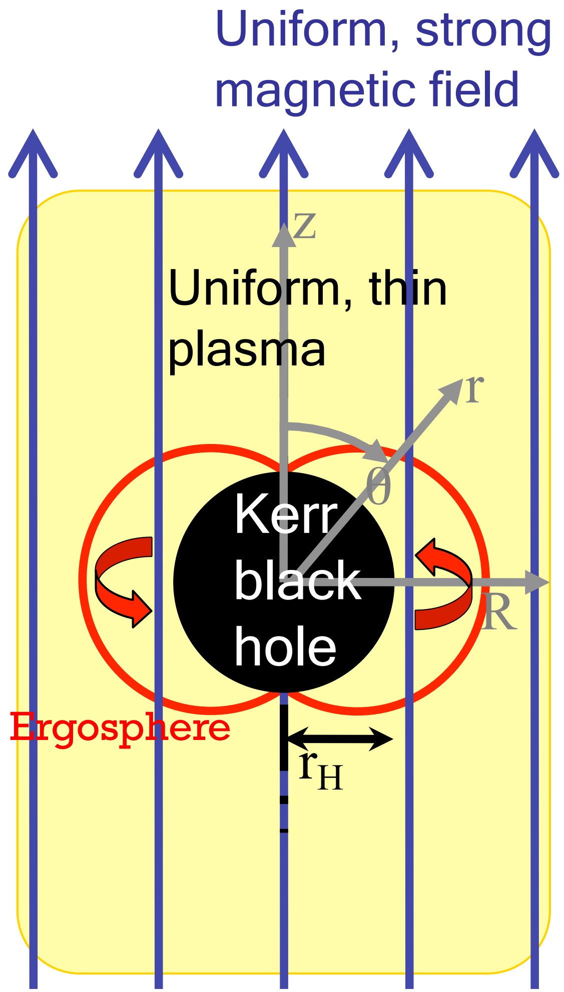
Even with uniform magnetic field as its initial condition・・・

Uniform magnetic field around rotating black hole with No Accretion disk : Initial condition

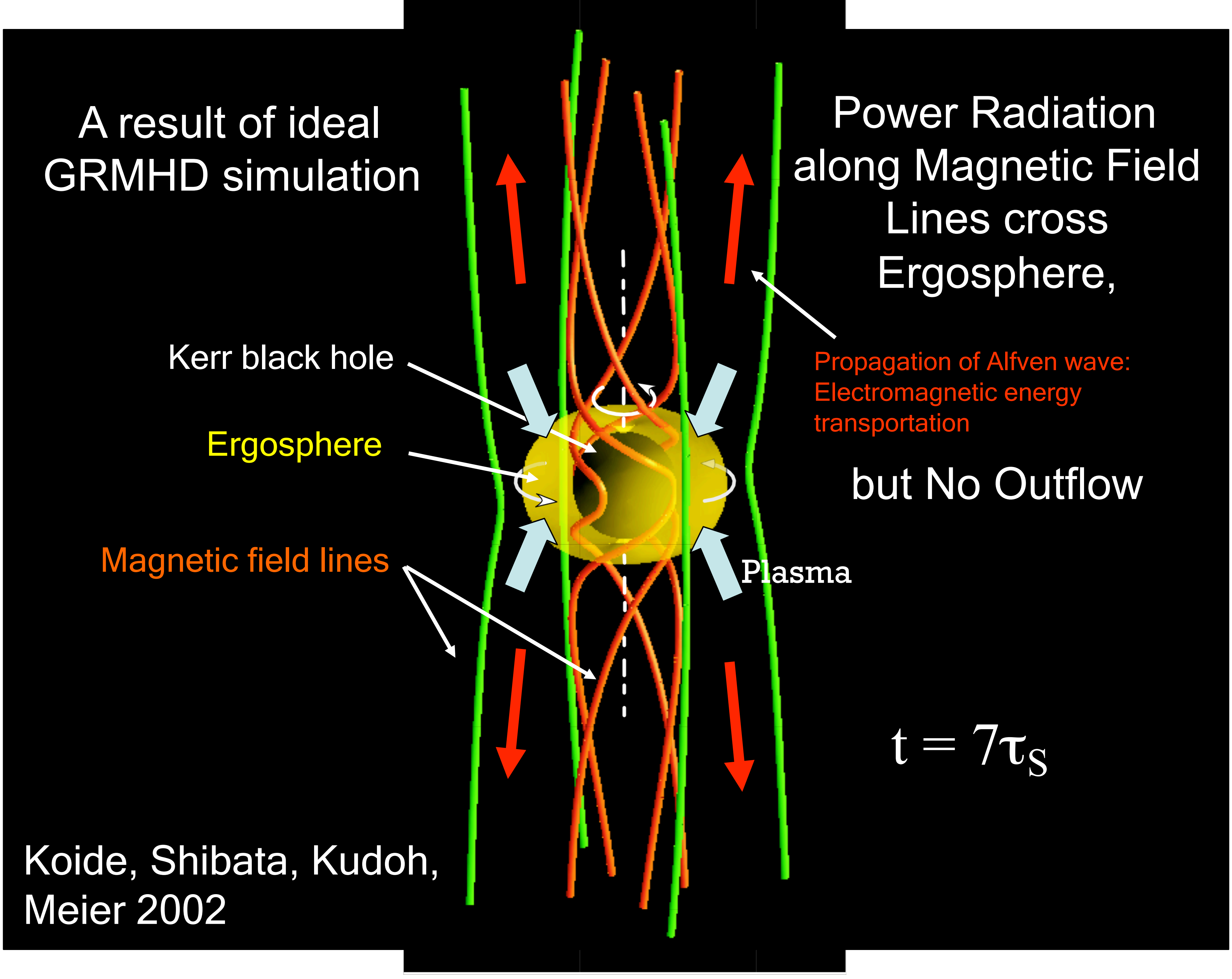
(1) Kerr black hole: maximally rotating rotation parameter, $a=J/J_{max}=0.99995$

(2) Magnetic field: Uniform around Kerr black hole (Wald solution)

(3) Plasma: zero momentum, uniform, low density and pressure $\rho_0=0.1B_0^2/c^2$, $p_0=0.06\rho_0 c^2$

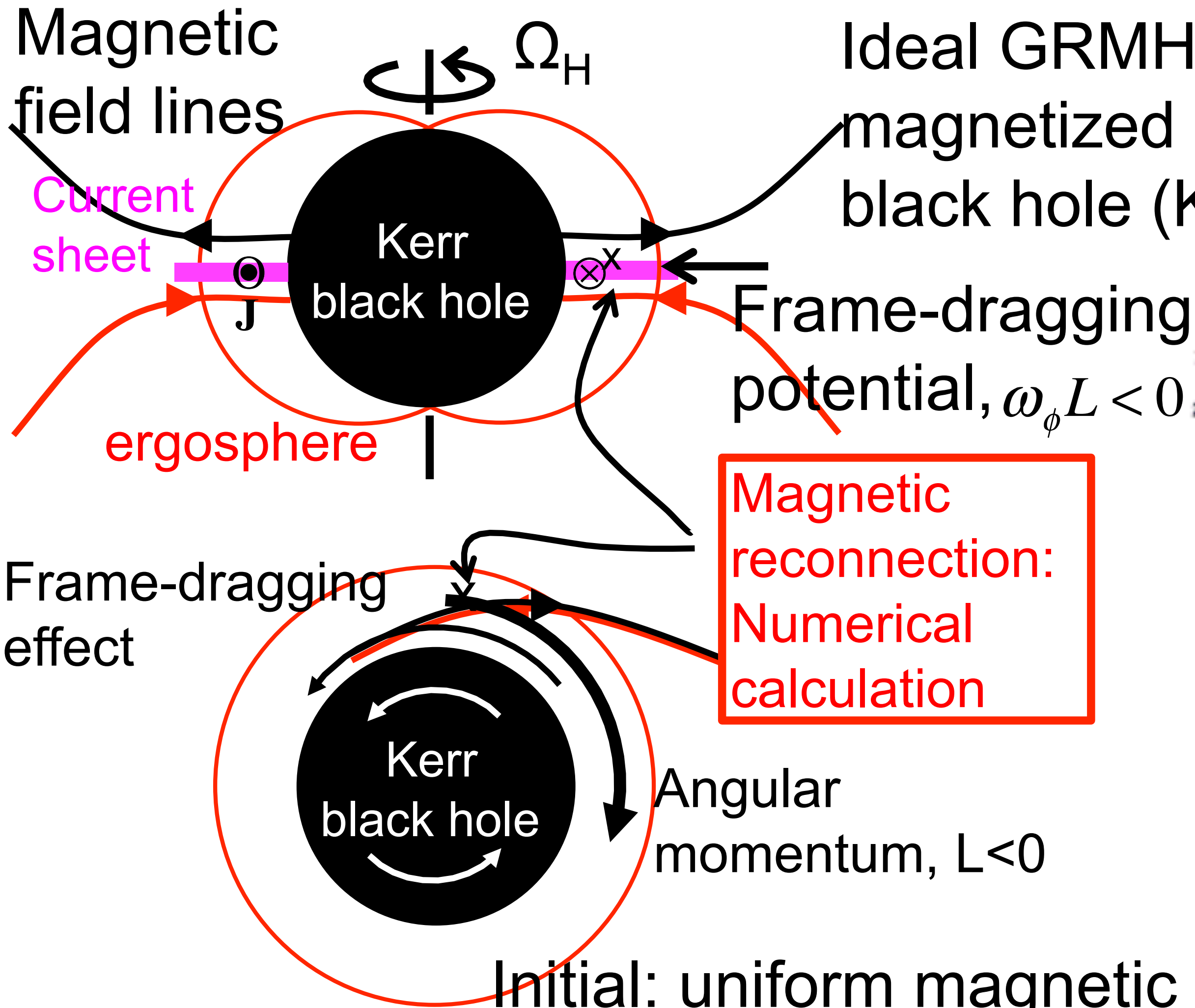


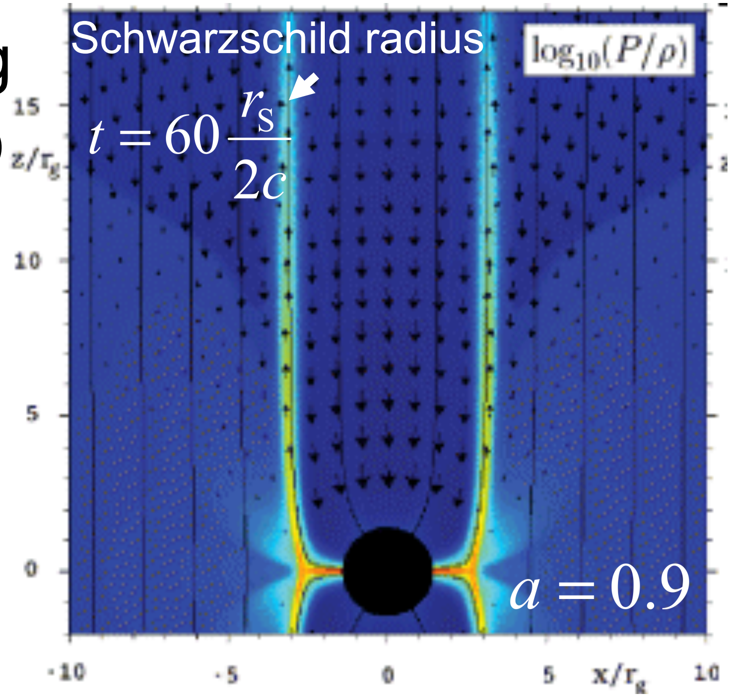
(Koide, Shibata, Kudoh, Meier 2002)


A result of ideal
GRMHD simulation
Power Radiation
along Magnetic Field
Lines cross
Ergosphere,
Kerr black hole
Ergosphere
Magnetic field lines
Propagation of Alfven wave:
Electromagnetic energy
transportation
but No Outflow
Plasma
$t = 7\tau_S$
Koide, Shibata, Kudoh,
Meier 2002

Longer term simulation of uniformly magnetized plasmas around Kerr BH

Ideal GRMHD simulation of uniformly magnetized plasma around rotating black hole (Komissarov 2005)





Initial: uniform magnetic field (Wald solution)

⇒ sprit monopole-like magnetic field in ergosphere

⇒ magnetic reconnection in ergosphere

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Covariant form of standard resistive GRMHD equations

Unit system $\left[\begin{array}{l} c=1 \\ \mu_0 = 1 \end{array}\right.$

- General relativistic equations of conservation laws:

proper particle number density

$$\nabla_\nu \left(n U^\nu \right) = 0 \quad \text{(particle number)}$$

4-velocity

$$\nabla_\nu T^{\mu\nu} = 0 \quad \text{(energy and momentum)}$$

Energy-momentum tensor

Maxwell equations:

Field strength tensor

4-current density

$$\partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} + \partial_\lambda F_{\mu\nu} = 0 \qquad \nabla_\nu F^{\mu\nu} = -J^\mu$$

Ohm's law with resistivity: $F_{\mu\nu} U^\nu = \eta \left(J_\mu + \left(U_\nu J^\nu \right) U_\mu \right)$

resistivity

Kerr Metric: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$

$$g_{00} = -h_0^2; \quad g_{ii} = h_i^2; \quad g_{0i} = -h_i^2 \omega_i \quad (i = 1,2,3); \quad g_{ij} = 0 \quad (i \neq j)$$

Lapse function: $\alpha = \sqrt{h_0^2 + \sum_i \left(h_i \omega_i \right)^2}$

(gravitational time delay)

Shift vector: $\beta_i = h_i \omega_i / \alpha$

$\boldsymbol{\beta} = \left(\beta_1, \beta_2, \beta_3 \right)$

(velocity of dragged frame)

3+1 form of resistive GRMHD equations for numerical calculations

Special relativistic mass density, γρδ

$$\frac{\partial D}{\partial t} = -\nabla \cdot [\alpha D(\mathbf{v} + c\vec{\beta})]$$

(conservation of particle number)

general relativistic effect

Special relativistic total momentum density

$$\frac{\partial \boldsymbol{P}}{\partial t} = -\nabla \cdot [\alpha(\boldsymbol{T} + c\boldsymbol{\beta}\boldsymbol{P})] - \left(D + \frac{\varepsilon}{c^2}\right)\nabla(c^2\alpha) + \alpha \boldsymbol{f}_{\mathrm{curv}} - \boldsymbol{P} : \boldsymbol{\sigma}$$

(equation of motion)

special relativistic effect

Shear of frame-dragging

Special relativistic total energy density

$$\frac{\partial \varepsilon}{\partial t} = -\nabla \cdot [\alpha(c^2\boldsymbol{P} - Dc^2\boldsymbol{v} + \varepsilon c\boldsymbol{\beta})] - (\nabla\alpha)\cdot c^2\boldsymbol{P} - \boldsymbol{T} : \boldsymbol{\sigma}$$

(equation of energy)

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times [\alpha(\boldsymbol{E} - c\boldsymbol{\beta} \times \boldsymbol{B})] \qquad \alpha(\boldsymbol{J} + \rho_e c\boldsymbol{\beta}) + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = \nabla \times \left[\alpha\left(\boldsymbol{B} + \frac{\boldsymbol{\beta}}{c} \times \boldsymbol{E}\right)\right]$$

$$\nabla \cdot \boldsymbol{B} = 0 \qquad \rho_e = \frac{\alpha}{c^2}\nabla \cdot \boldsymbol{E}$$

(Maxwell equations)

$$\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} = \frac{\eta}{\gamma}\left[\boldsymbol{J} - \gamma^2\left(\rho_e - \frac{1}{c^2}(\boldsymbol{v}\cdot\boldsymbol{J})\right)\boldsymbol{v}\right]$$

(Ohm's law with finite resistivity)

All of these equations are coupled! We have to consider Ampere's law and Gauss's law of electric charge. We use the numerical method of Watanabe & Yokoyama (2006).

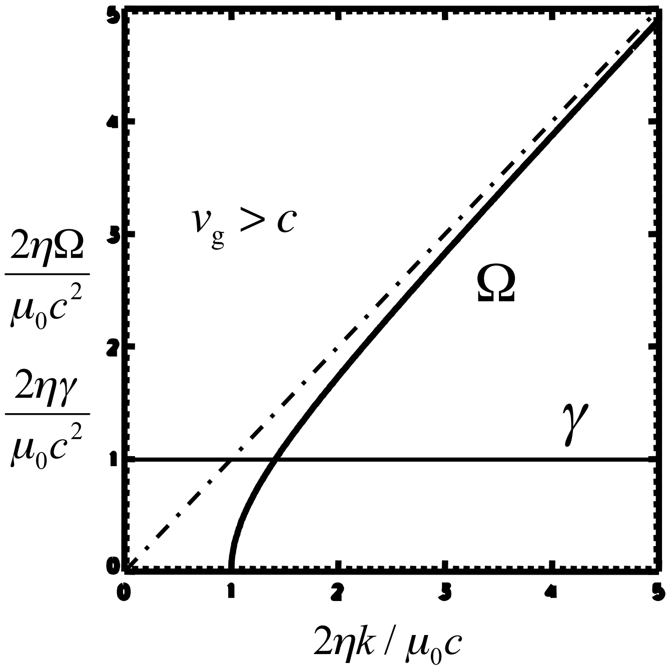
"Standard" resistive GRMHD equations※

=Relativistic one-fluid model with resistivity

- Conservation of particle number, momentum, and energy
- Maxwell equations
- Standard relativistic Ohm's law：

$$U^{\nu} F^{\mu}{}_{\nu} = \eta \left(J^{\nu} - \rho_{\mathrm{e}} U^{\nu} \right)$$

Resistivity



Dispersion relation of electro-magnetic wave in resistive plasma

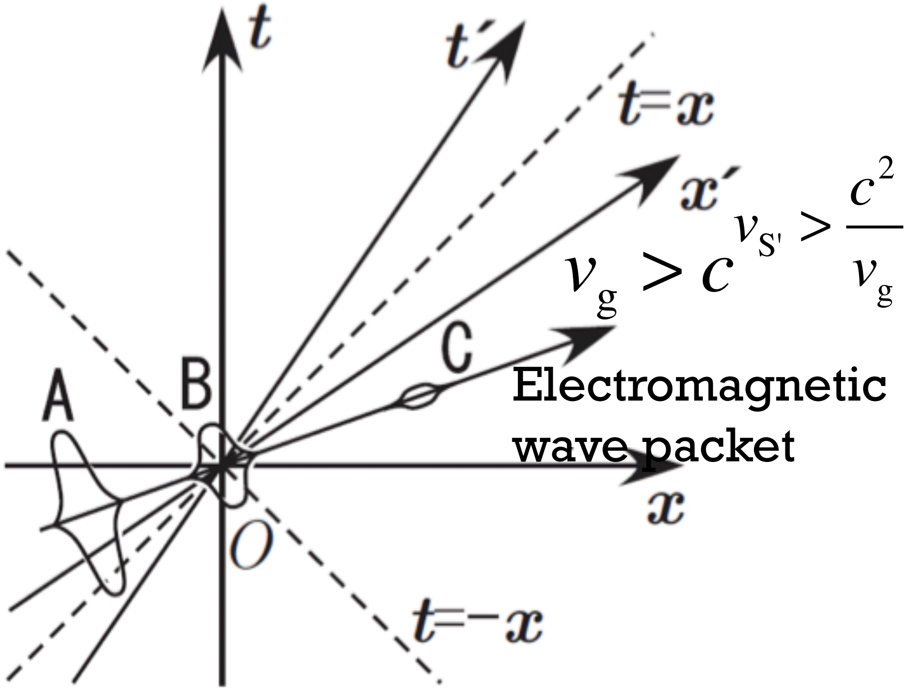
Group velocity of electro-magnetic wave is greater than light speed

Causality broken?

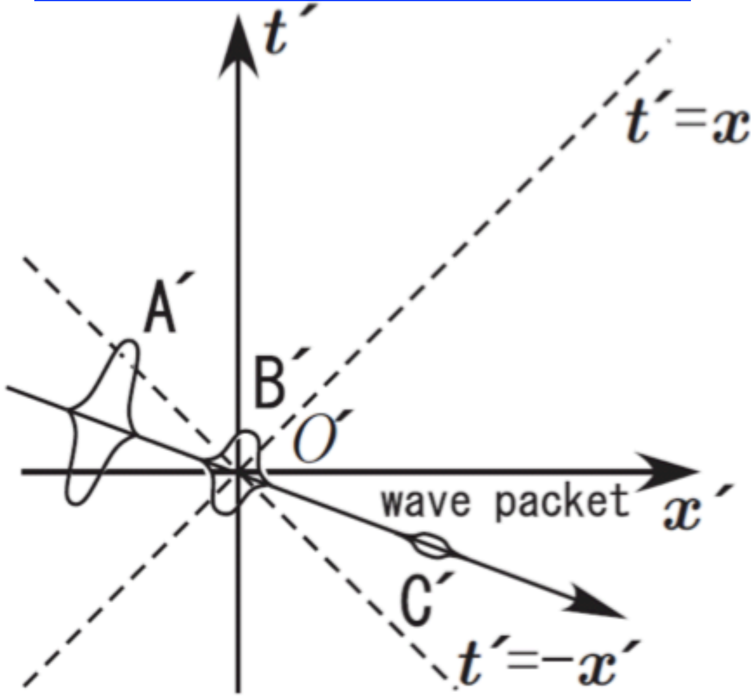
※ Now, numerical calculations are limited in special relativistic version.

Superluminal information transportation: Break down of causality?: Spontaneous decrease in entropy

Plasma rest frame



New coordinates frame



increase in entropy

electromagnetic energy
⇒ thermal energy

decrease in entropy

thermal energy ⇒
electromagnetic energy

Generalized GRMHD equati

$$\nabla_\nu \left(\rho U^\nu\right) = 0$$

$\nabla_\nu \Big[hU^\mu U$

gy

Equation of 3+1 formalism required for numerical calculation of the generalized GRMHD are much complicated so that no one wants to perform calculation with them.

To clarify the limitation of the standard resistive GRMHD equations, we tried to perform acausal phenomena expected (not welcome) in the standard equations.

Therm

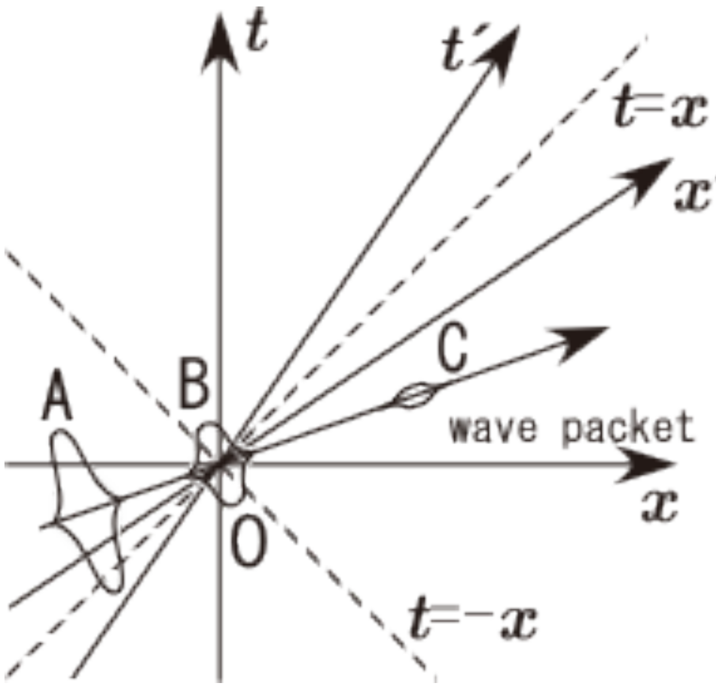
ndard Ohm's law

general relativistic Ohm's law

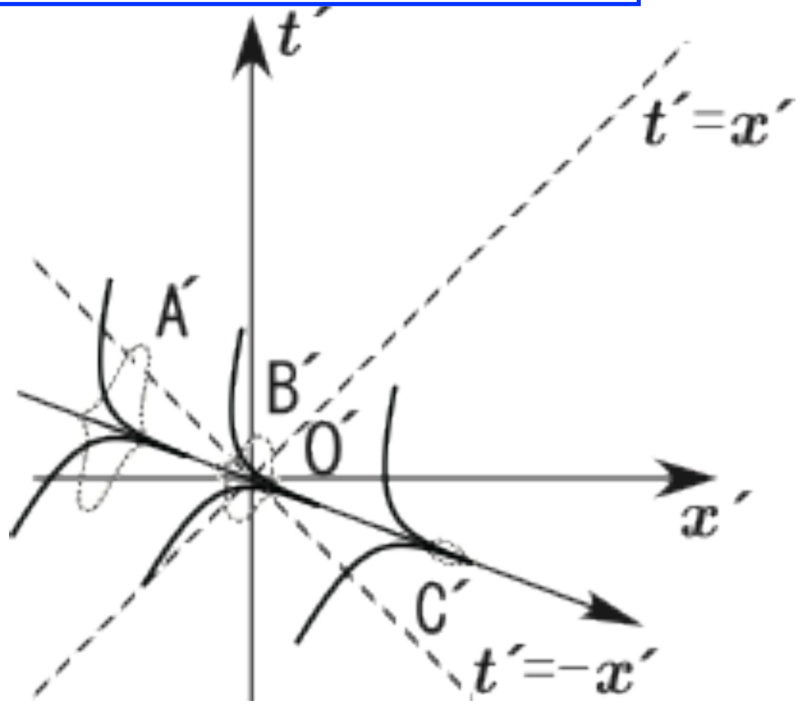
$$\eta < \quad (2.5 \sim 3)\frac{m}{e\rho}\sqrt{\mu h^\dagger} \quad \Rightarrow \quad v_{\rm g} \le c$$

In the new coordinates with fast plasma flow, the wave packet has monotonic amplitude profile and then could not send a signal

Plasma rest frame



New coordinates frame



It seems that monotonic wave decreases. This shape of the wave is not “wave packet” anymore. The wave packet is not Lorentz invariant.

Remarks on causality problem of RRMHD equations

- Wave-packet is not Lorentz invariant, and then cannot be accept as a relativistic object.
- The group velocity, which represents propagation velocity of wave packet, is not relativistic quantity. In fact, the definition of the group velocity, $v_g=\partial\omega/\partial k$, is not scalar nor covariant variable.
- We cannot discuss causality using the group velocity.
- Instead of group velocity, "head velocity" $v_h=\lim_{k\to\infty} \omega/k$ presents the propagation velocity of information. In resistive RMHD, the head velocity is light speed, and Lorentz invariant.

Concerning causality, there is no limitation of standard resistive GRMHD!

- Koide & Morino, Phys. Rev. D, 84, 083009, Jackson, Classical Electrodynamics (1962).

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Test calculations of resistive GRMHD

- Interaction between plasma and uniform magnetic field around black hole

Plasma fall in uniform magnetic field into a Schwarzschild black hole

$$\eta = 10^{-2} r_S c$$

(~ ideal RMHD)

Color: pressure

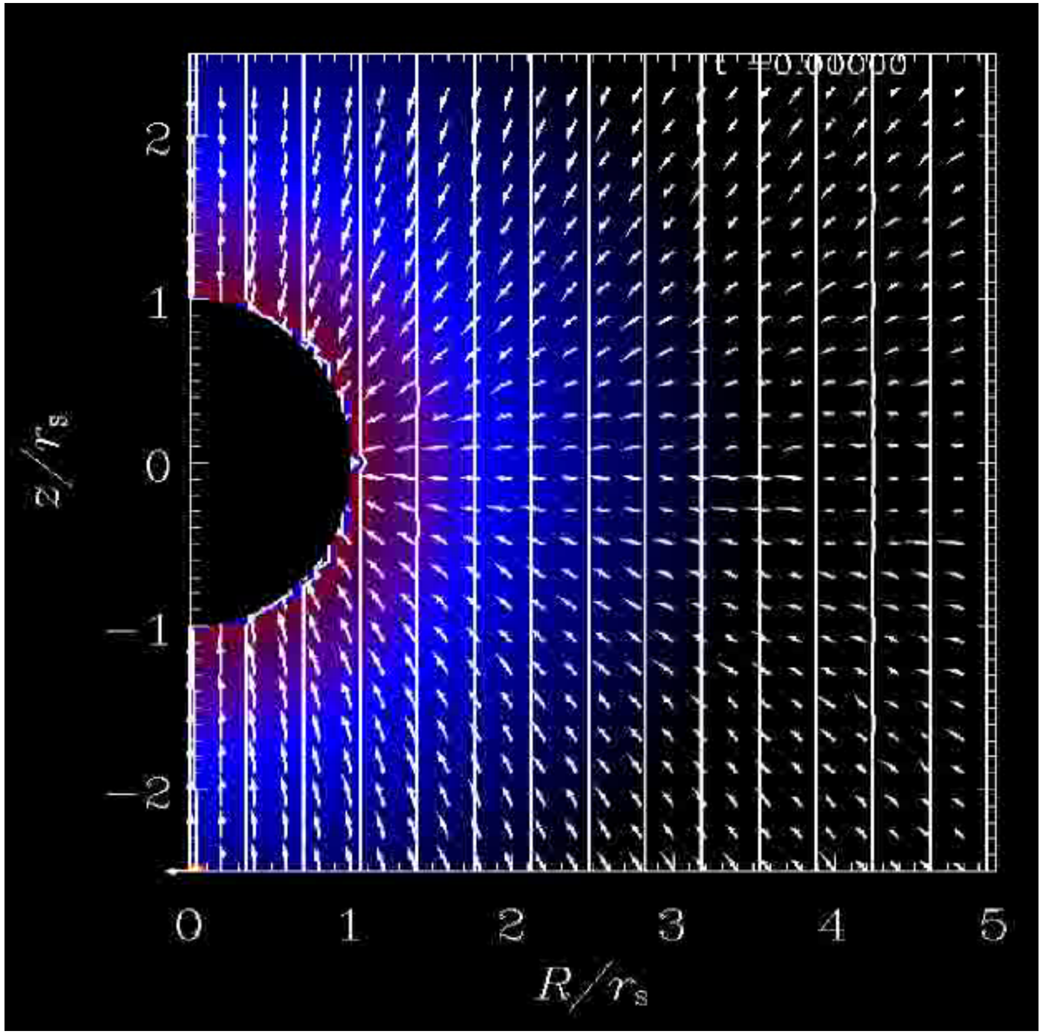
$$a = 0$$

Unit of length

$$r_S = \frac{2GM}{c^2}$$

Unit of time

$$\tau_S = \frac{r_S}{c}$$



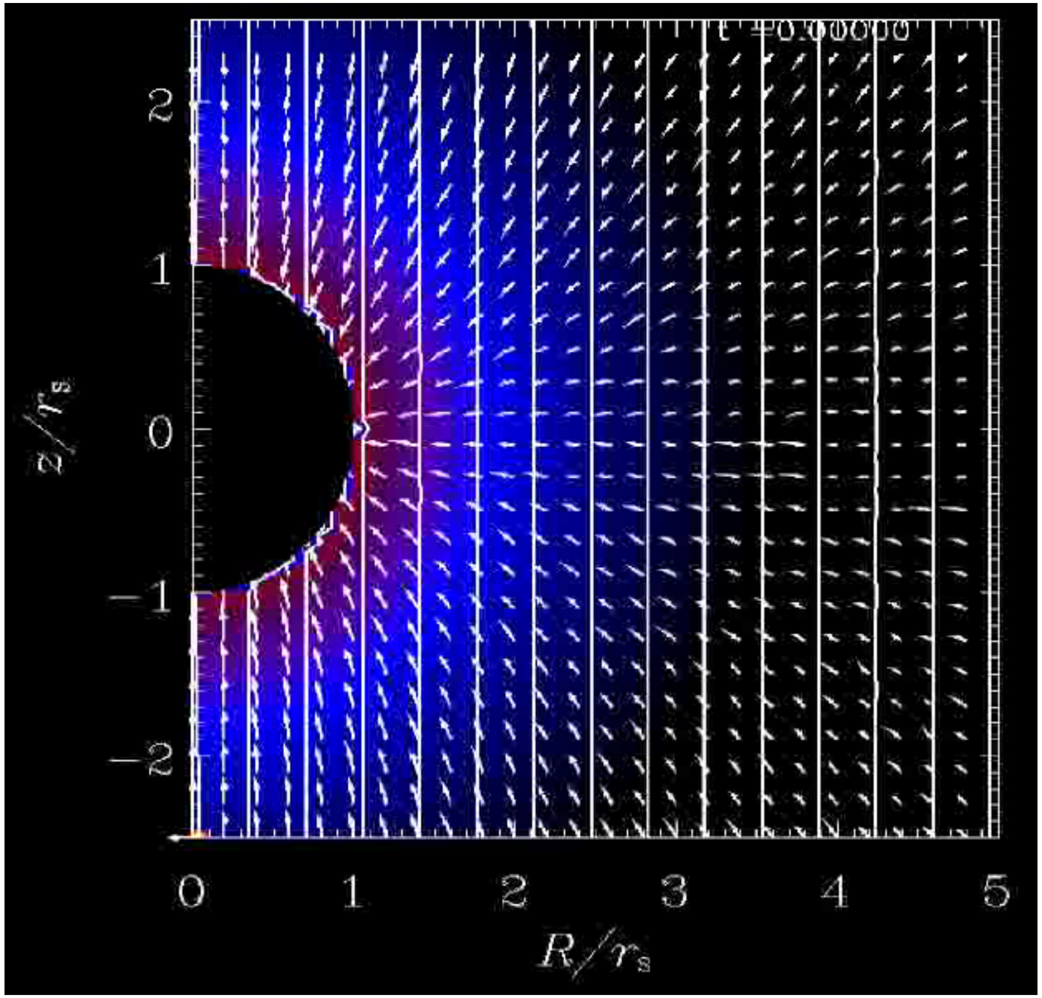
Plasma fall in uniform magnetic field into a Schwarzschild black hole

$\eta = 10^3 r_S c$

(～Vacuum)

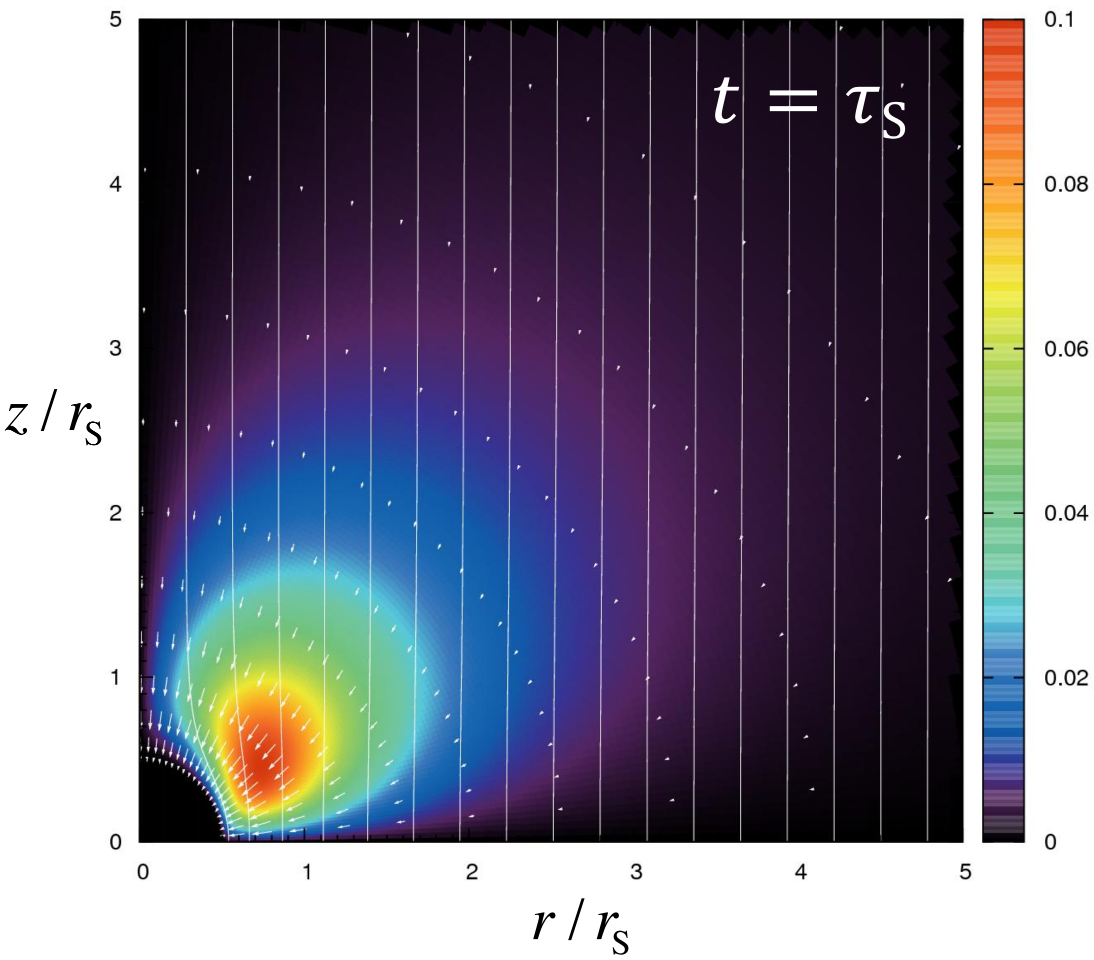
Color: pressure

$a = 0$



Test calculation: Frame-dragging dynamo effects

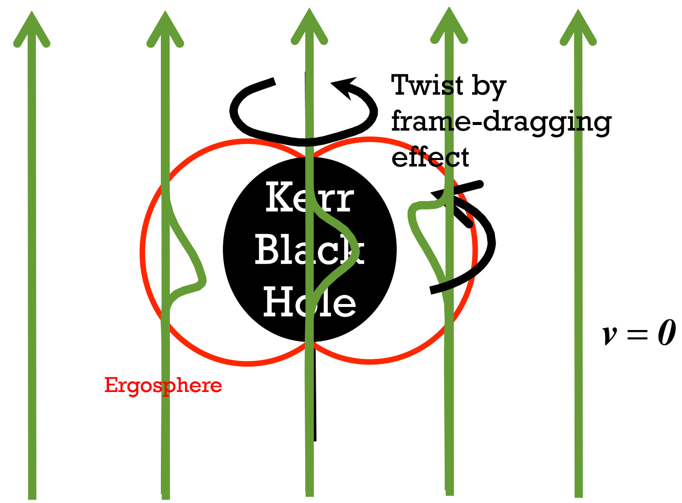
B_ϕ / B_0



$$\eta = 10^{-3} r_S c$$

$$a \equiv \frac{J}{J_{\max}} = 1$$

$$J_{\max} = \frac{GM^2}{c}$$



HLL+MUSCL scheme

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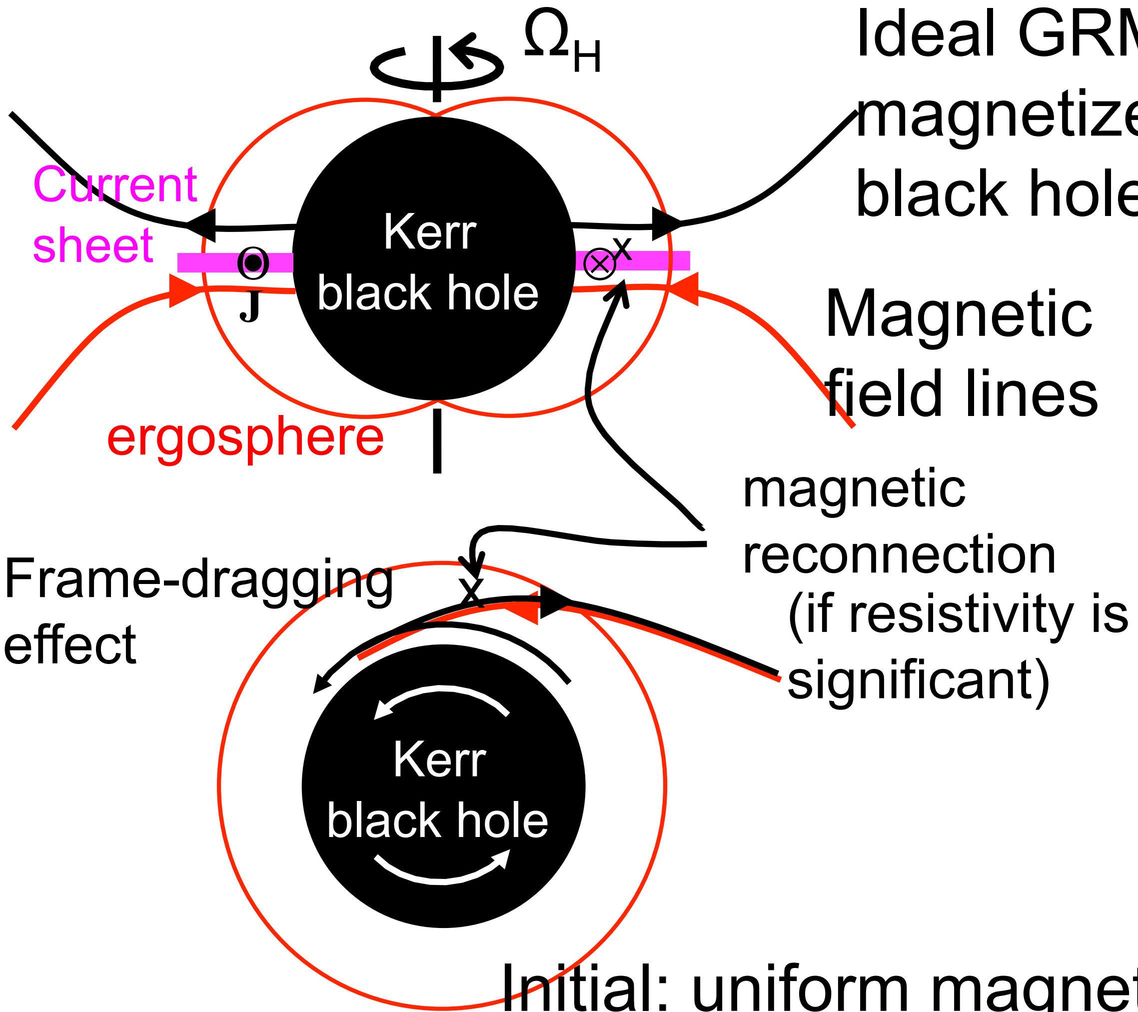
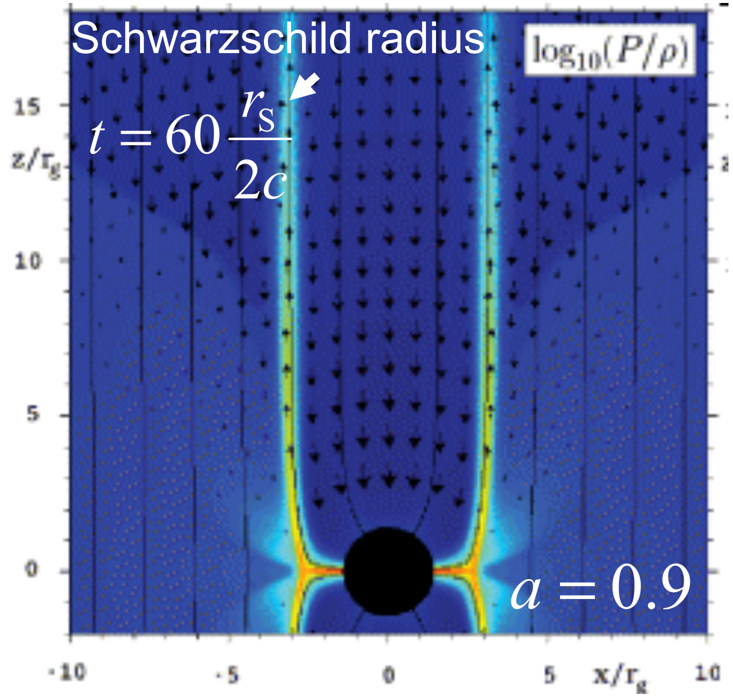
  - Resistive GRMHD simulation with initial uniform magnetic field around a rotating black hole (2 trial)
  - Resistive GRMHD simulation with initial split monopole magnetic field around a rotating black hole
- Summary and Future plans

Longer term simulation of uniformly magnetized plasmas around Kerr BH



Ideal GRMHD simulation of uniformly magnetized plasma around rotating black hole (Komissarov 2005)



Initial: uniform magnetic field (Wald solution)

⇒ sprit monopole-like magnetic field in ergosphere

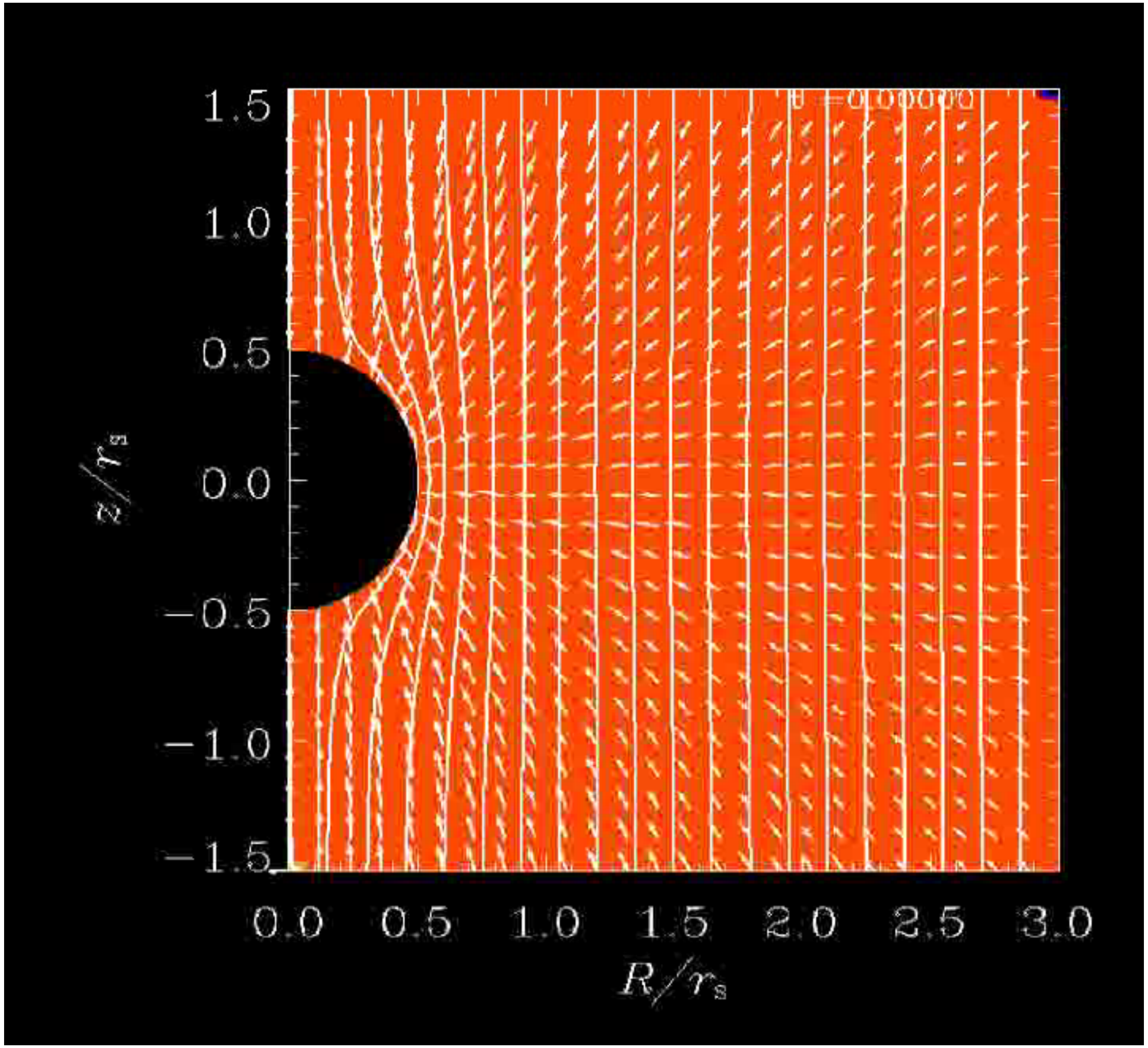
⇒ magnetic reconnection in ergosphere

Trial calculation with uniform magnetic field as initial field

$$a = 0.99995$$

$$\beta = 0.025$$

Color: $B_\phi / \rho^{1/2}$
Lines: magnetic field
Arrows: velocity

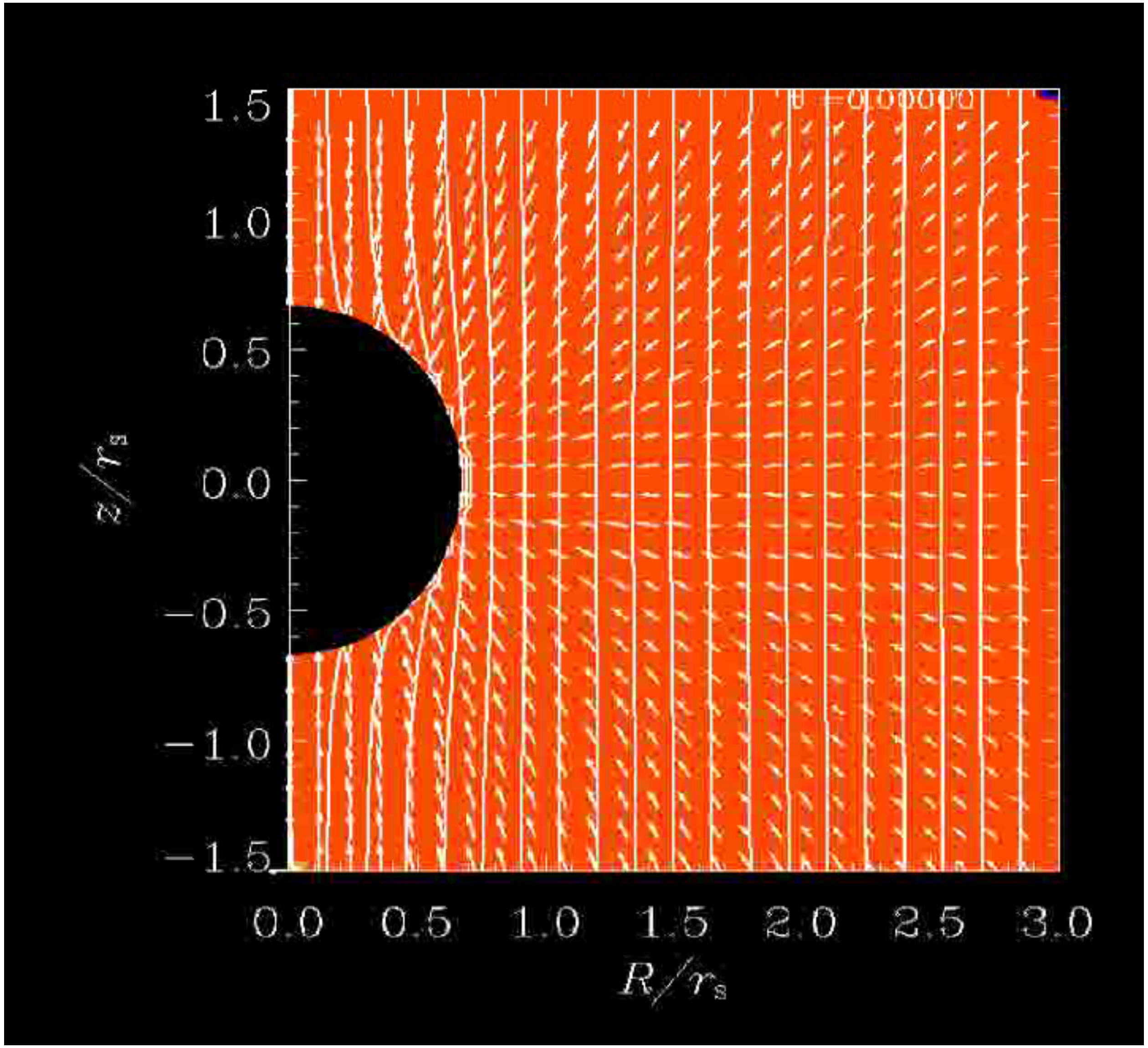


Trial calculation with uniform magnetic field (try #2)

$a = 0.94$

$\beta = 0.025$

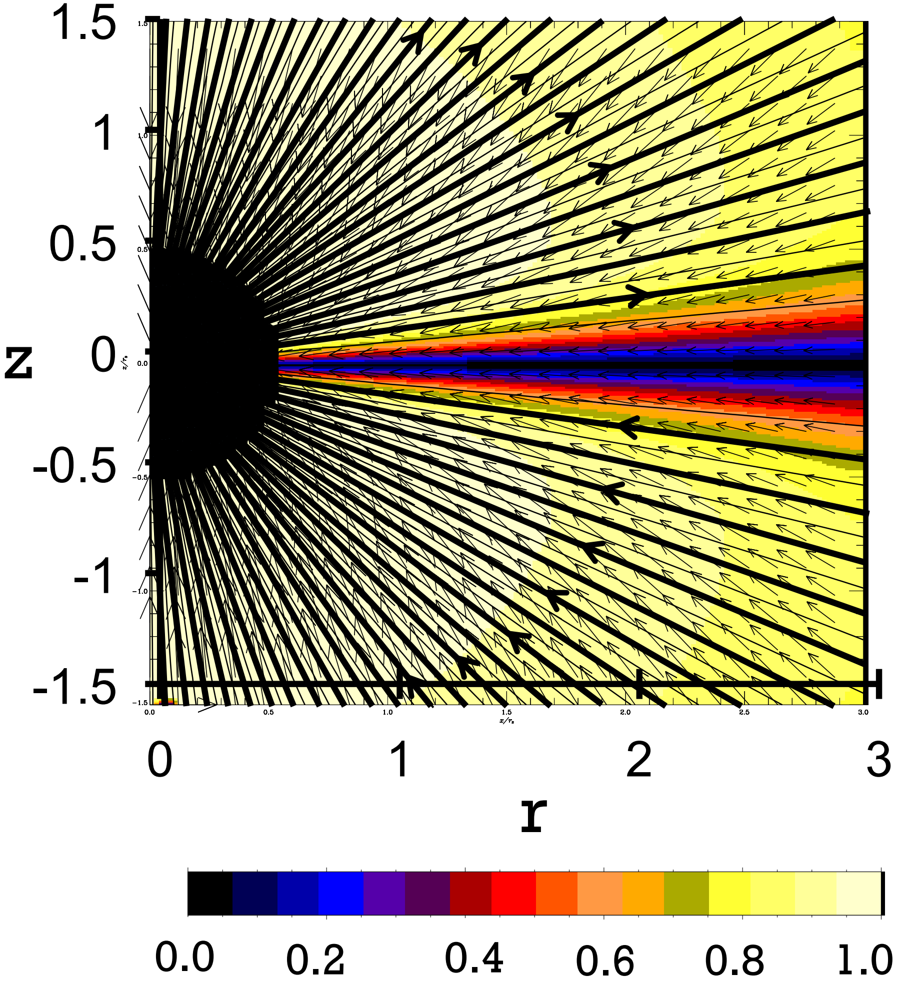
Color: $B_\phi / \rho^{1/2}$
Lines: magnetic field
Arrows: velocity



Initial condition of magnetic field of a simple case: Split monopole field

$t = 0$

$a = 0.99995$



Resistivity

$$\eta = 0.01 r_S$$

Relativistic Alfven velcoty

$$v_A = \frac{B}{\sqrt{\rho c^2 + \frac{\Gamma}{\Gamma - 1} p + B^2}} c$$

Split-monopole magnetic field

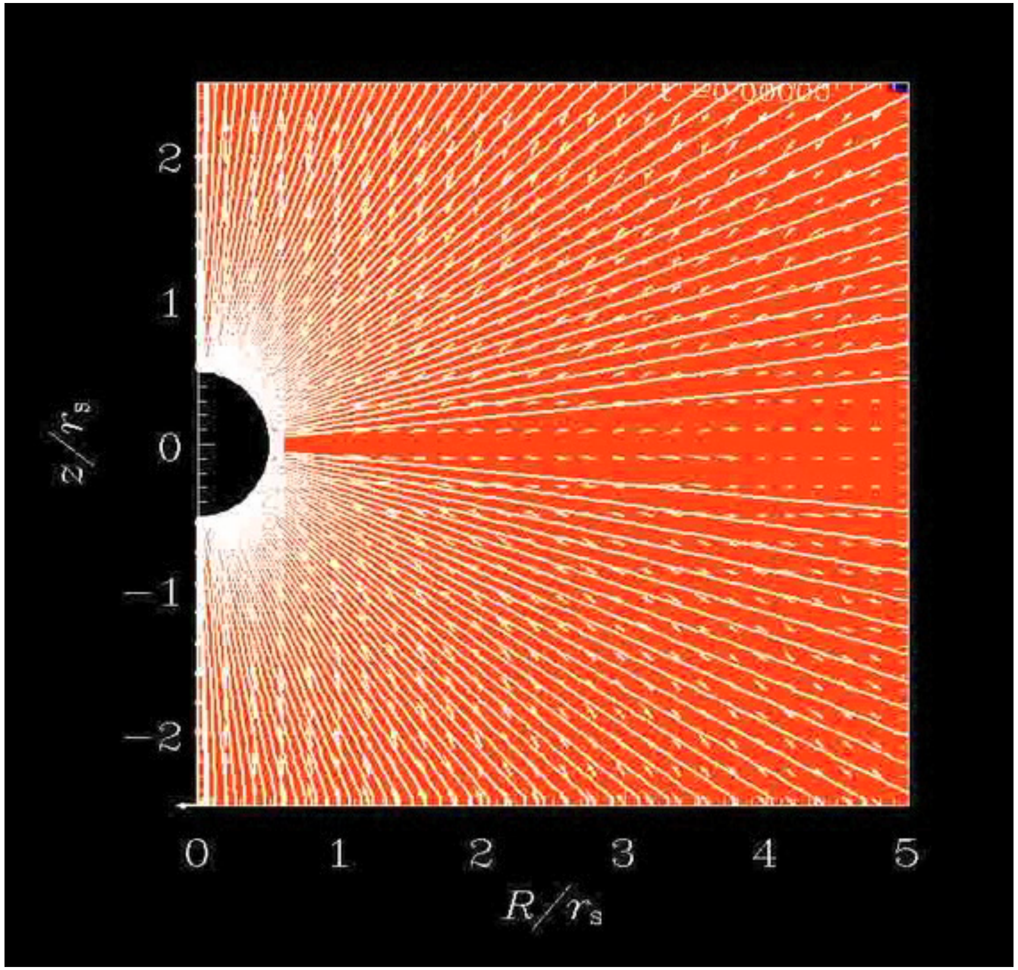
Relativistic magnetic reconnection

Magnetic reconnection in split-monopole magnetic field around black holes

$\eta = 0.01 r_S$

- Rapidly rotating (Kerr) black hole case:

$a = 0.99995$

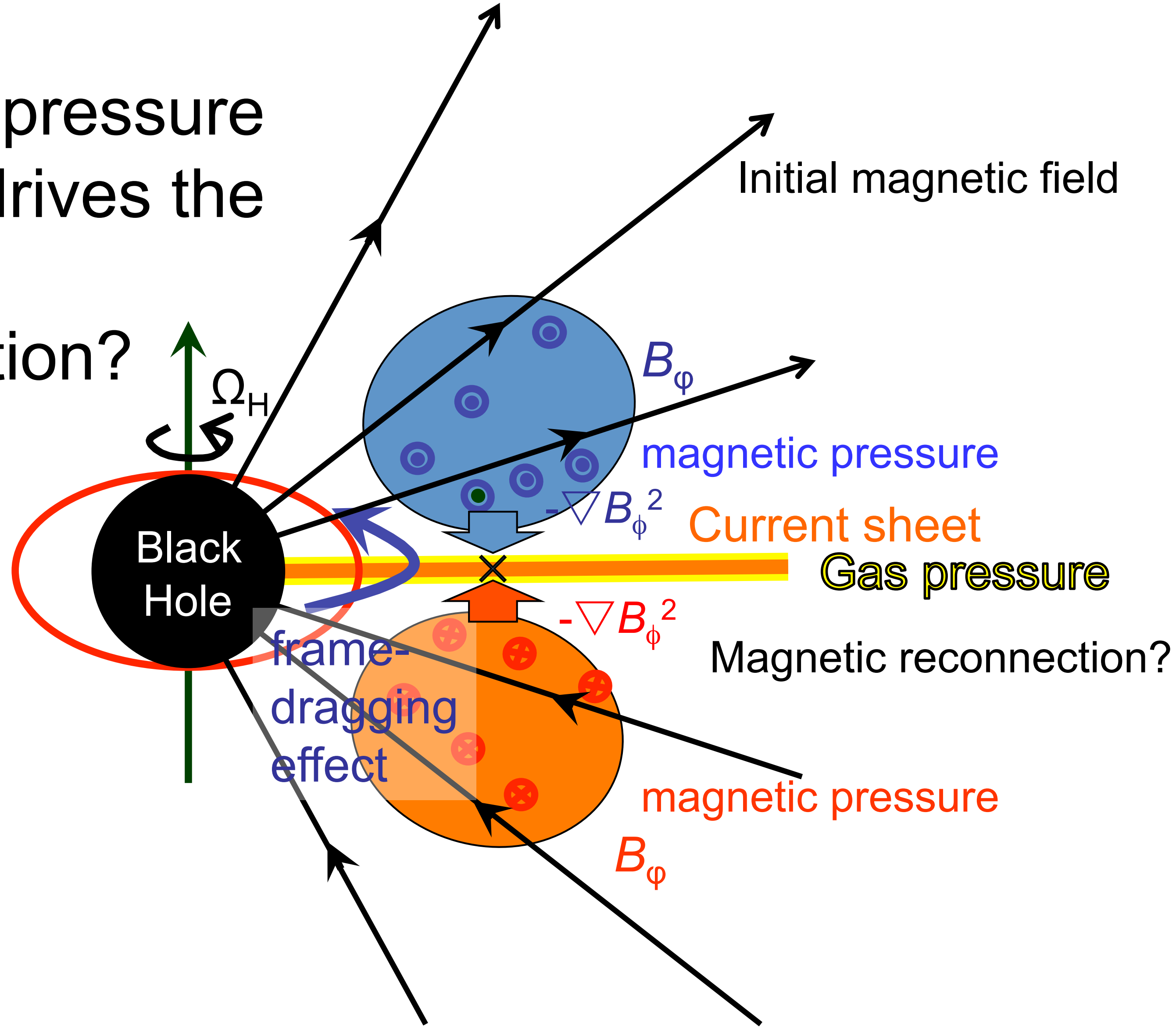


Color: $B_\phi / \rho^{1/2}$
Lines: magnetic field
Arrows: velocity

What mechanism causes the magnetic reconnection in the ergosphere?

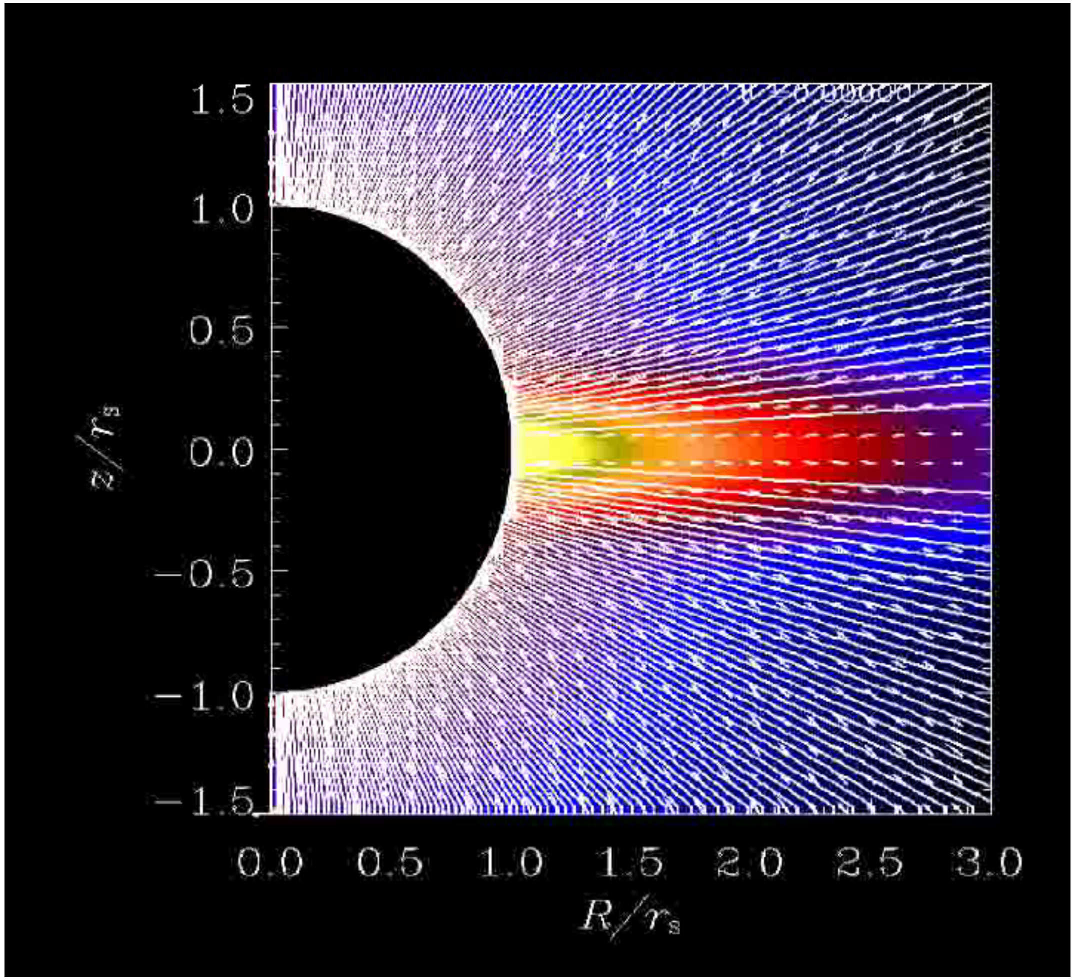
A expected mechanism of magnetic reconnection around Kerr black hole

Magnetic pressure gradient drives the magnetic reconnection?



Magnetic reconnection in split-monopole magnetic field around black holes

- Schwarzschild black hole case:



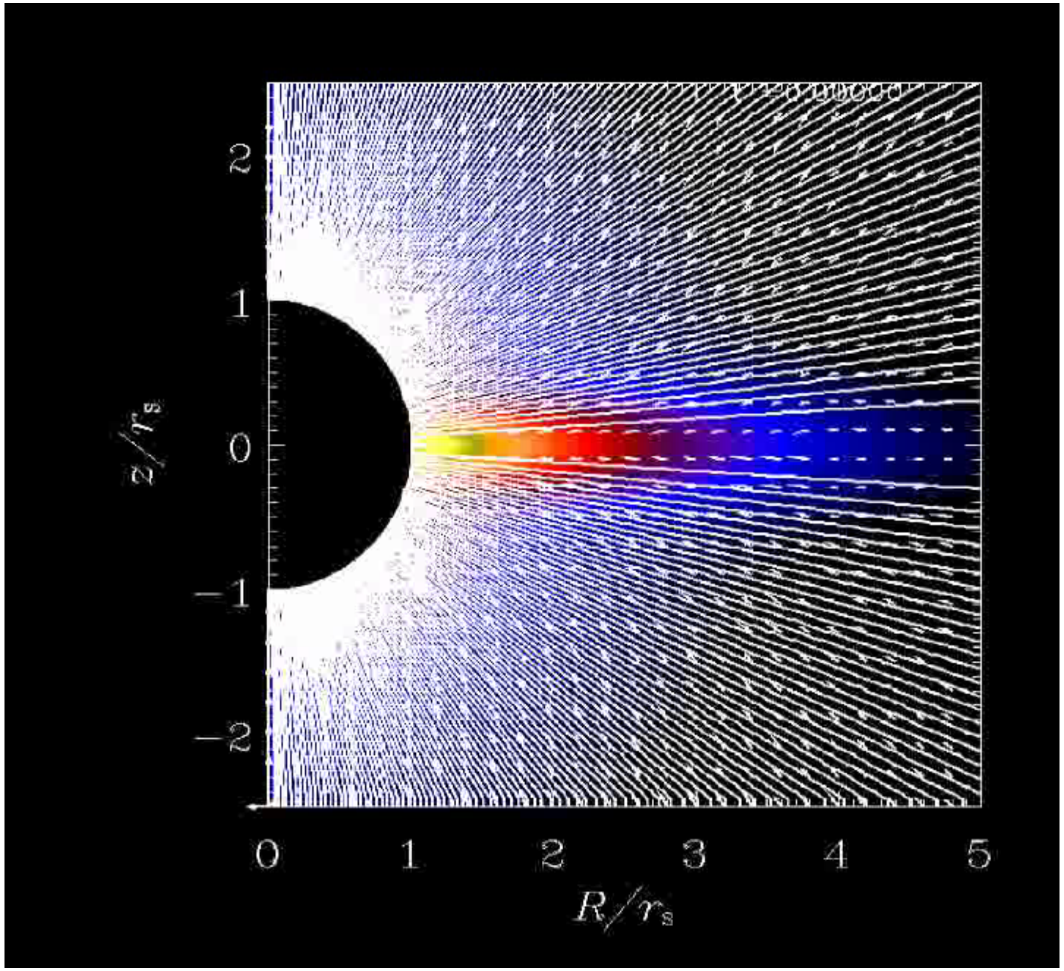
$a = 0$

$\eta = 0.01 r_S$

Color: pressure
Lines: magnetic field
Arrows: velocity

Magnetic reconnection in split-monopole magnetic field around black holes

- Schwarzschild black hole case:



$a = 0$

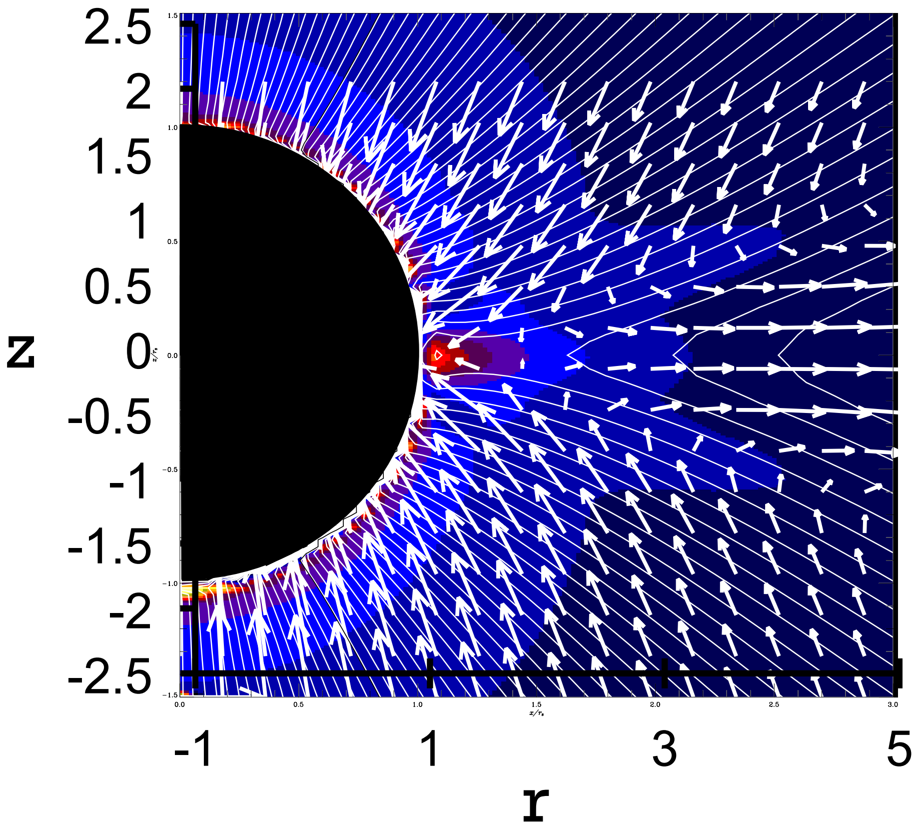
$\eta = 0.01 r_S$

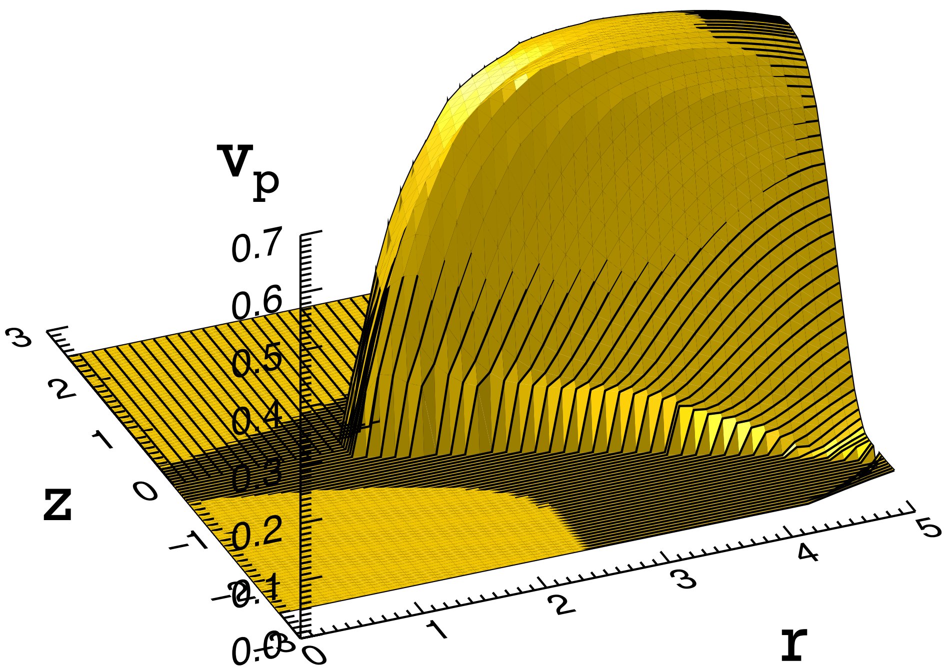
Color: pressure
Lines: magnetic field
Arrows: velocity

Magnetic reconnection in split-monopole magnetic field around black holes

- Schwarzschild black hole case:

arrow: velocity, color: density



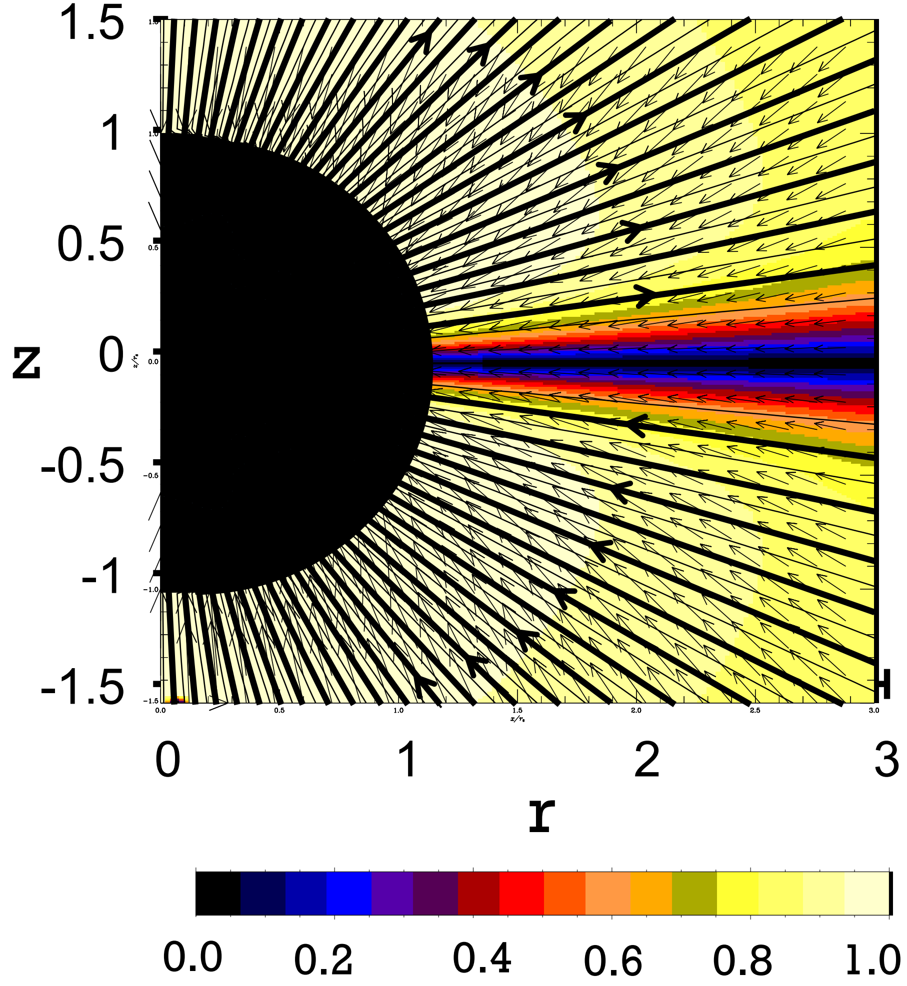


$v_{p,\,max}$=0.62c

Initial condition of magnetic field

$t = 0$

$a = 0$



Relativistic Alfven velcoty

$$v_{\mathrm{A}} = \frac{B}{\sqrt{\rho c^2 + \frac{\Gamma}{\Gamma - 1} p + B^2}} c$$

Relativistic magnetic reconnection

Split-monopole magnetic field

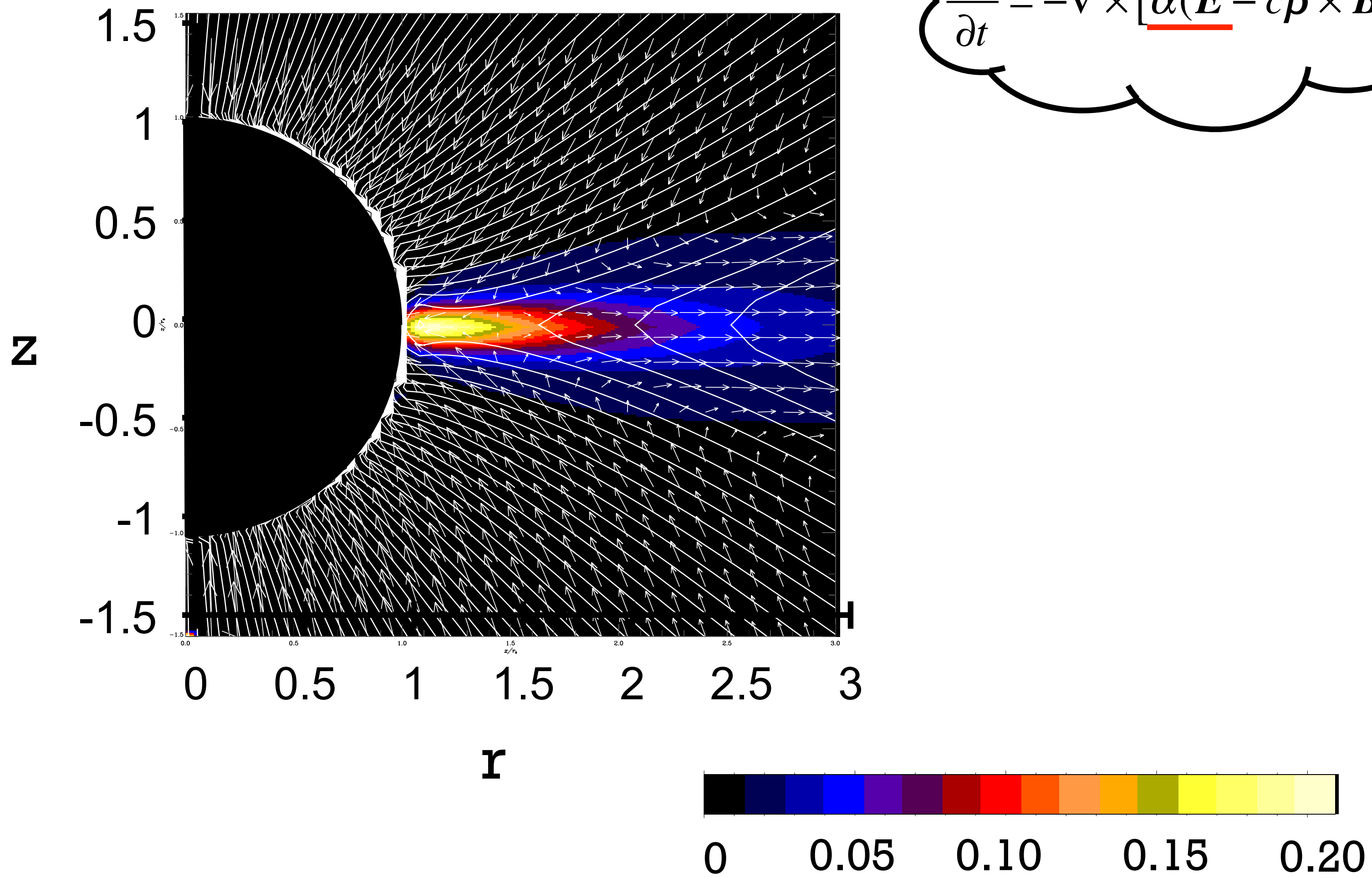
Magnetic reconnection rate

Lapse function

$$a = 0 \qquad \alpha E_\phi = \alpha \eta J_\phi$$

General relativistic
Faraday's law

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \left[\underline{\alpha(\boldsymbol{E}} - c\boldsymbol{\beta} \times \boldsymbol{B}) \right]$$



Evaluation of tearing instability

Maximum growth rate of tearing instability:

$$\omega_{\max} \cong \tau_A^{-1} R^{-1/2} \quad \text{(Non-relativistic)}$$

$$\eta = 0.01 r_{\mathrm{S}}, \quad a_{\mathrm{CS}} = 0.1, \quad v_{\mathrm{A}} = 1$$

Thickness of current sheet

$$\tau_{\mathrm{A}} = 0.1 \tau_S, \quad R = 10$$

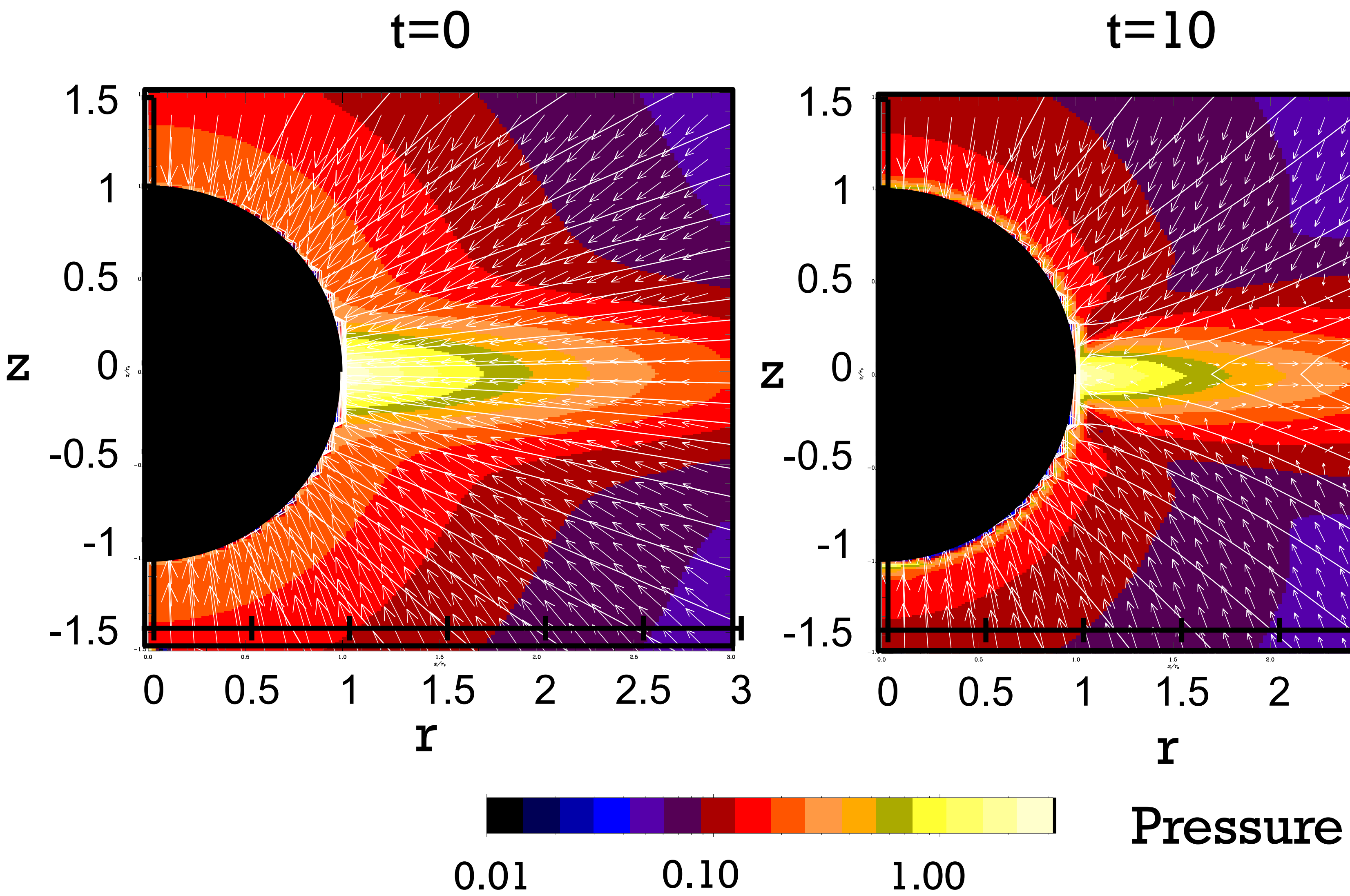
$$\boxed{\omega_{\max} = 3\tau_{\mathrm{s}}^{-1}} \qquad \boxed{\lambda_{\max} = \frac{2\pi}{k_{\max}} \approx r_{\mathrm{S}}}$$

$$\because \quad \omega \cong \tau_A^{-1} \left(a_{\mathrm{CS}} k\right)^{-2/5} R^{-3/5}, \quad \left(k a_{\mathrm{CS}}\right)_{\max} \cong R^{-1/4}$$

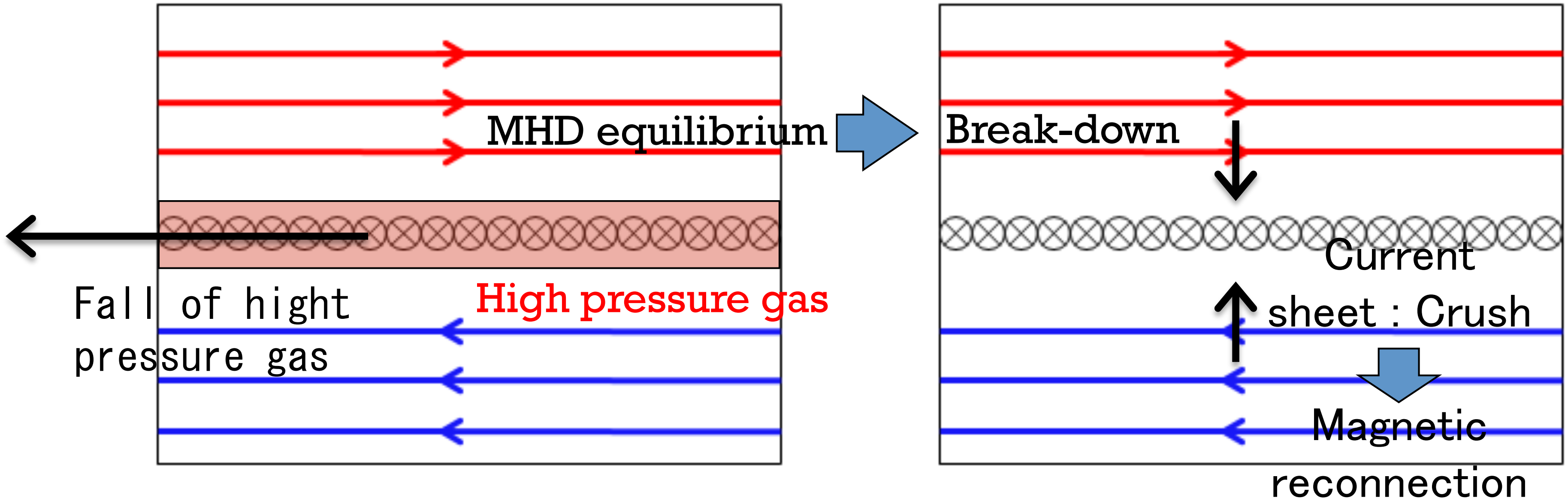
$$\tau_A = \frac{a_{\mathrm{CS}}}{v_A} \quad \text{(Alfven transit time)} \qquad R = \frac{a_{\mathrm{CS}} v_{\mathrm{A}}}{\eta} \quad \text{(Magnetic Reynolds number)}$$

However, the magnetic reconnection does not dependent on the perturbation of the initial velocity at the current sheet. This indicates that the tearing instability has no essential role in the magnetic reconnection.

Analysis of numerical results of magnetic reconnection around Schwarzschild black hole

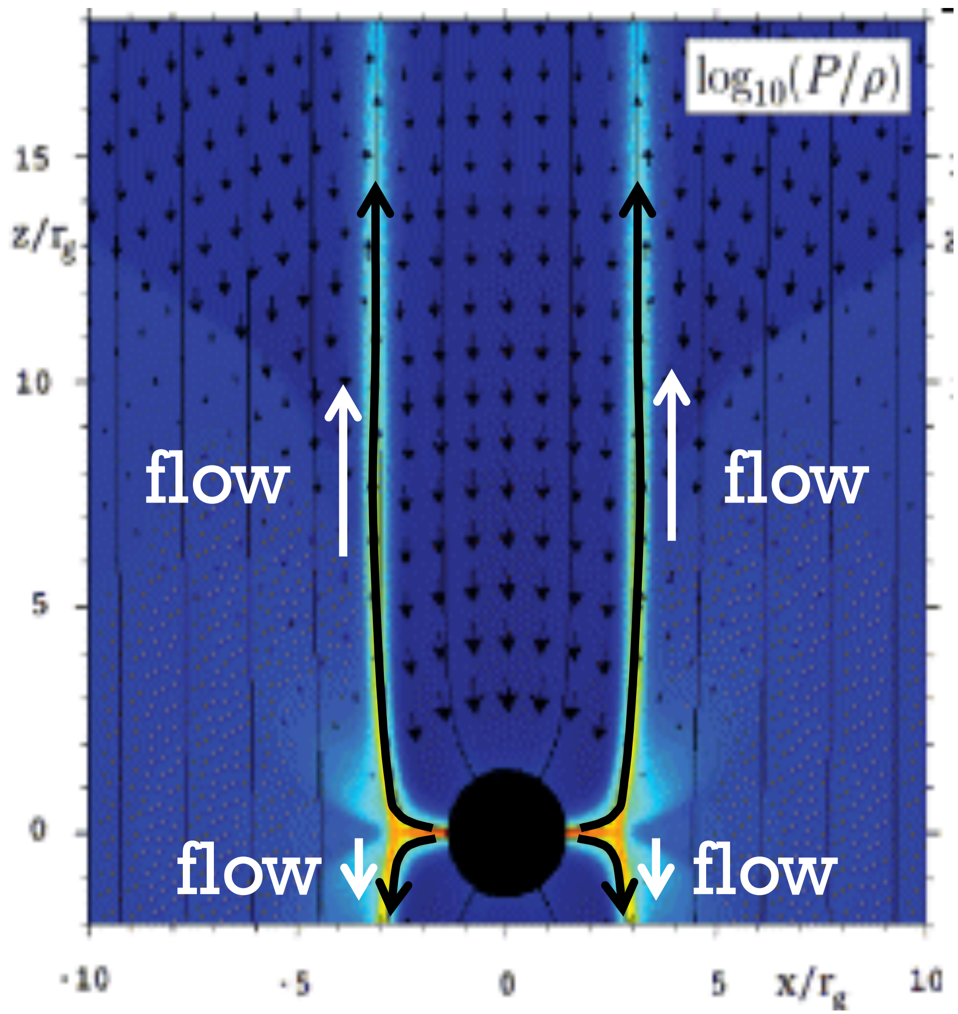


Preliminary model of the magnetic reconnection in ergoshpere: Mechanism of “Daruma-Otoshi” ?



Daruma-otoshi:
Japanese traditional toy.
When one piece is hit to shit out, the pieces upper and lower are connected.

Expected flow from ergosphere driven by magnetic reconnection in initial uniform magnetic field case

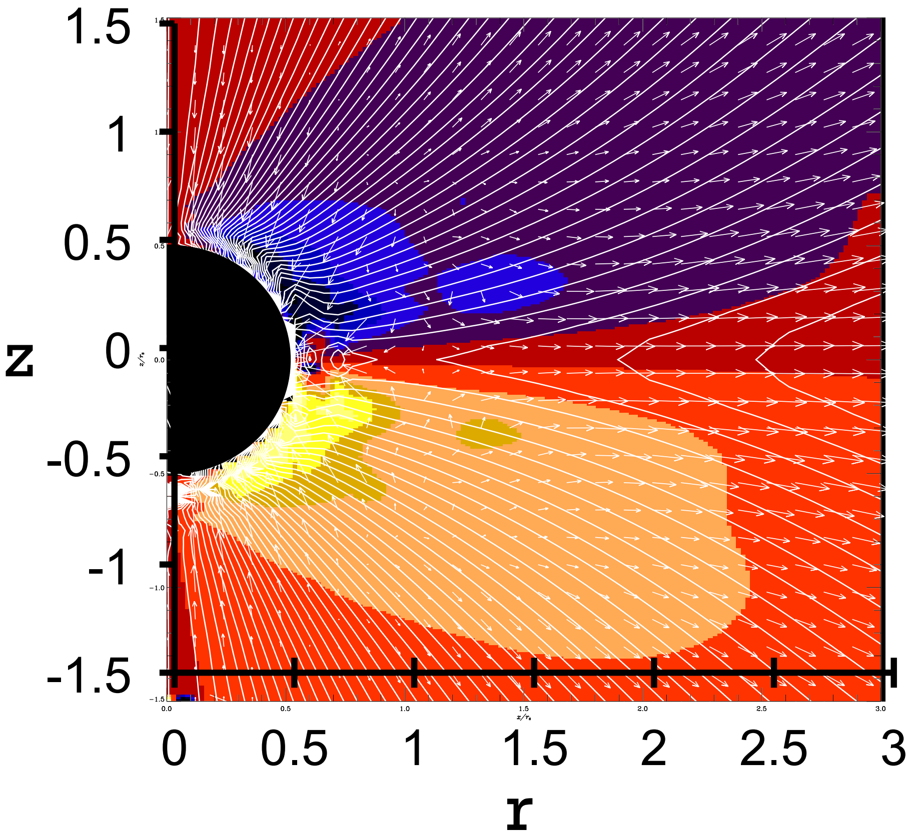


Ideal GRMHD simulation of uniformly magnetized plasma around rotating black hole (Komissarov 2005)

Even in the initial uniform magnetic field case, around the rotating black hole, magnetic reconnection is caused spontaneously and hollow flow from the ergosphere is expected to appear.

Kerr black hole case

$$a = 0.99995$$



Color: azimuthal component of magnetic field, B_ϕ
Solid lines: magnetic field lines
Arrows: velocity of plasma

In Kerr black hole case:
$v_{p,max}$=0.71c (at t=10 τ_s)

In Schwarzschild black hole case:
$v_{p,max}$=0.63c (at t=10 τ_s)

More explosive magnetic reconnection is occurred in Kerr black hole case compared with the case of Schwarzschild black hole case.

Summary

- I present the basis of resistive GRMHD, with respect to plasma dynamics around black holes, the standard equations, and its causality.
- I performed simulations of resistive GRMHD with split-monopole magnetic field around black holes. The primary results showed spontaneous magnetic reconnection.
- I proposed a model, “daruma-otoshi” mechanism of magnetic reconnection around black holes.
- The numerical results suggest even in initial uniform magnetic field case, the magnetic reconnection is caused spontaneously around the rotating black hole.
  In conclusion, in black hole magnetosphere, it is full of magnetic reconnection!

Future plans

- Further calculations and analysis of magnetic reconnection around black holes are needed to confirm the mechanism of the magnetic reconnection near the black hole.
- To perform numerical simulations with longer-term, more realistic magnetic field configuration and clarify importance of magnetic reconnection around astrophysical black holes, numerical code with more precise numerical methods, such as HLLD, are required.
- Radiation should be included in GRMHD simulations. I consider “test EM wave” calculation with resistive GRMHD.
- Comparison between magnetic field configurations suggested by our future precise numerical simulations and forthcoming radio observations with high spatial resolution.