

# *High Energy Emission from Pulsar Magnetospheres*

**Kouichi HIROTANI**

***ASIAA/TIARA-NTHU, Taiwan***

**Workshop on BH Magnetospheres**

**March 1, 2012**

# §1 $\gamma$ -ray Pulsar Observations

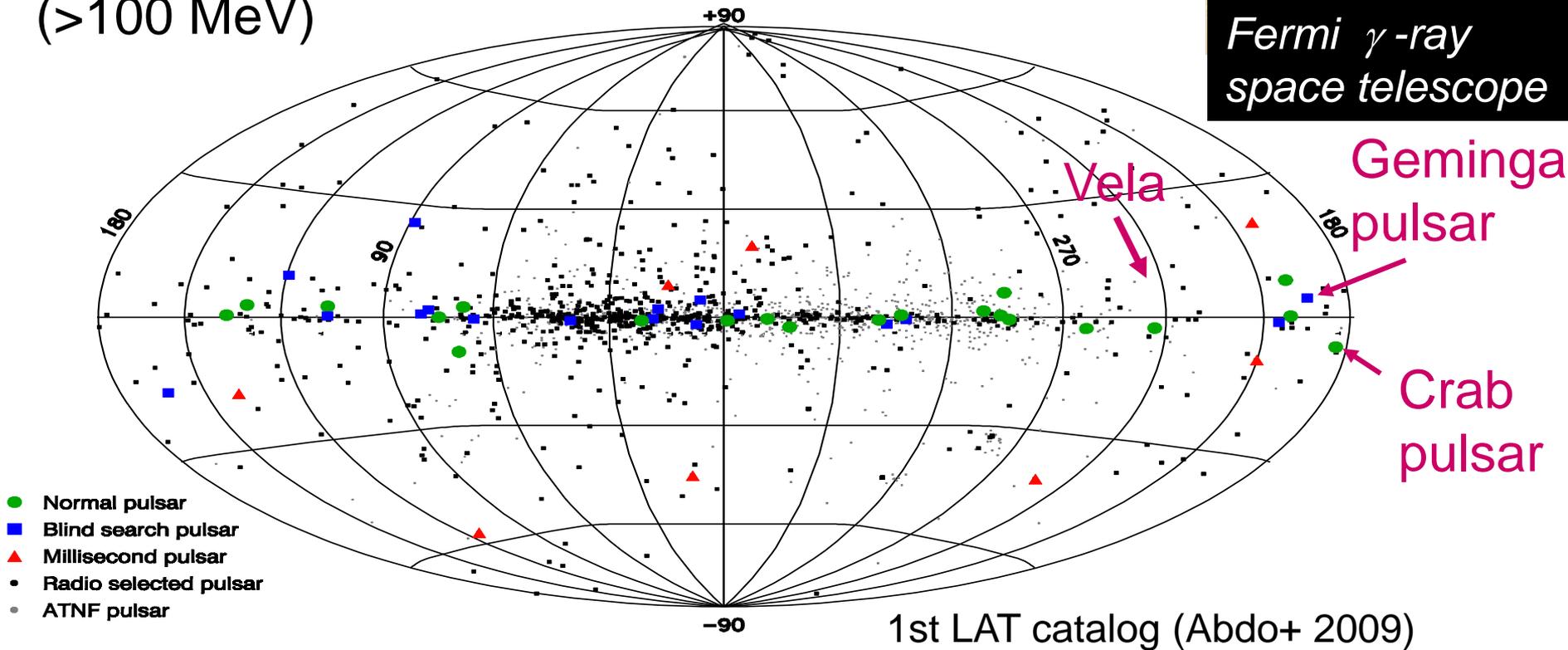
Fermi/LAT has detected more than **121** pulsars above 100 MeV.

Fermi/LAT point sources (>100 MeV)



Large Area Telescope

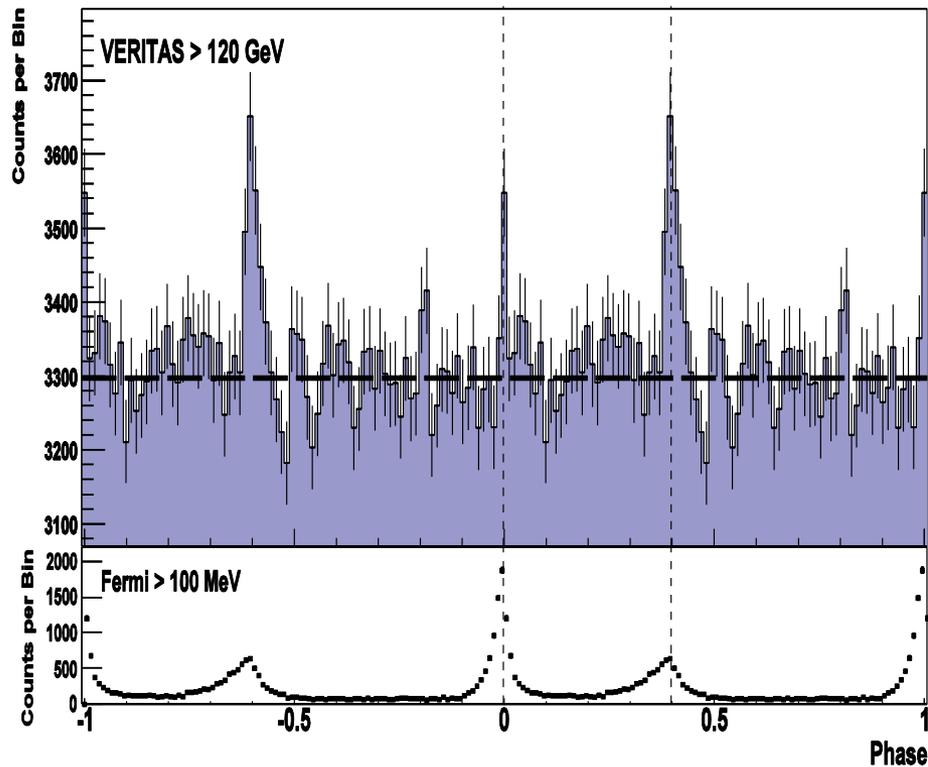
Fermi  $\gamma$ -ray space telescope



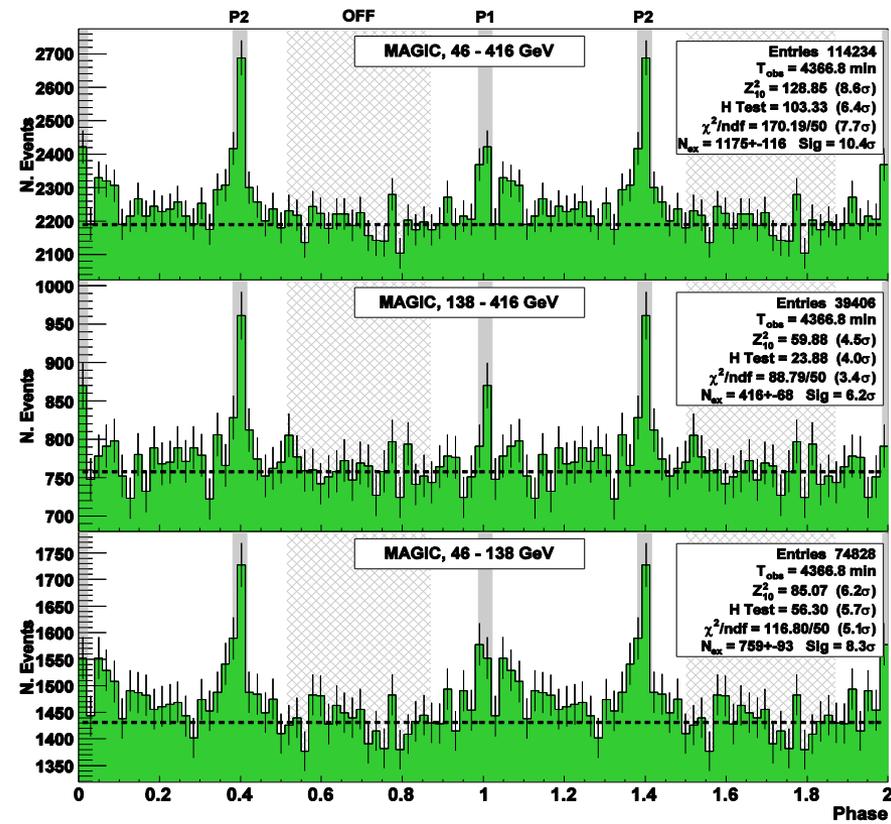
# §1 $\gamma$ -ray Pulsar Observations

Recent IACTs found pulsed emission in 25-400 GeV from the Crab pulsar.

VERITAS ( $> 120$  GeV)  
Aliu+ (2011, Science 334, 69)



MAGIC (25–416 GeV)  
Aleksić+ (2011a,b)



# §1 Introduction

$L_{\text{spin}} (< 10^{39} \text{ erg s}^{-1})$  is dissipated at ...

- Inner magnetosphere

( $r \sim 30 \text{ km}$ )

close to the NS surface

- Outer magnetosphere

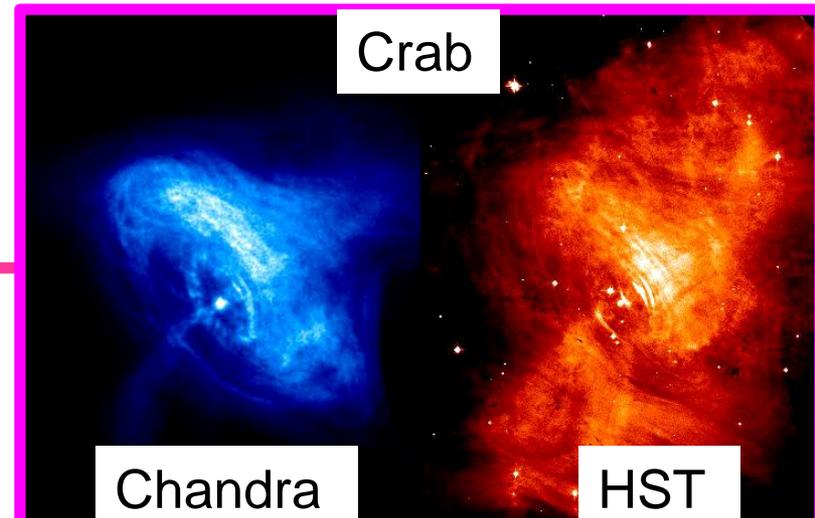
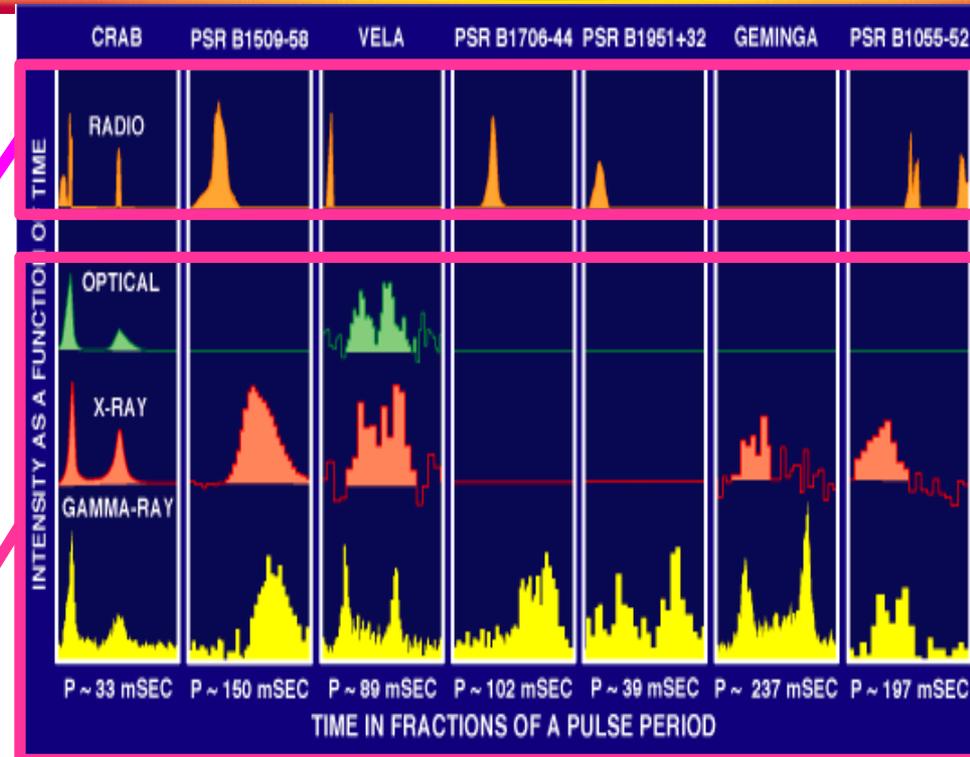
( $r \sim 1000 \text{ km}$ )

near the 'light cylinder'

- Wind region

( $r \sim 10^{-1 \sim 0} \text{ pc}$ )

in the inter-stellar medium



# §1 Introduction

$L_{\text{spin}} (< 10^{39} \text{ erg s}^{-1})$  is  
dissipated at ...

- Inner magnetosphere  $\sim 0.001\%$   
( $r \sim 30 \text{ km}$ )  $\rightarrow$  pulsed radio  
 $E_{\parallel}$  arises along  $\mathbf{B}$  in a  
limited volume near the PC  
surface.
- Outer magnetosphere  $0.1\text{--}20\%$   
( $r \sim 1000 \text{ km}$ )  $\rightarrow$  pulsed X,  $\gamma$ -ray  
 $E_{\parallel}$  is screened much less  
efficiently.
- Wind region  $80\text{--}99.9\%$   
( $r \sim 10^{-1\sim 0} \text{ pc}$ )  $\rightarrow$  PWN emission

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- Wind region  $80\text{--}99.9\%$  (slightly outside the LC)  
( $r \sim \text{pc}$ )  $\rightarrow$  PWN emission

**Pulsed  
emissions**

# §1 Introduction

Today, I compare several representative models and consider a quantitative extension of the OG model in detail.

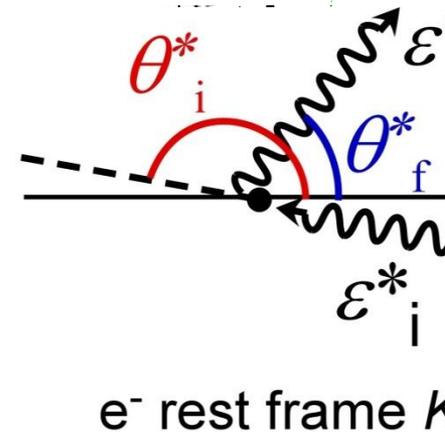
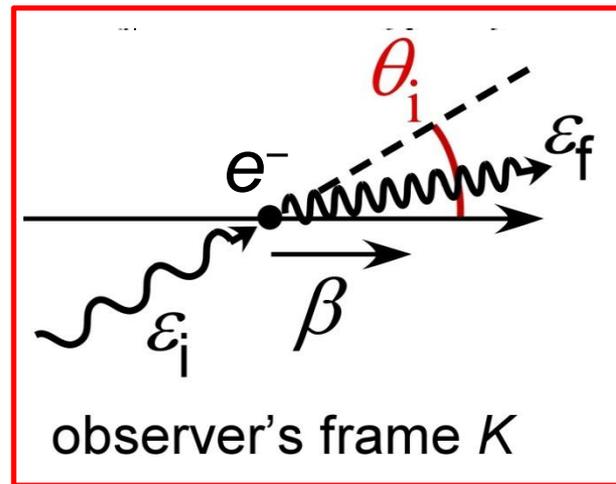
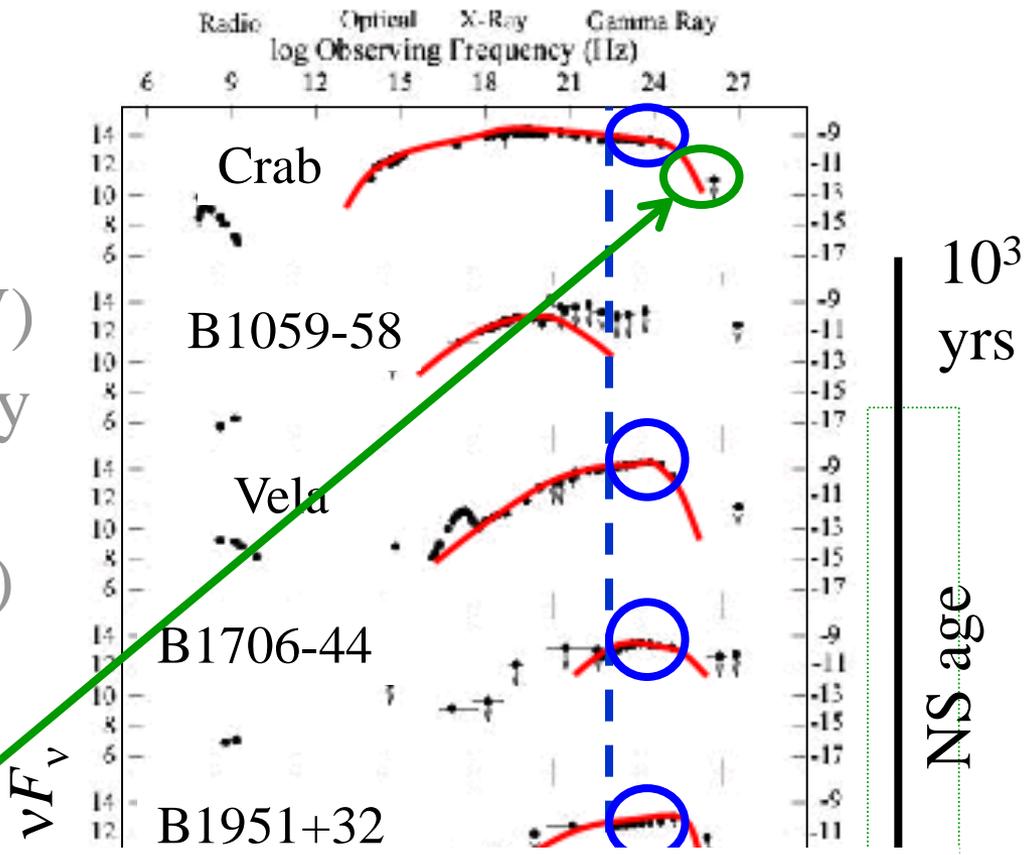
- Inner magnetosphere  $\sim 0.001\%$  Polar-cap (PC) model  
( $r \sim 30$  km)  $\rightarrow$  pulsed radio
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( $r \sim 1000$  km)  $\rightarrow$  pulsed X,  $\gamma$ -ray Slot-gap (SG) model  
Pair-starved PC model
- Wind region  $80\text{--}99.9\%$   
( $r \sim 10^{-1\sim 0}$  pc)  $\rightarrow$  PWN emission



# Broad-band spectra (pulsed)

● High-energy ( $> 100\text{MeV}$ ) photons are emitted mainly via **curvature** process by ultra-relativistic ( $\sim 10\text{TeV}$ )  $e^\pm$ 's accelerated in pulsar magnetosphere.

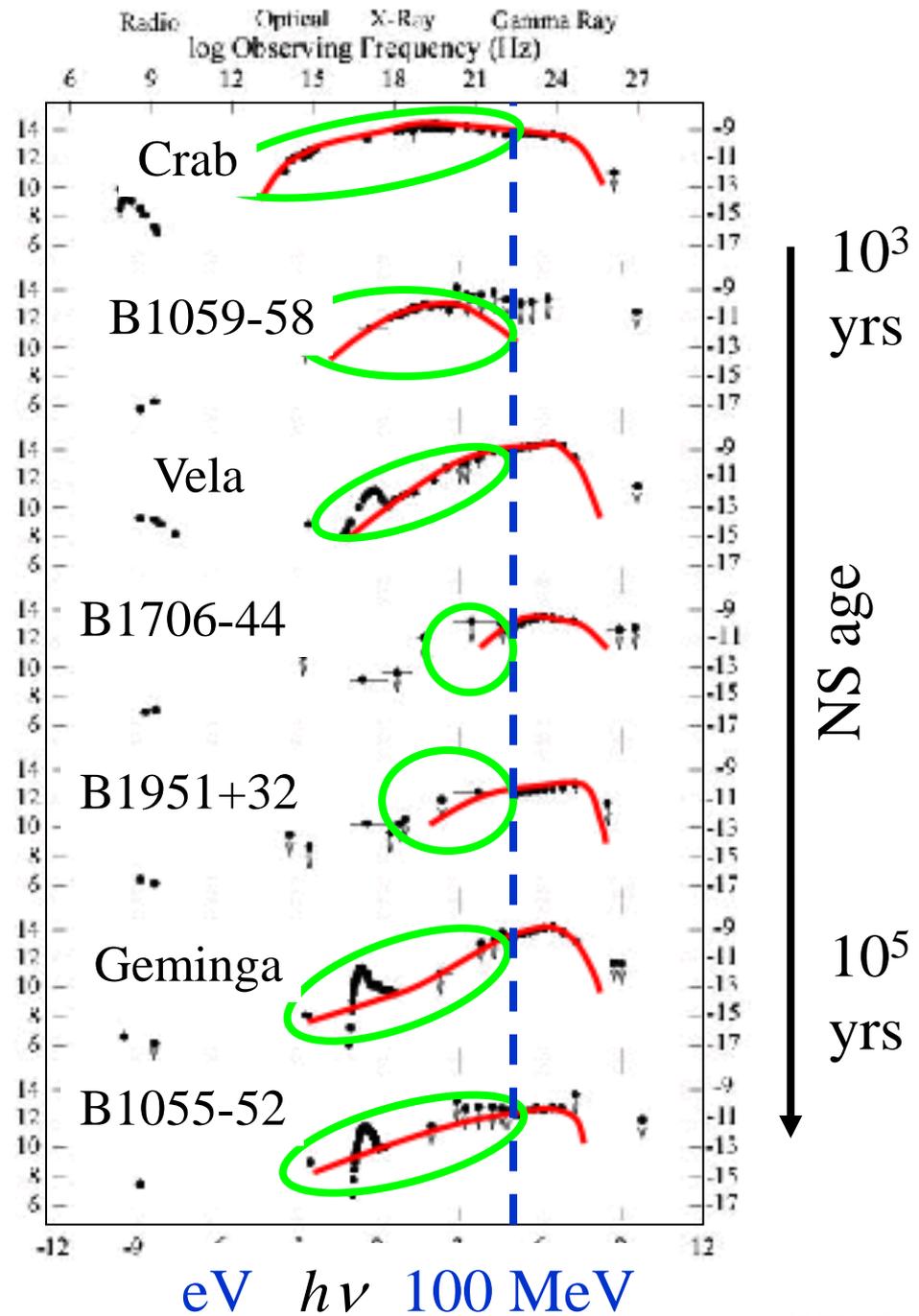
● Above  $20\text{GeV}$ , **ICS** by **secondary/tertiary pairs** contributes.



# Broad-band spectra (pulsed)

● High-energy ( $> 100\text{MeV}$ ) photons are emitted mainly via **curvature** process by ultra-relativistic ( $\sim 10\text{TeV}$ )  $e^\pm$ 's accelerated in pulsar magnetosphere.

● Some of the **primary  $\gamma$ -rays** are absorbed in the NS magnetosphere and **reprocessed** in lower energies via **synchrotron** process.



# *§1 Introduction*

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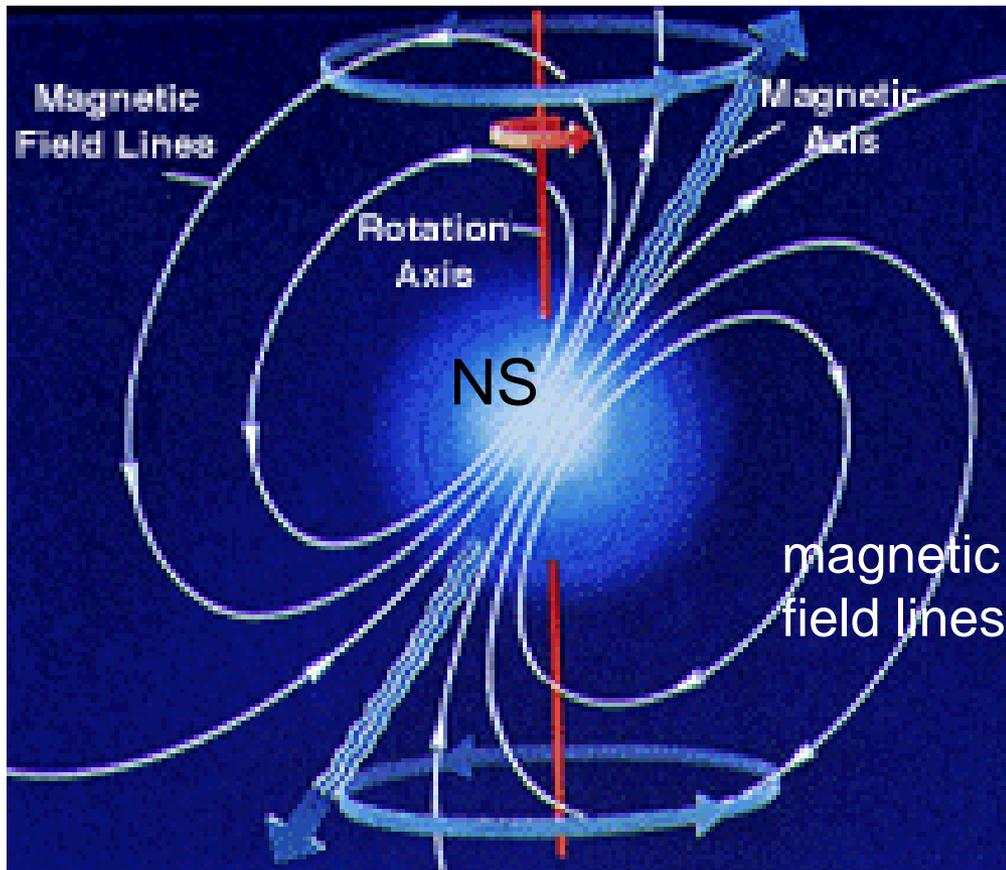
Such high-energy emissions are realized when the rotational energy of the NS is electro-dynamically extracted and partly dissipated in the inner and/or outer magnetosphere.

(e.g., unipolar inductor)

## §2 Rotating NS Magnetosphere

### Pulsars:

rapidly rotating, highly magnetized neutrons stars (NS)



Magnetic and rotation axes are misaligned.

Pulsar turns on and off as the beam sweeps our line of sight  
(e.g., lighthouse).

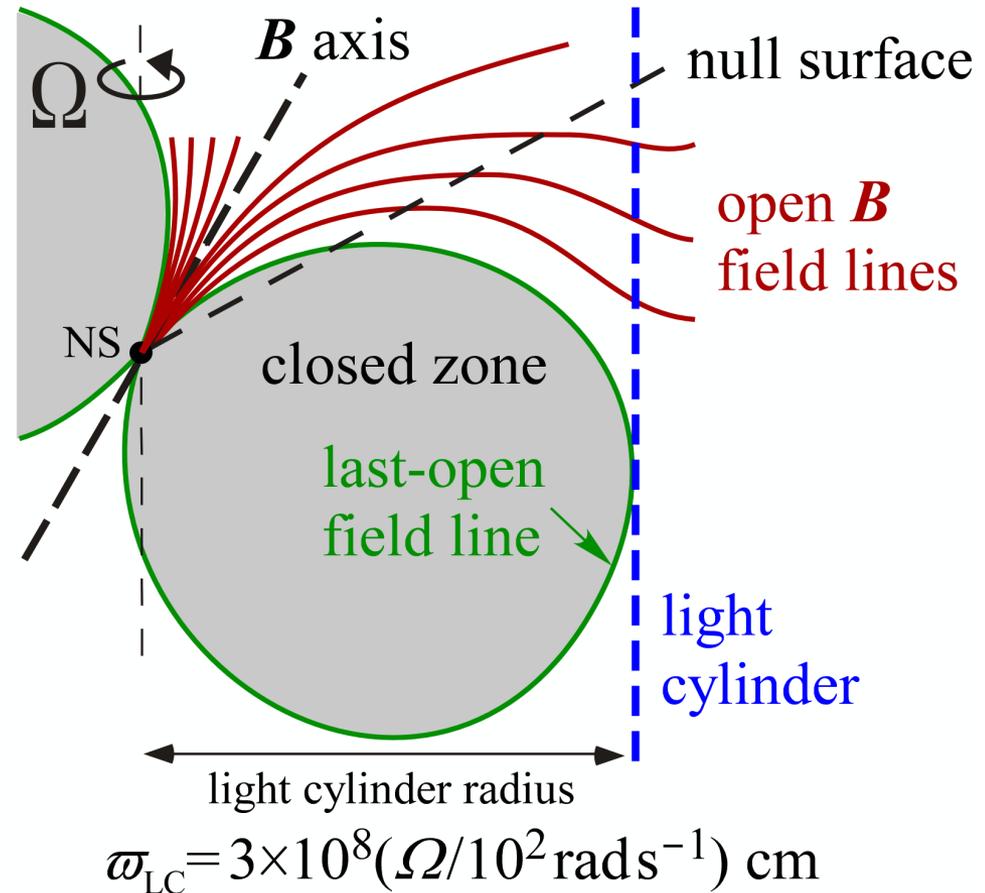
## §2 Rotating NS Magnetosphere

A rotating NS magnetosphere can be divided into **open** and **closed zones**.

**Last-open field lines** form the boundary of them.

In the open zone,  $e^\pm$ 's escape through the **light cylinder** as a pulsar wind.

In the closed zone, on the other hand, an  $E_{\parallel}$  would be very quickly screened by the dense plasmas.



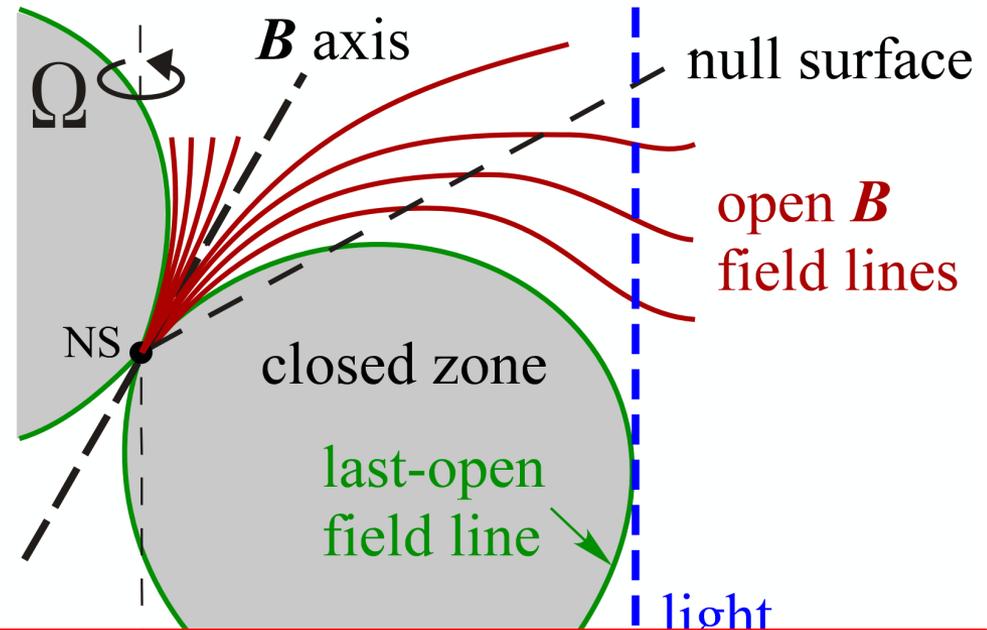
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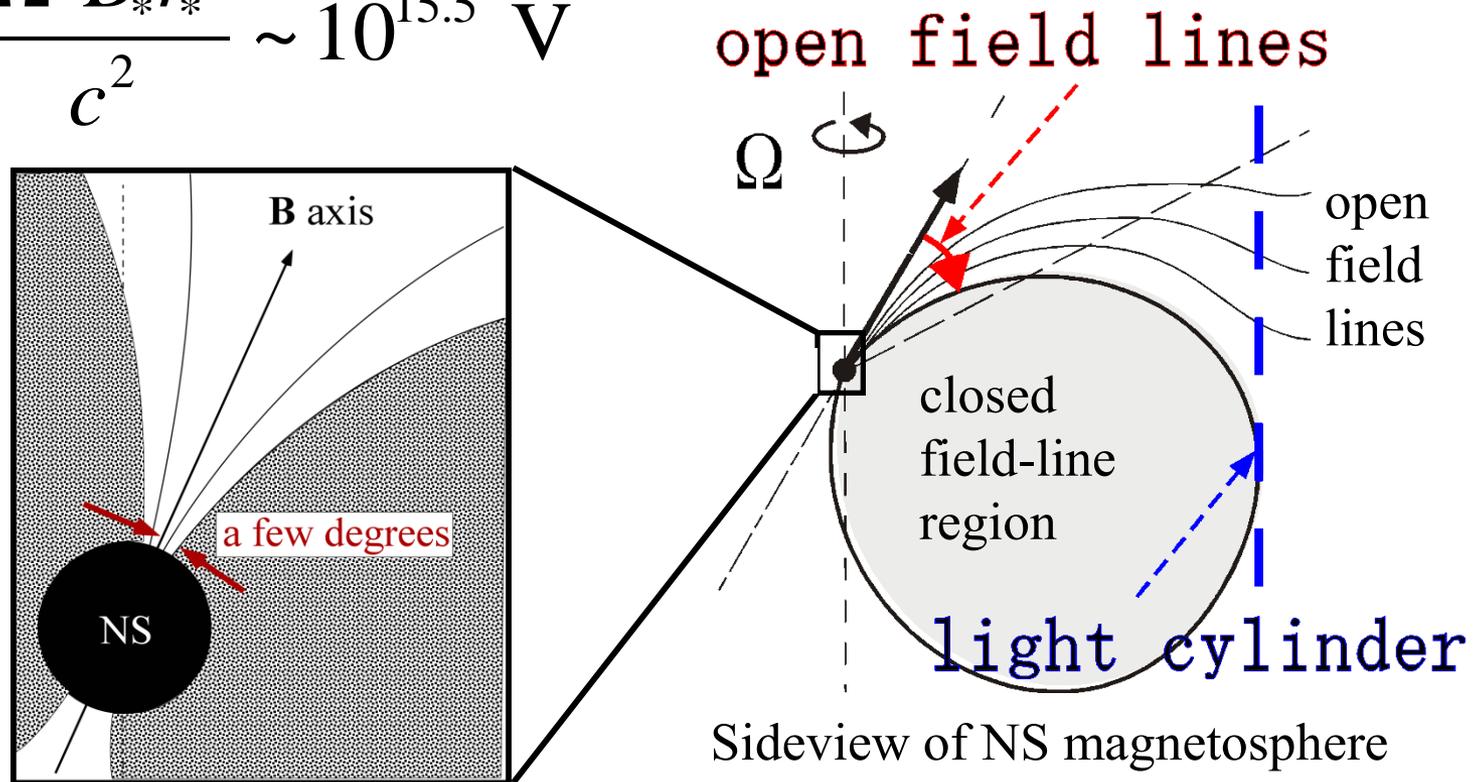
Thus, in all pulsar emission models, particle acceleration takes place only within the open zone.

## §2 Rotating NS Magnetosphere

For typical high-energy pulsars, **open zone** occupies only a few degrees from **B** axis on the PC surface.

Available voltage in the open zone:

$$\text{EMF} \sim \frac{\Omega^2 B_* r_*^3}{c^2} \sim 10^{15.5} \text{ V}$$



## §2 Rotating NS Magnetosphere

In a rotating NS magnetosphere, the **Goldreich-Julian charge density** is induced for a static observer. The inhomogeneous part of Maxwell eqs. give

$$\nabla \cdot \mathbf{E}_{\parallel} = 4\pi(\rho - \rho_{\text{GJ}}),$$

where  $\mathbf{E}_{\parallel} \equiv \mathbf{E} \cdot \mathbf{B}$ ,  $\rho \equiv e(n_+ - n_-)$  and

$$\rho_{\text{GJ}} \equiv \frac{1}{4\pi} \nabla \cdot \mathbf{E}_{\perp} = -\frac{\boldsymbol{\Omega} \cdot \mathbf{B}}{2\pi c}.$$

Thus,  $E_{\parallel}$  arises if  $\rho \neq \rho_{\text{GJ}}$ .

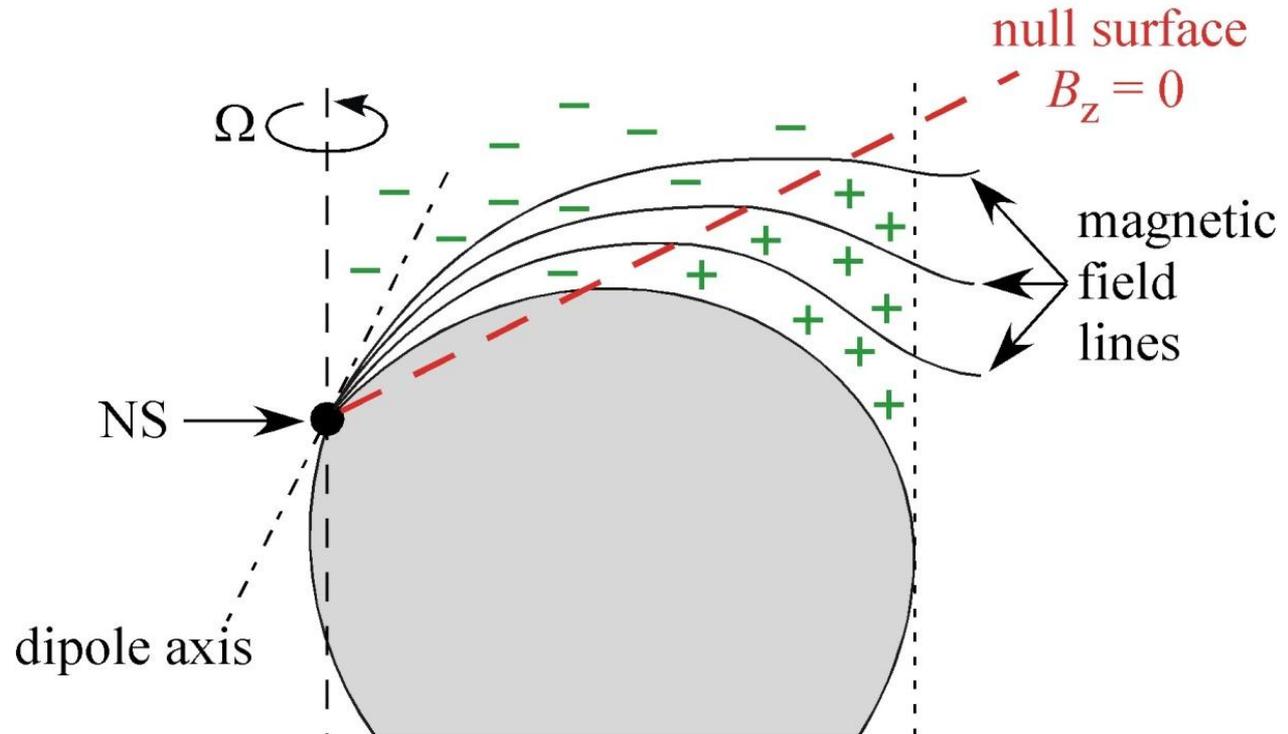
Note that  $\rho_{\text{GJ}}$  is uniquely determined by  $\mathbf{B}$ -field geometry. For example, it changes sign at the so-called ‘null-charge surface’.

## §2 Rotating NS Magnetosphere

Note that  $\rho_{\text{GJ}}$  is uniquely determined by  $\mathbf{B}$ -field geometry. For example, it changes sign at the so-called ‘null-charge surface’.

$$\rho_{\text{GJ}} \equiv \frac{1}{4\pi} \nabla \cdot \mathbf{E}_{\perp}$$

$$= -\frac{\boldsymbol{\Omega} \cdot \mathbf{B}}{2\pi c}.$$



$$\rho_{\text{GJ}} \equiv \frac{c^2}{4\pi\sqrt{-g}} \partial_{\mu} \left[ \frac{\sqrt{-g}}{\rho_w^2} g^{\mu\nu} g_{\phi\phi} (\Omega_F - \omega) F_{\phi\nu} \right]$$

## §3 *Traditional Emission Models*

Next question:

**Where** is the particle accelerator, in which  $E_{\parallel}$  arises?

In this section, we geometrically consider three representative pulsar **high-energy emission models**:

(historical order)

1. Inner-gap (or polar-cap) model, 1982-1990's
2. Outer-gap model, 1986-present
3. Slot-gap model 2003-2009

Note: Polar-cap model still survives as a phenomenological model of coherent radio emissions.

# §3 Traditional Emission Models

Schematic view of the three representative emission models:

- **polar gap (inner gap)**

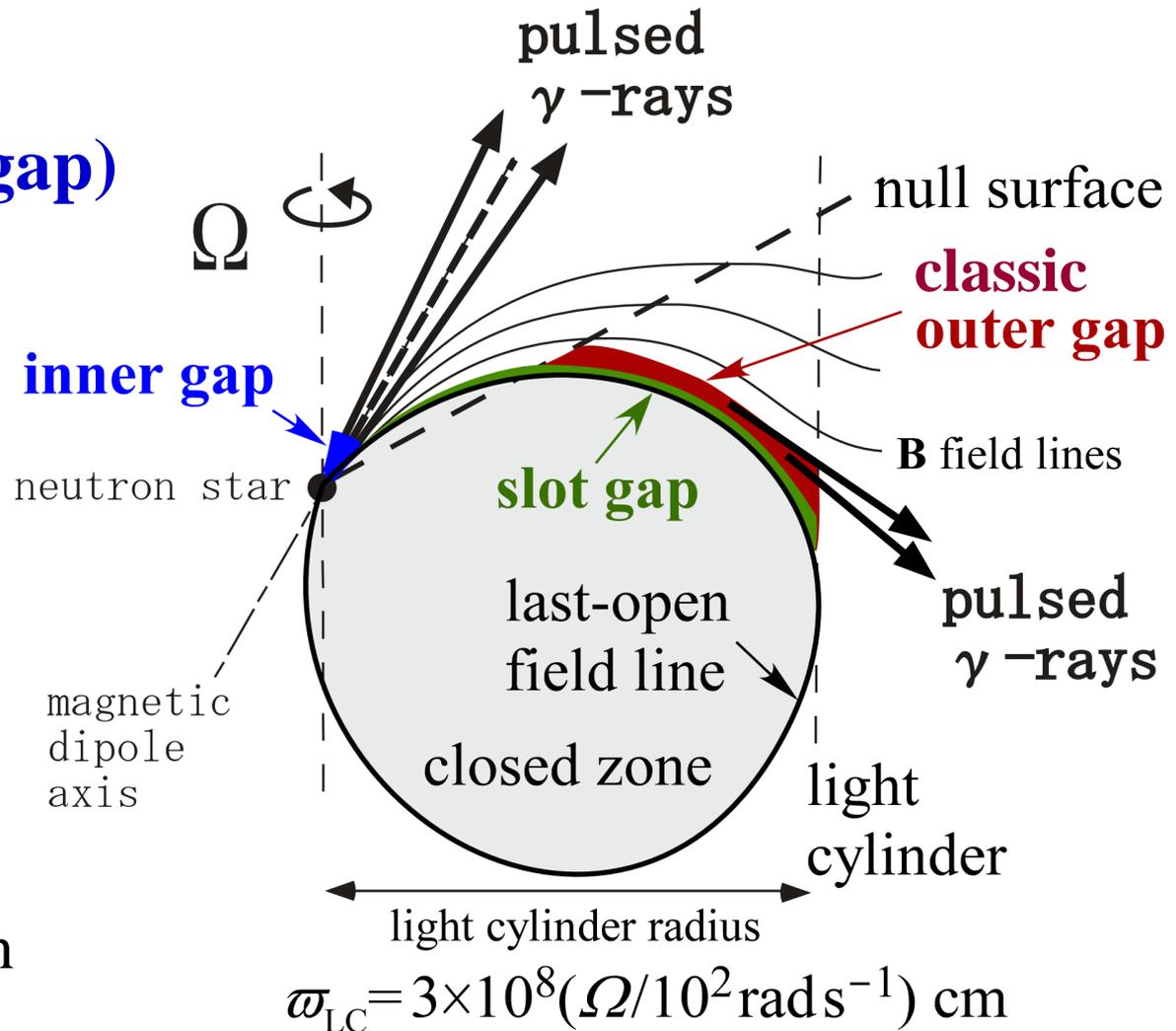
(Daugherty & Harding  
1996 ApJ 458, 278)

- **slot gap**

(Arons  
1983, ApJ 302, 301;  
Muslimov & Harding  
2004, ApJ 606, 1143)

- **outer gap**

(Cheng, Ho, Ruderman  
1986, ApJ 300, 500)

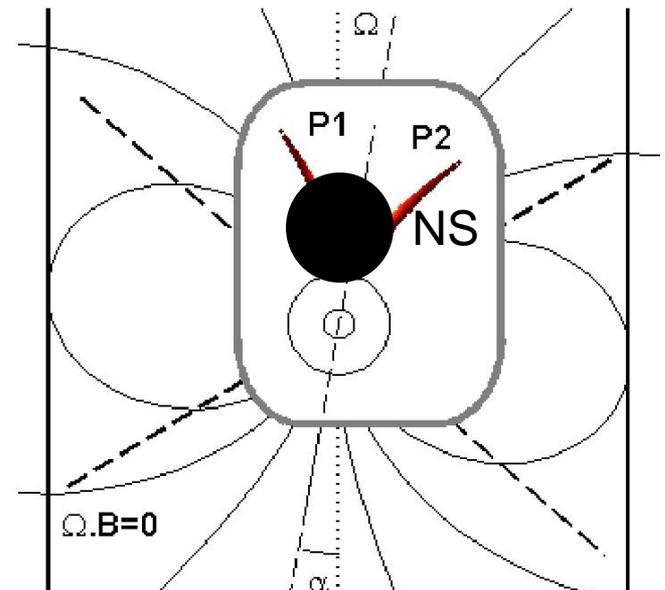
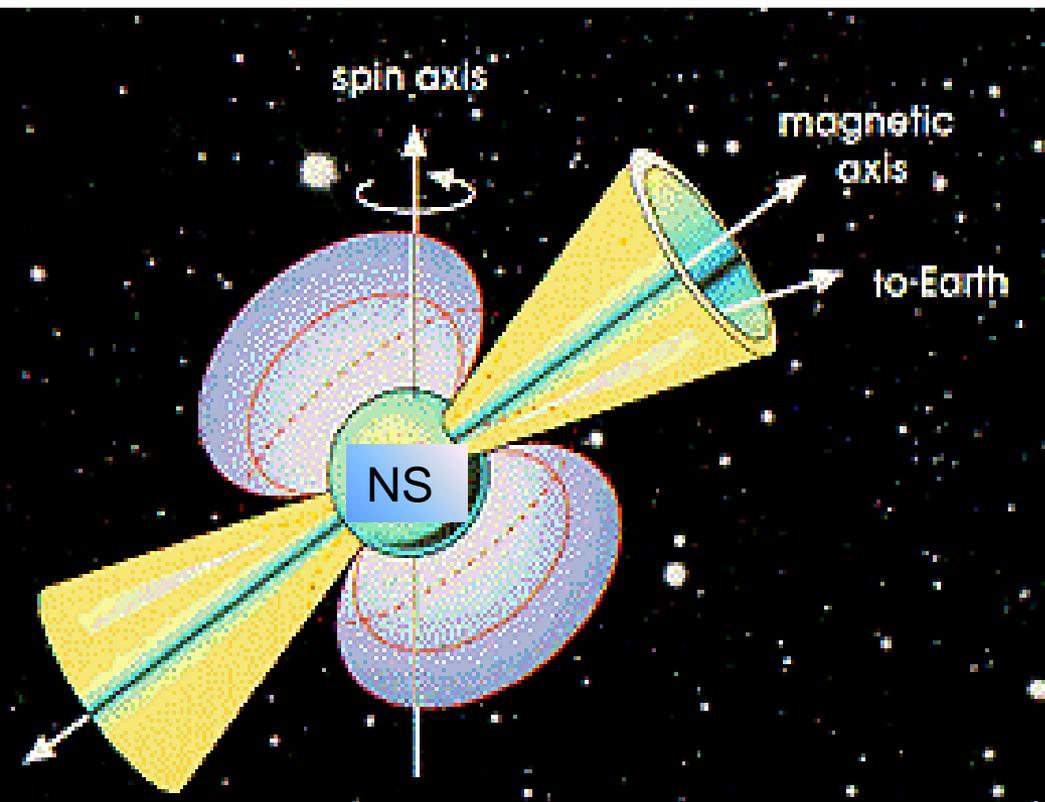


## §3 Traditional Emission Models

Early 80's, the **polar-cap (PC) model** was proposed.

(Daugherty & Harding ApJ 252, 337, 1982)

Emission altitude  $< 3r_{\text{NS}}$   $\longrightarrow$  pencil beam ( $\Delta\Omega \ll 1$  ster)



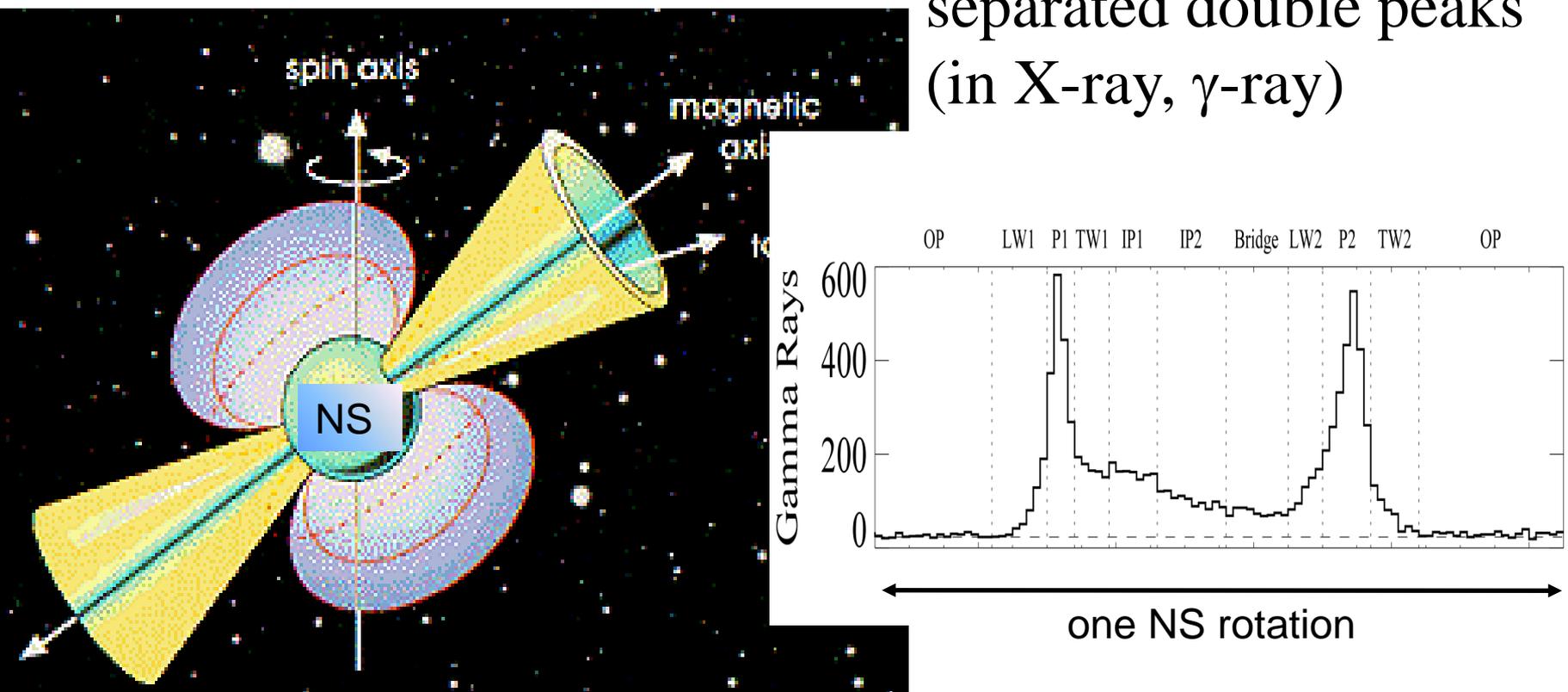
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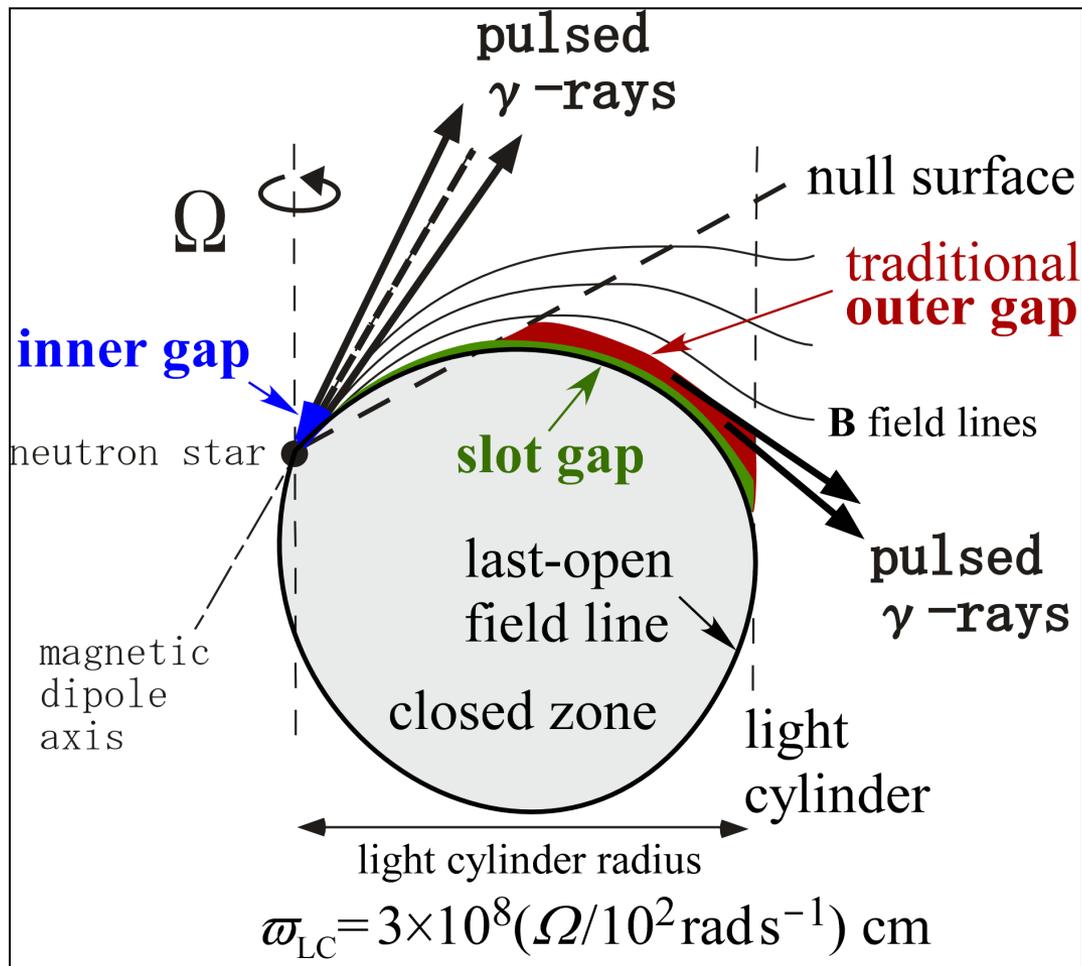
Thus, a **high-altitude emission model** drew attention.

# §3 Traditional Emission Models

Mid 80's, the **outer-gap (OG) model** was proposed.

(Cheng, Ho, Ruderman ApJ 300, 500, 1986)

Emission altitude  $> 100 r_{\text{NS}}$   $\longrightarrow$  hollow cone emission  
( $\Delta\Omega > 1$  ster)



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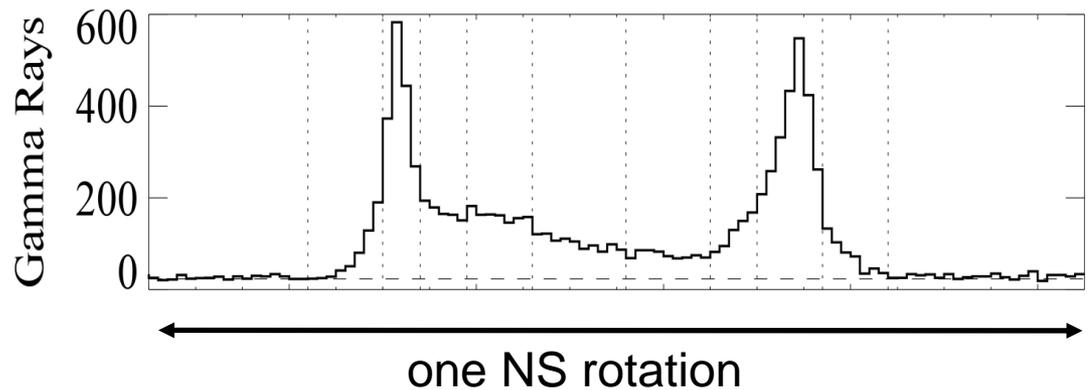
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Emission altitude  $> 100 r_{\text{NS}}$   $\longrightarrow$  hollow cone emission  
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Mid 90s', the outer-gap model was further developed by taking account of **special relativistic effects**.

(Romani ApJ 470, 469)

$\longrightarrow$  Explains wide-separated double peaks.

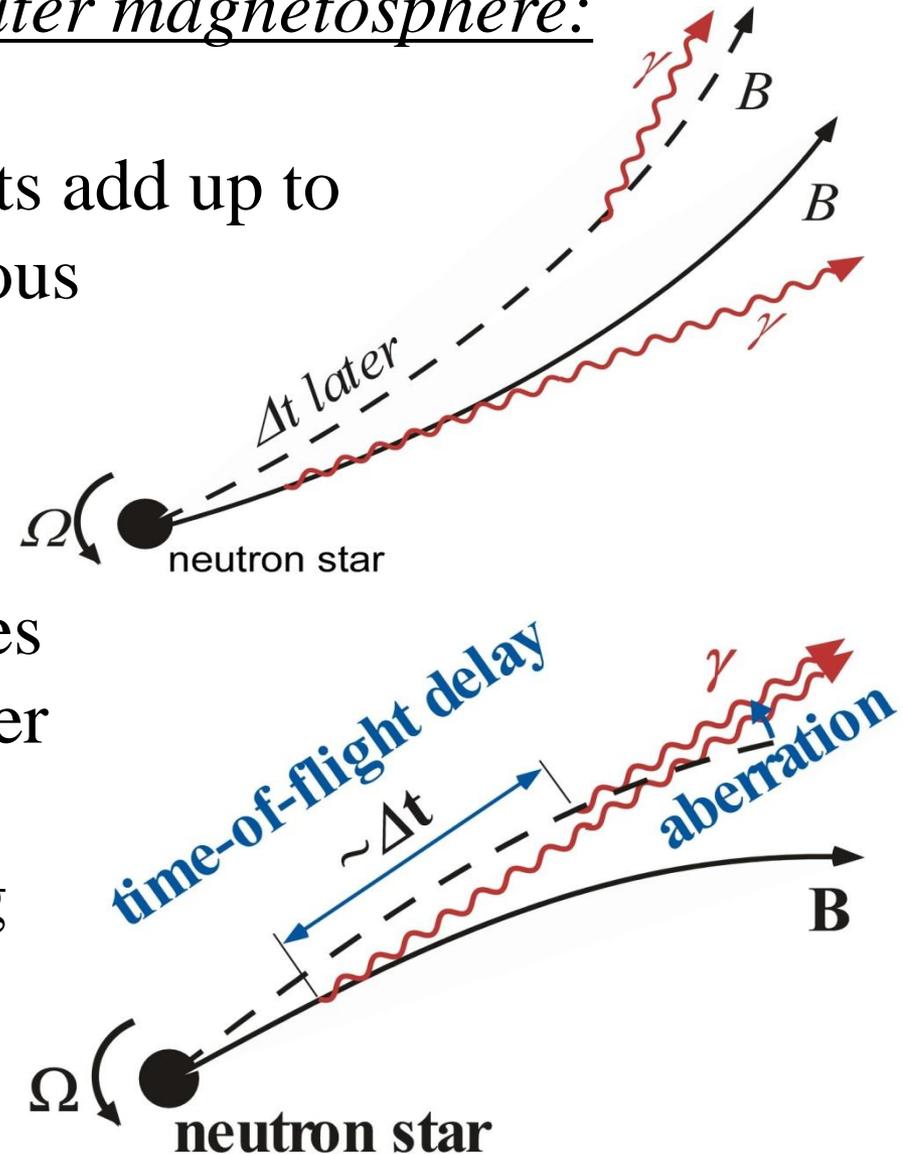


# §3 Traditional Emission Models

## Special relativistic effects in outer magnetosphere:

On the **leading** side, phase shifts add up to **spread** photons emitted at various altitudes over  $140^\circ$  in phase.

On the **trailing** side, photons emitted earlier at lower altitudes catch up with those emitted later at higher altitudes to **focus** in a small phase range  $30^\circ$ , forming caustics (strong intensity) in the phase plot.



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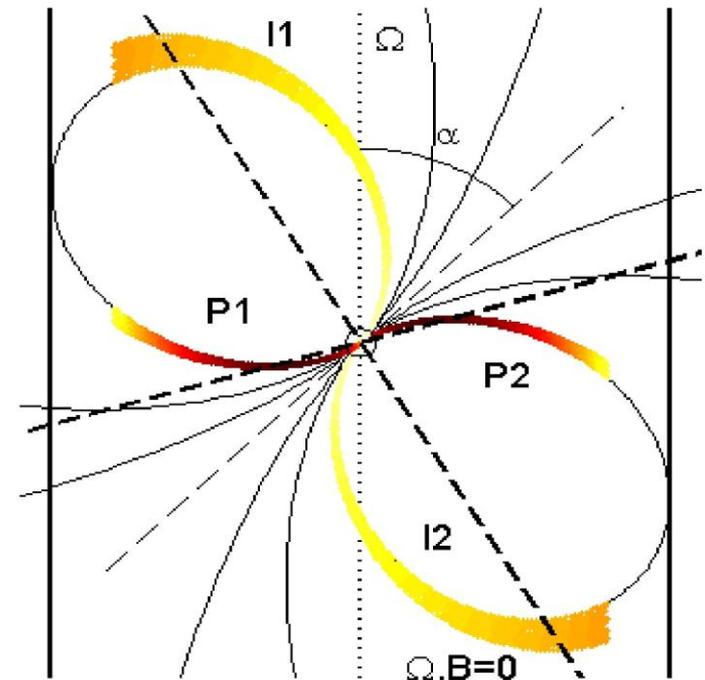
Outer-gap model became promising.

## §3 *Traditional Emission Models*

Early 00's, an alternative mode, **slot-gap model**, was proposed. (Muslimov & Harding ApJ 588, 430, 2003)

They revisited the original idea of Arons (1983), extending his lower-altitude slot-gap model into the higher altitudes (by hand).

Due to special relativistic effects, wide-separated double peaks also appear, in the same way as in the outer-gap model (although the peak formation mechanism is slightly different).



## §3 *Traditional Emission Models*

However, a SG, which is geometrically thin, cannot reproduce the observed  $\gamma$ -ray fluxes.

In the Crab pulsar ( $\Omega=190 \text{ rad s}^{-1}$ ):

KH (2008) ApJ 688, L25

For **OG** model ( $f\sim 0.14$ ,  $\kappa\sim 0.3$ ,  $\mu=4\times 10^{30} \text{ G cm}^3$ ),

$$(\nu F_\nu)_{\text{peak}} \sim 4\times 10^{-4} \text{ MeV s}^{-1} \text{ cm}^{-2} \sim \text{Fermi flux.}$$

For **SG** model ( $f\sim 0.04$ ,  $\kappa\sim 0.2$ , large  $\mu$ ),

$$(\nu F_\nu)_{\text{peak}} \sim 3\times 10^{-5} (\mu/8 \times 10^{30})^2 \text{ MeV s}^{-1} \text{ cm}^{-2} \\ < 0.1 \text{ Fermi flux.}$$

For other pulsars, the thin SG model predicts further smaller fluxes than (typically  $< 1\%$  of) observed.

## §3 *Traditional Emission Models*

However, a SG, which is geometrically **thin**, cannot reproduce the observed  $\gamma$ -ray fluxes.

In fact, classic **thin** OG model has the same difficulty.

On these grounds, **all** the pulsar emission **models** adopt transversely **thick** accelerator geometry.

\_\_\_\_\_ occupying a good fraction of open zone

## §3 *Traditional Emission Models*

To contrive a geometrically thick gap, the **pair-starved PC (PSPC) model** was proposed (Venter + 2009 ApJ 707, 800).

Large  $e^\pm$  production (by extracted  $e^-$ 's)  $\rightarrow$  SG

Medium  $e^\pm$  production (by self-sustained  $e^\pm$ 's)  $\rightarrow$  OG

Small  $e^\pm$  production (by extracted  $e^-$ 's)  $\rightarrow$  PSPC

Both OG and PSPC models can produce the observed  $\gamma$ -ray fluxes.

## §3 *Traditional Emission Models*

Two major high-energy emission models:

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Outer-gap model

Arise because of pair starvation in outer  $\mathbf{B}$ -sphere.

Pairs produced by  $\gamma\gamma \rightarrow ee$  and discharge by  $E_{\parallel} > 0$ .

$e^+$ 's accelerated outwards.

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Pair-starved PC model

Arise because of pair starvation near NS surface.

$e^-$ 's extracted from NS surface by  $E_{\parallel} < 0$ .

$e^-$ 's accelerated outwards.

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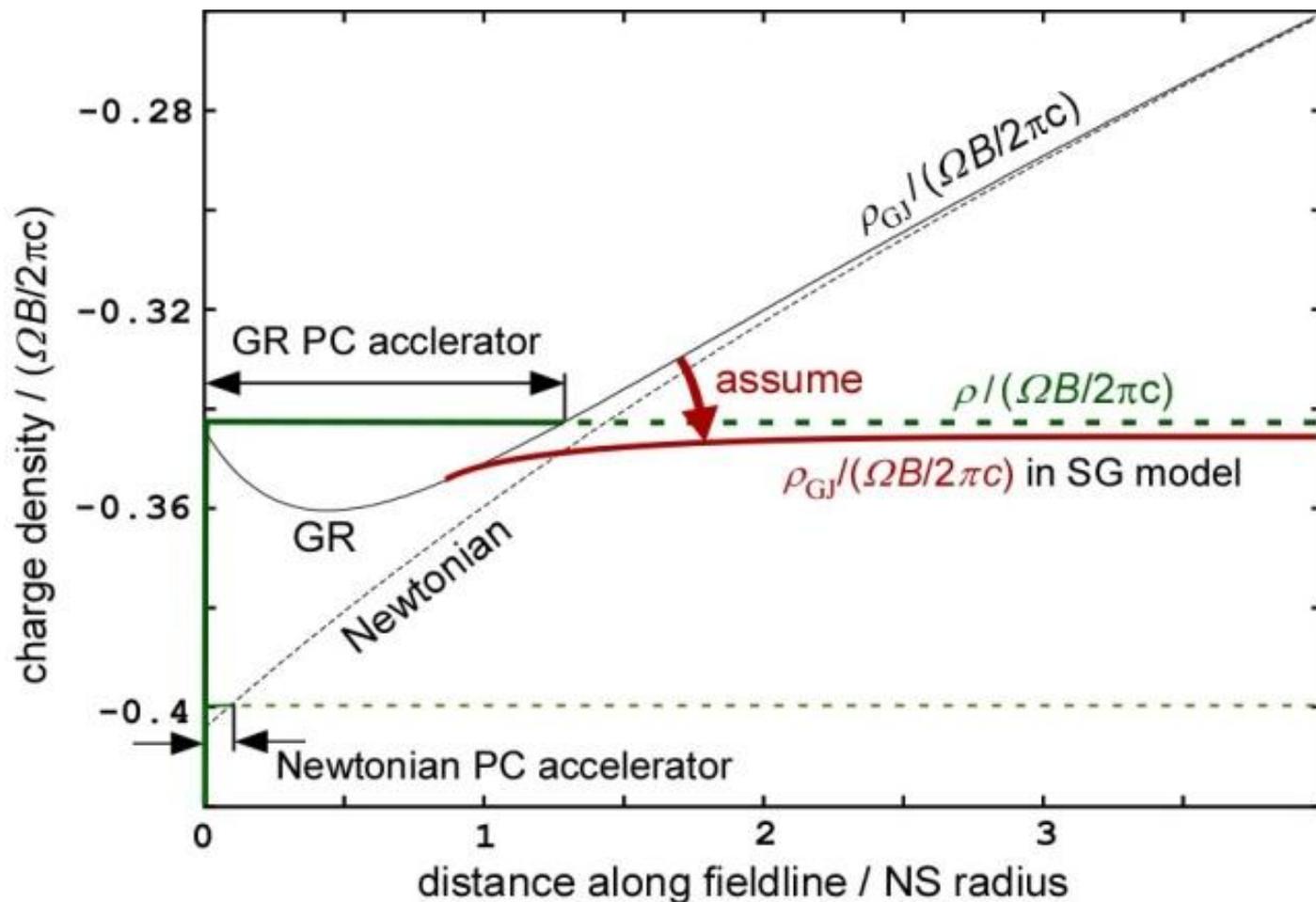
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However, there is a serious electro-dynamical inconsistency in the PSPC model

(KH 2011, in HE emission from PSRs and their systems, p. 117).

## §5 Difficulties in SG Model: Unphysical GJ Charge

To prevent a sign reversal of  $E_{\parallel}$ , they assumed that the GJ charge density (per  $B$  flux) tends to a constant in the higher altitudes



## §3 *Traditional Emission Models*

Two major high-energy emission models:

Outer-gap model

Arise because of pair starvation in outer **B**-sphere.

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$e^+$ 's accelerated outwards.

Pair-starved PC model

Arise because of pair starvation near NS surface.

$e^-$ 's extracted from NS surface by  $E_{\parallel} < 0$ .

$e^-$ 's accelerated outwards.

So, we concentrate on the OG emission in this talk.

## §3 *Traditional Emission Models*

Various attempts have been made on recent OG model:

3-D geometrical model

- phase-resolved spectra (Cheng + '00; Tang + '08)
- atlas of light curves for PC, OG, SG models  
(Watters + '08)

2-D self-consistent solution (Takata + '06; KH '06)

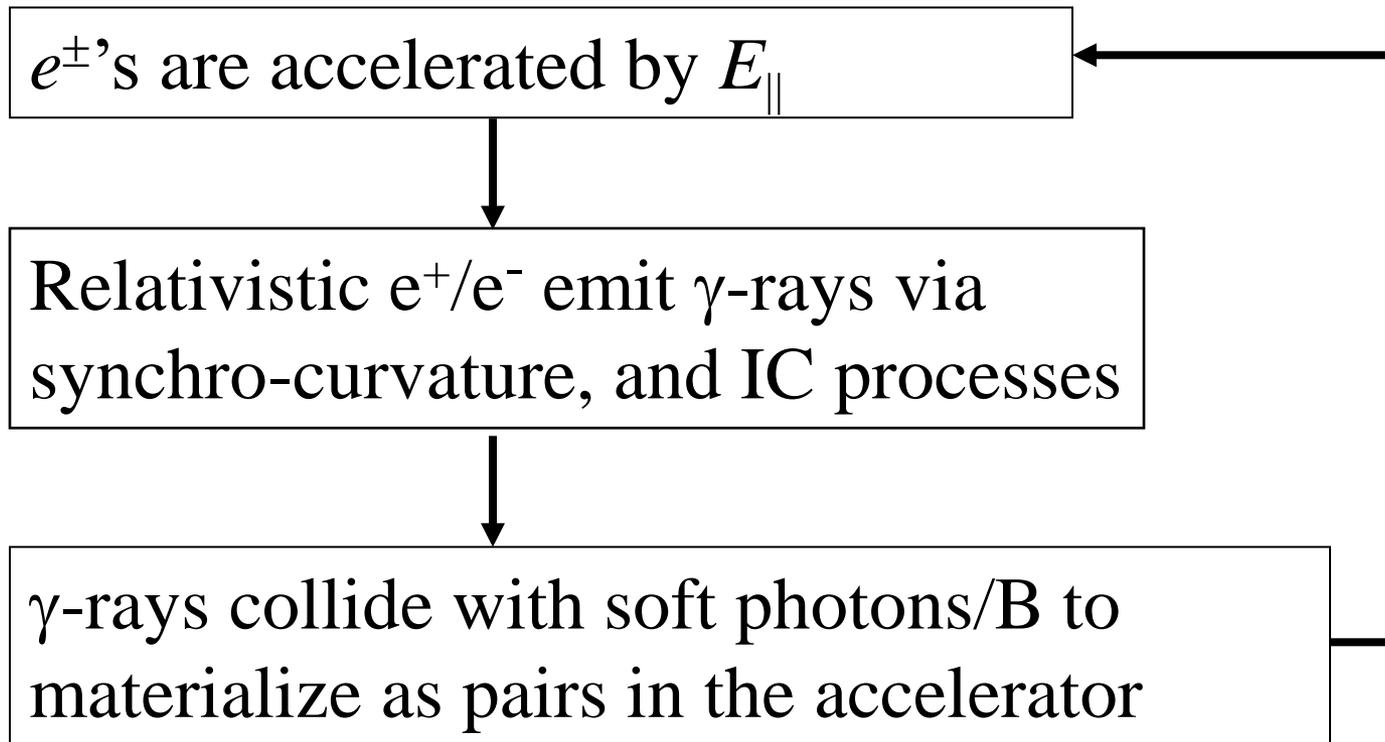
3-D self-consistent solution

- phase-resolved spectra, absolute luminosity  
if we give only  $P$ ,  $dP/dt$ ,  $\alpha$ ,  $kT$  (+ $\zeta$ ) (this talk)

In this talk, I'll present the most recent results obtained in my 3-D version of self-consistent OG calculations.

## §4 Modern Outer-gap Model

Self-sustained pair-production cascade in a rotating NS magnetosphere:



## §4 Modern Outer-gap Model

The Poisson equation for the electrostatic potential  $\psi$  is given by

$$-\nabla^2\psi = 4\pi(\rho - \rho_{\text{GJ}}),$$

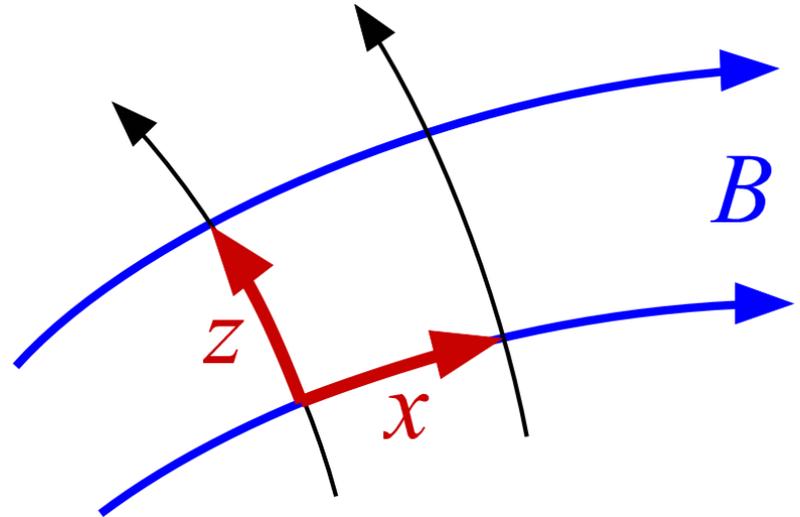
where

$$E_{\parallel} \equiv -\frac{\partial\Psi}{\partial x}, \quad \rho_{\text{GJ}} \equiv -\frac{\mathbf{\Omega}\cdot\mathbf{B}}{2\pi c},$$

$$\rho \equiv e \int_0^{\infty} d\mathbf{p}^3 [N_+(\mathbf{x}, \mathbf{p}) - N_-(\mathbf{x}, \mathbf{p})] + \rho_{\text{ion}}.$$

$N_+/N_-$ : distrib. func. of  $e^+/e^-$

$\mathbf{p}$ : momentum of  $e^+/e^-$



## §4 Modern Outer-gap Model

Assuming  $\partial_t + \Omega \partial_\phi = 0$ , we solve the  $e^\pm$ 's Boltzmann eqs.

$$\frac{\partial N_\pm}{\partial t} + \vec{v} \cdot \nabla N_\pm + \left( e \vec{E}_\parallel + \frac{\vec{v}}{c} \times \vec{B} \right) \cdot \frac{\partial N_\pm}{\partial \vec{p}} = S_{IC} + S_{SC} + \int \alpha_\nu d\nu \int \frac{I_\nu}{h\nu} d\omega$$

together with the radiative transfer equation,

$$\frac{dI_\nu}{dl} = -\alpha_\nu I_\nu + j_\nu$$

$N_\pm$ : positronic/electronic spatial # density,

$E_\parallel$ : magnetic-field-aligned electric field,

$S_{IC}$ : ICS re-distribution function,  $d\omega$ : solid angle element,

$I_\nu$ : specific intensity,  $l$ : path length along the ray

$\alpha_\nu$ : absorption coefficient,  $j_\nu$ : emission coefficient

## §4 Modern Outer-gap Model

Specify the three parameters: (period  $P$  is known)

- magnetic inclination (e.g.,  $\alpha_{\text{inc}}=45^\circ, 75^\circ$ ),
- magnetic dipole moment of NS (e.g.,  $\mu=4\times 10^{30}\text{G cm}^3$ )
- neutron-star surface temperature (e.g.,  $kT_{\text{NS}}=50\text{ eV}$ )

Solve Poisson eq. + Boltzmann eqs + radiative transf. eq.

I first solved (in **6-D** phase space)

- 3-D gap geometry,
  - acceleration electric field distribution,  $E_{\parallel}$ ,
  - particle density and energy spectrum,
  - photon specific intensity ( $\rightarrow$  predicts  $\gamma$ -ray properties),
- by specifying these three parameters, assuming  $\mathbf{B}$ -field structure by vacuum rotating dipole solution (Cheng + '00).

## §5 *Gamma-ray and X-ray Luminosities of MSPs*

Wang & KH (2011), ApJ 736, 127

Let us first examine **millisecond pulsars**.

Before presenting numerical results, we first analytically derive the **death line** of MSPs on  $(P, \dot{P})$  plane..

For MSPs, **pair-production threshold** constrains the death line. Typical Lorentz factor of in-falling  $e^-$ 's at PC:

$$\gamma \approx 2.5 \times 10^7 P^{1/3} \left[ \ln \left( \frac{r_{\text{null}}}{r_*} \right) \right]^{-1/3}$$

Equating  $\sigma T_{\text{max}}^4$  with the energy flux deposited by the bombardment, we obtain the maximum temperature of the heated PC surface:

$$kT_{\text{max}} \approx 426 \text{ eV } \gamma^{1/4} P^{-1/4} \mu_{30}^{1/4}$$

## §5 Gamma-ray and X-ray Luminosities of MSPs

Wang & KH (2011), ApJ 736, 127

Thus, thermal photons will have energies around keV,

$$h\nu_X \approx 3kT_{\max} \approx 0.0023 m_e c^2 P^{-1/6} \mu_{30}^{1/4} \quad (\sim \text{keV})$$

While the curvature  $\gamma$ -rays have energies around GeV,

$$h\nu_\gamma \approx 160 m_e c^2 P^{-7/4} \mu_{30}^{3/4} \quad (\sim \text{a few GeV})$$

Therefore, the pair-production threshold,

$h\nu_X \times h\nu_\gamma > (m_e c^2)^2$  gives the **death line**,

$$\lg P > -14.1 + 2.83 \lg P$$

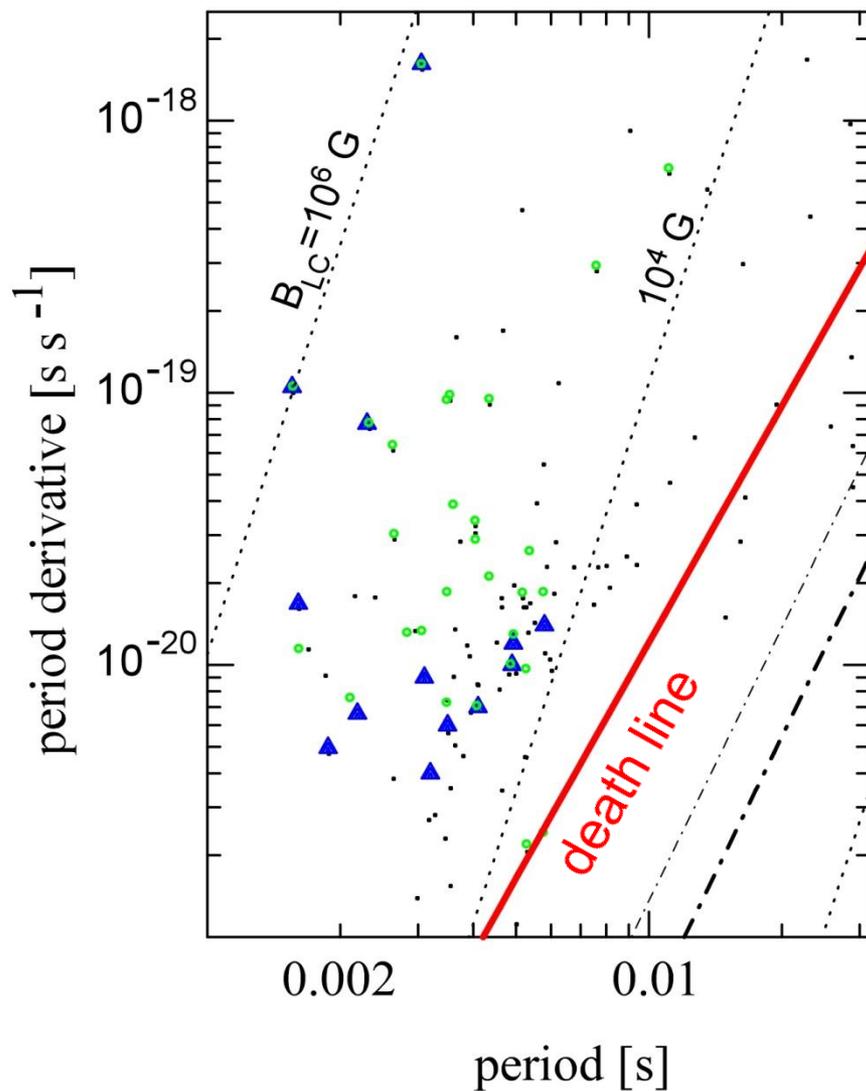
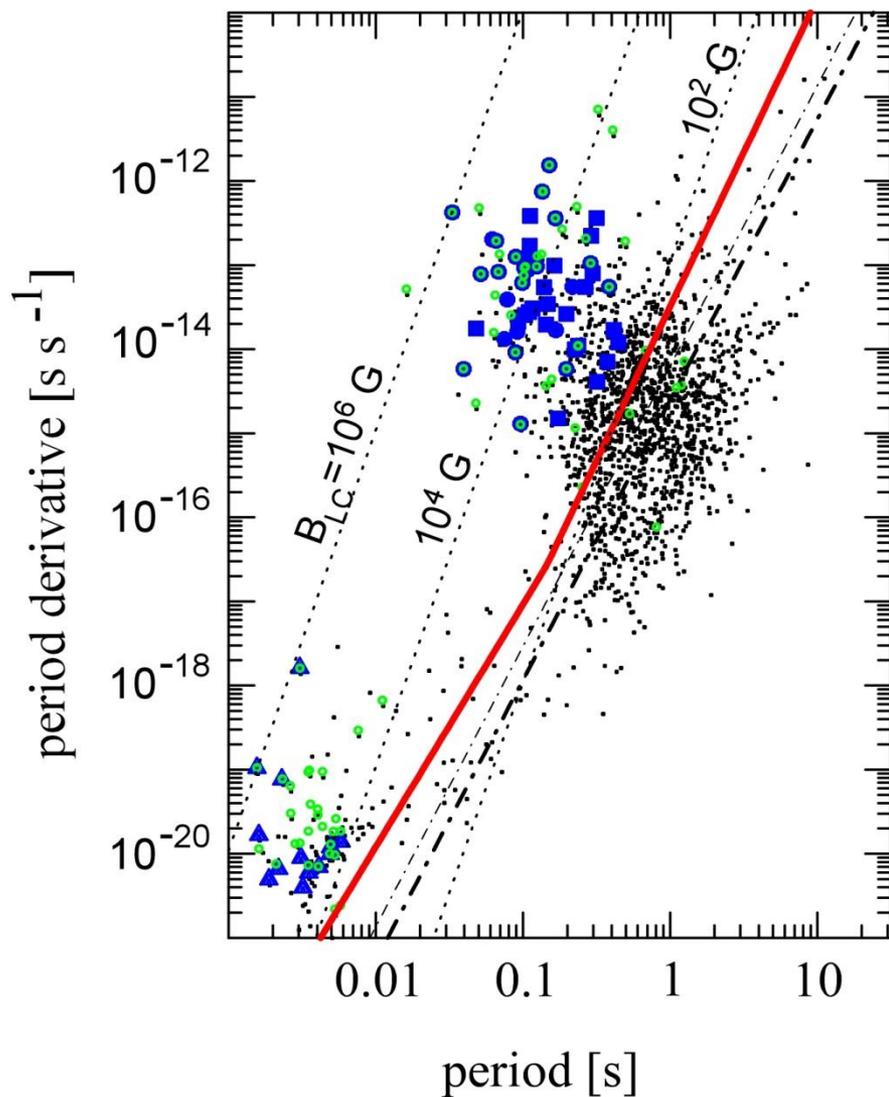
so that the pair production may be self-sustained.

# §5 Gamma-ray and X-ray Luminosities of MSPs

Death lines on  $(P, \dot{P})$  plane

Wang & KH (2011), ApJ 736, 127

Close-up of MSP region



## §5 *Gamma-ray and X-ray Luminosities of MSPs*

Next, let us numerically derive the **X-ray and  $\gamma$ -ray luminosities** on the  $(P, \dot{P})$  plane and confirm the analytical death line.

Apply the numerical scheme to typical MSP parameter sets.

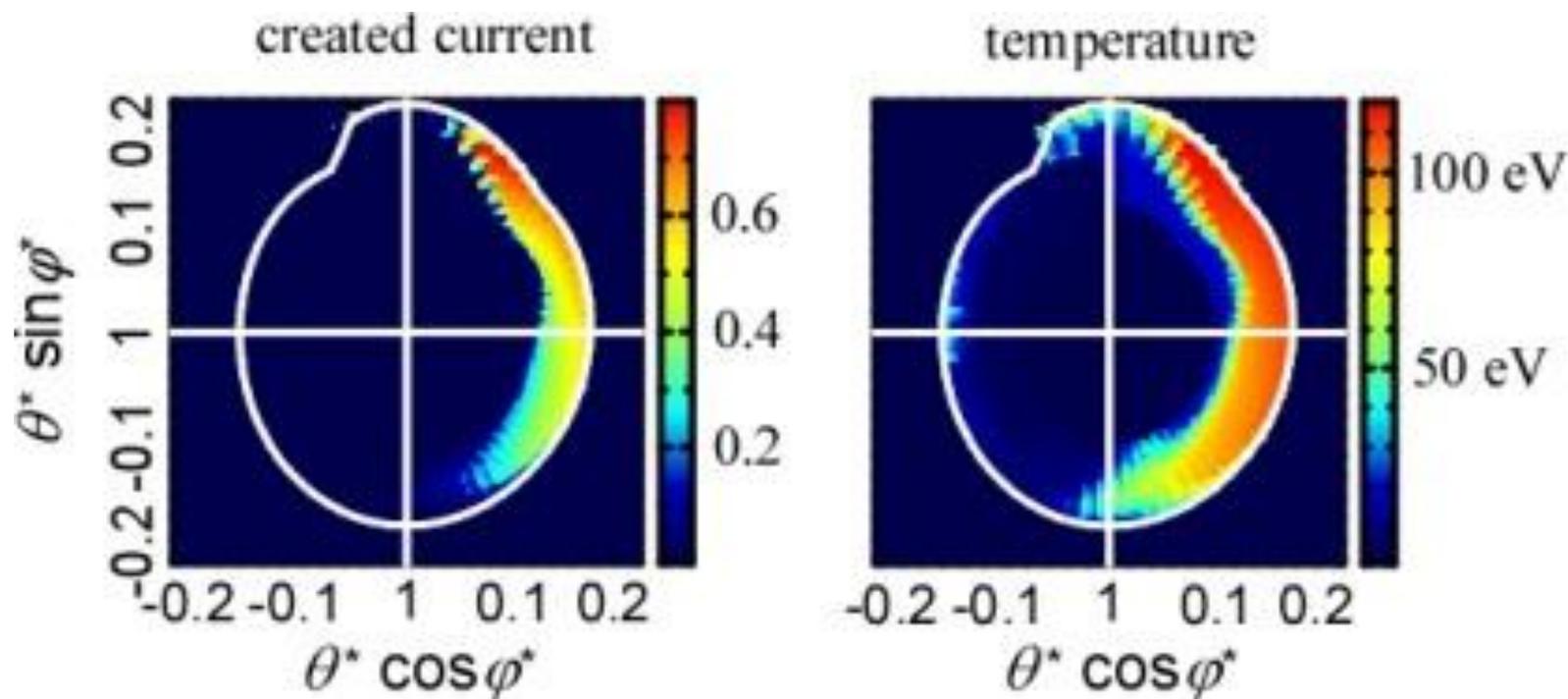
Advantage: For **MSPs**, only 3 parameters  $(P, dP/dt, \alpha)$ , or  $(P, \mu, \alpha)$  determines **OG geometry,  $E_{\parallel}$  distribution,  $e^{\pm}$  distribution functions, and specific intensity uniquely**, because the whole NS surface X-ray emission is not important.

## §5 Gamma-ray and X-ray Luminosities of MSPs

Wang & KH (2011), ApJ 736, 127

An example of the solved distribution of **created current density** and the **temperature** on the PC surface:

Ex)  $P=10$  ms,  $\dot{P}=10^{-19}$  s/s,  $\alpha=60^\circ$

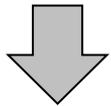


(Heat conduction in trans-**B** direction is not considered.)

# §5 Gamma-ray and X-ray Luminosities of MSPs

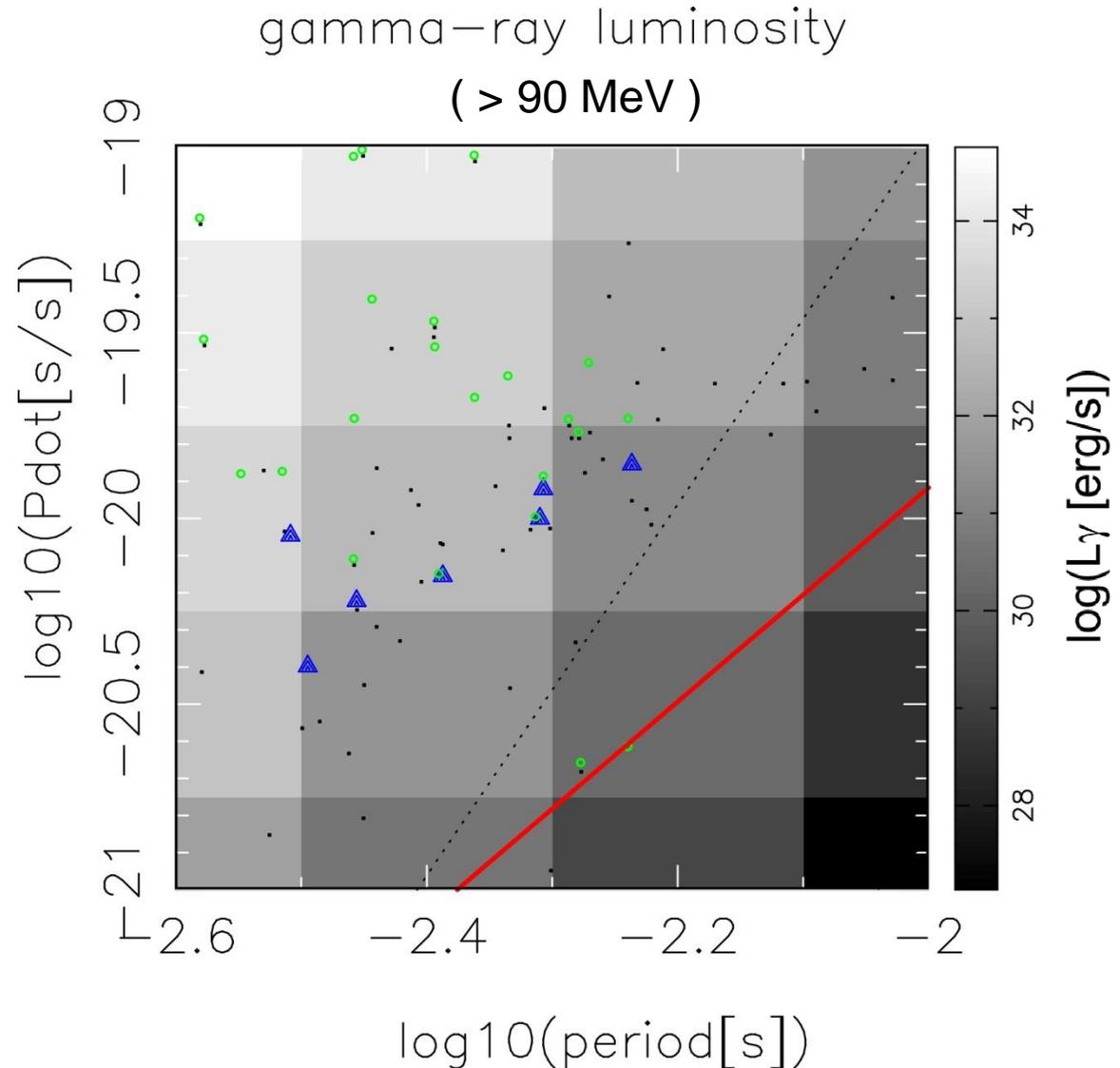
Solve OG for 20  
MSP parameter  
sets.

Distribution of  $\gamma$ -  
ray luminosity:



$\gamma$ -ray only MSPs?

(magnetospheric  
components only;  
PWS excluded)

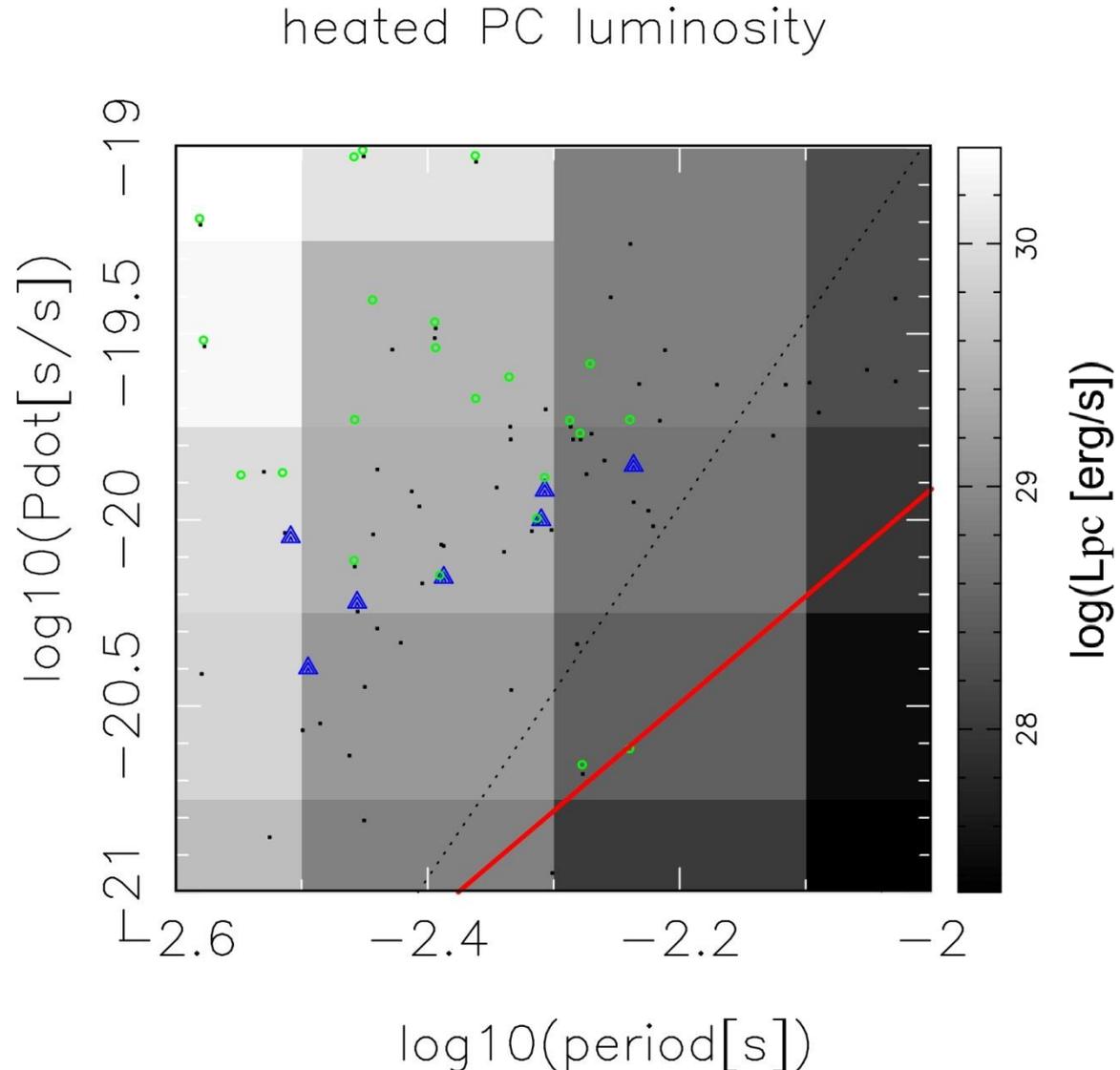


# §5 *Gamma-ray and X-ray Luminosities of MSPs*

Solve OG for 20  
MSP parameter  
sets.

Distribution of **X-**  
**ray luminosity of**  
**heated PC:**

(magnetospheric  
components only;  
PWS excluded)



## §6 Evolution of Spectral Hardness

**Next, let us show that the trailing peak has a harder spectrum than the leading peak in GeV energies.**

In the OG,  $\gamma$ -rays are emitted by curvature process.

Characteristic energy  
of curvature emission:

$$E_c = \frac{3hc}{4\pi} \frac{\gamma^3}{\rho_c}$$

$\gamma$  : Lorentz factor

Electro-static force balance

$$eE_{\parallel} = \frac{2e^2 \gamma^4}{3\rho_c^2}$$

$\rho_c$  : curvature radius

Eliminate  $\gamma \rightarrow$

$$E_c = \left(\frac{3}{2}\right)^{7/4} \frac{hc}{2\pi} \rho_c^{1/2} \left(\frac{E_{\parallel}}{e}\right)^{3/4}$$

# §6 Evolution of Spectral Hardness

KH (2011), ApJ 733, L49

Characteristic energy of curvature emission:

$$E_c = \left(\frac{3}{2}\right)^{7/4} \frac{hc}{2\pi} \rho_c^{1/2} \left(\frac{E_{\parallel}}{e}\right)^{3/4}$$

Above  $E_c$ , the flux density sharply declines as

$$F_{\nu} \propto \left(\frac{E}{E_c}\right)^{1/2} \exp\left(-\frac{E}{E_c}\right)$$

Thus, the cut-off energy,  $E_c$ , increases with increasing  $\rho_c$ .

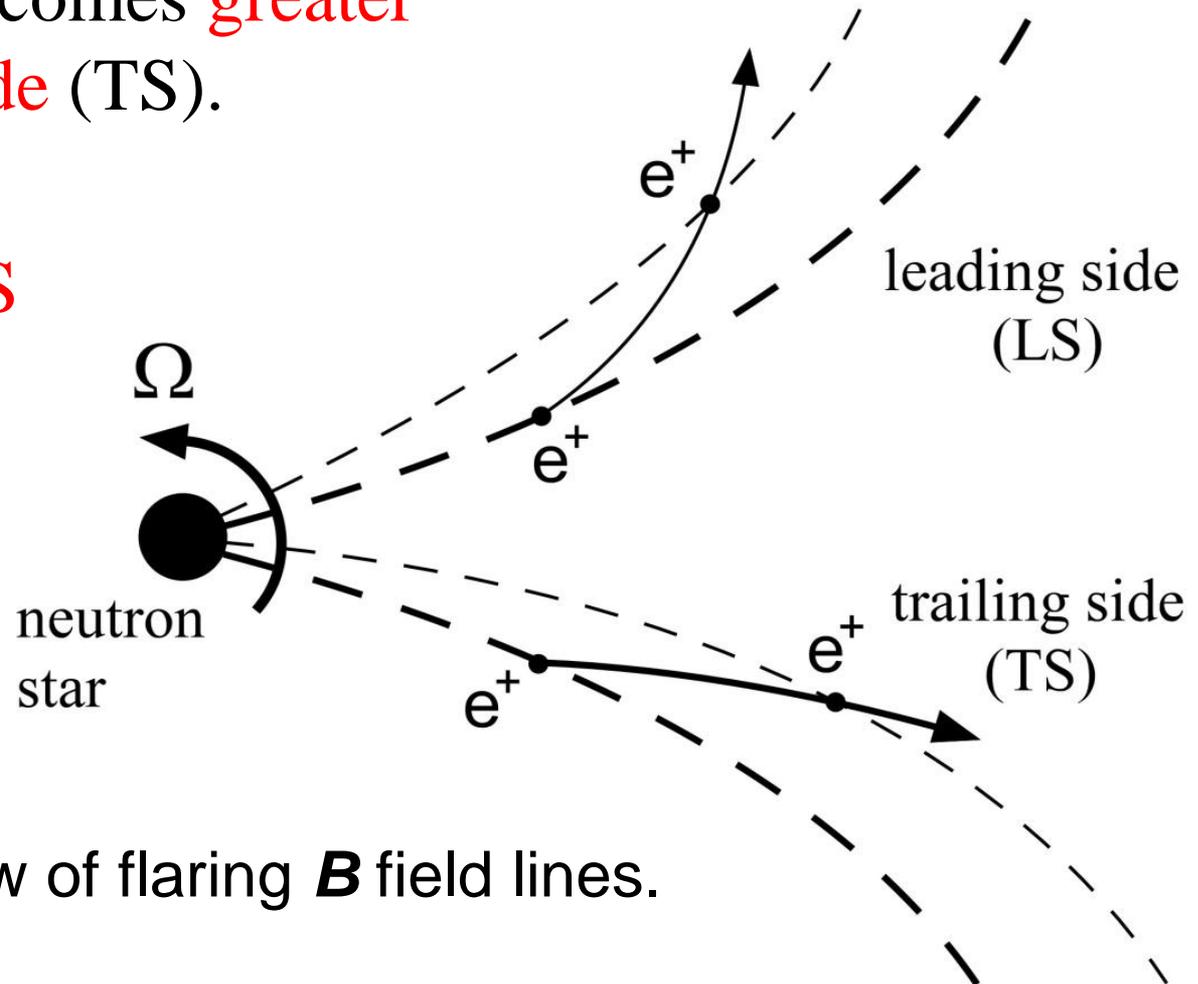
# §6 Evolution of Spectral Hardness

KH (2011), ApJ 733, L49

Because of rotation, the curvature radius,  $\rho_c$ , of particle paths becomes greater in the trailing side (TS).



Greater  $E_c$  in TS

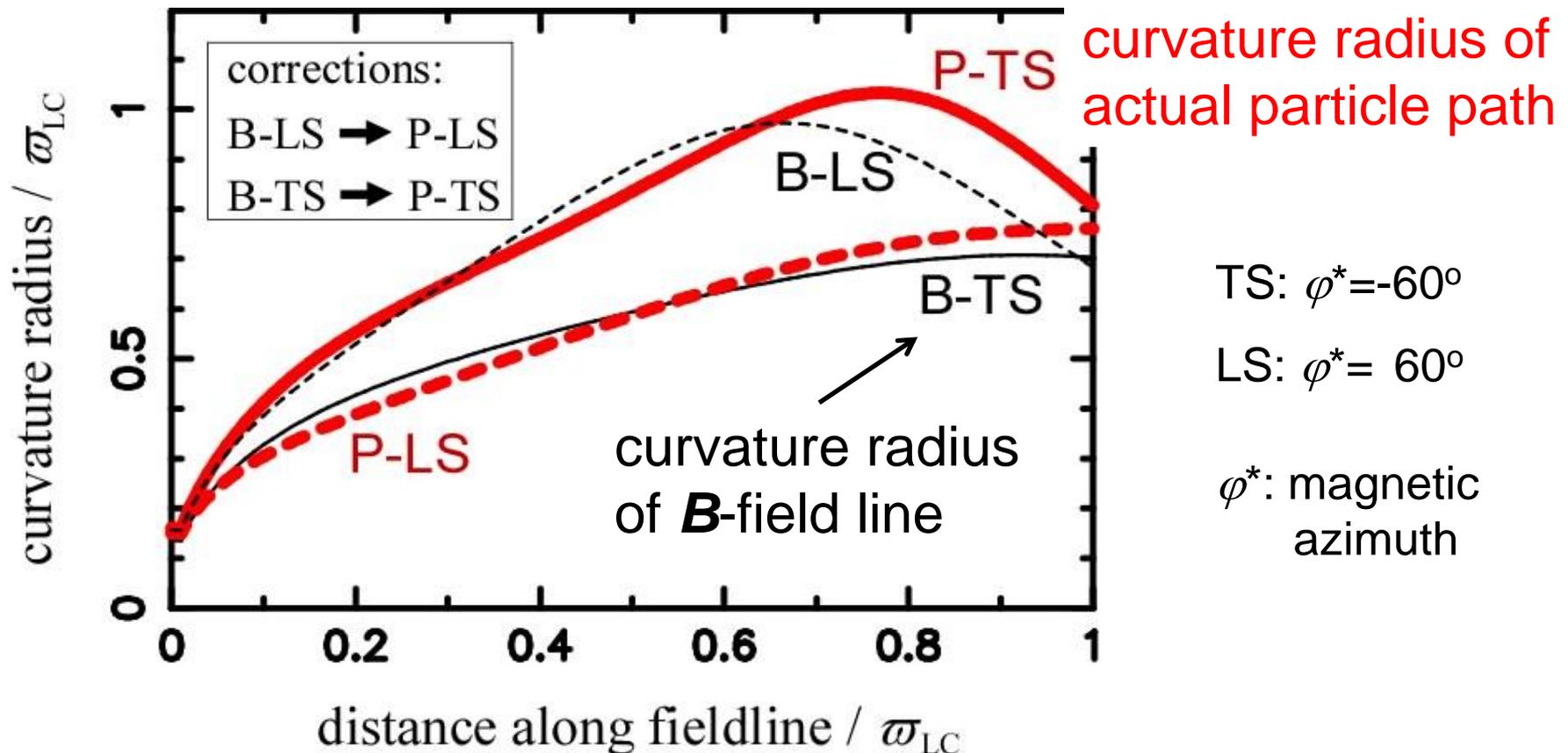


Top view of flaring  $\mathbf{B}$  field lines.

# §6 Evolution of Spectral Hardness

KH (2011), ApJ 733, L49

Quantitative discussion:  $\rho_c$  of actual particle paths.



# §6 Evolution of Spectral Hardness

KH (2011), ApJ 733, L49

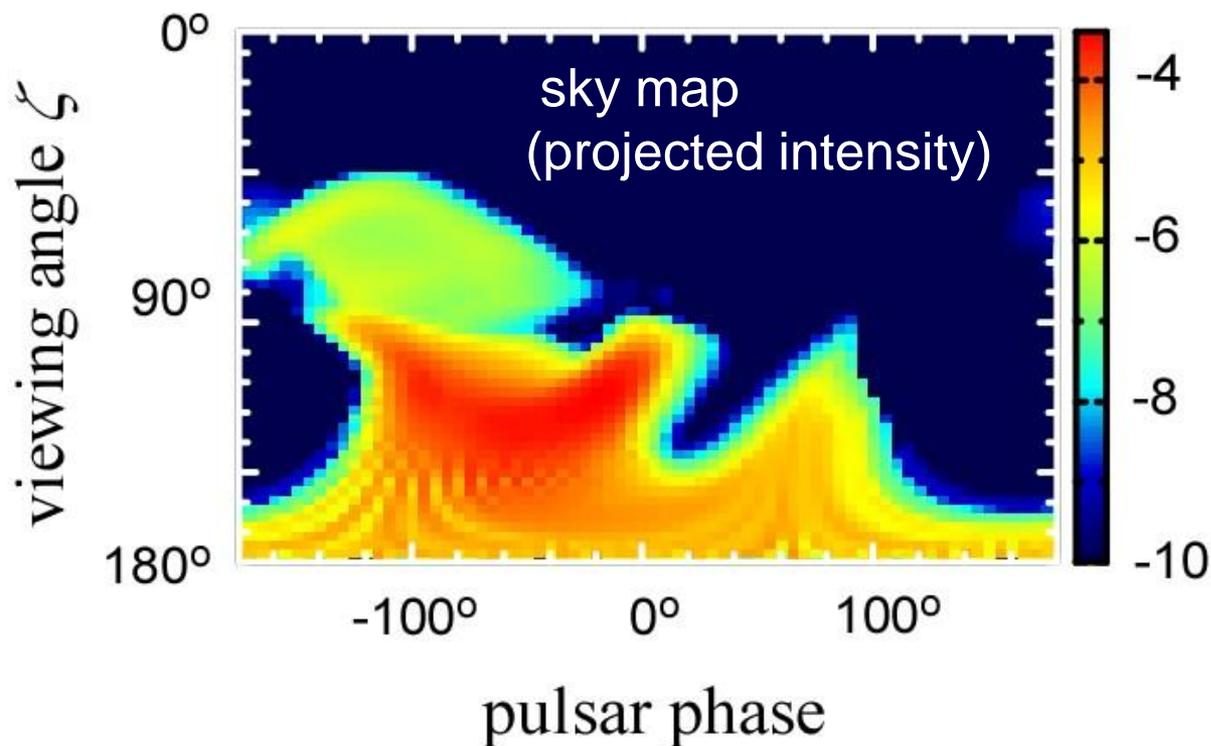
To compute  $E_c$ , we also need  $E_{\parallel}$ , in addition to  $\rho_c$ .

$$E_c = \left(\frac{3}{2}\right)^{7/4} \frac{hc}{2\pi} \rho_c^{1/2} \left(\frac{E_{\parallel}}{e}\right)^{3/4}$$

Solve  $E_{\parallel}$  from the set of Maxwell & Boltzmann eqs.



Photon specific intensity





# §6 Evolution of Spectral Hardness

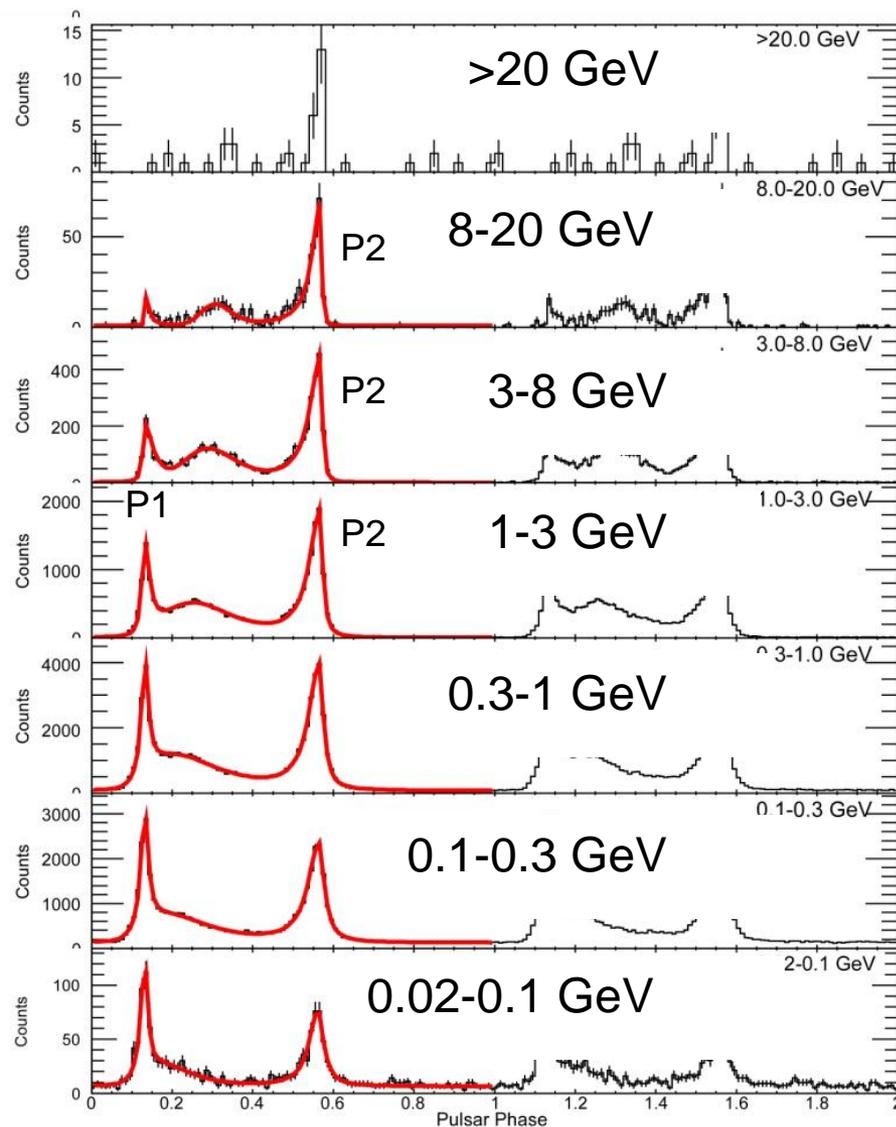
The hard P2 spectrum is consistent with observations.

So far, detailed phase-resolved spectra have been obtained from 7 pulsars with LAT.

Fitting spectra by power-law + exp.-cutoff,

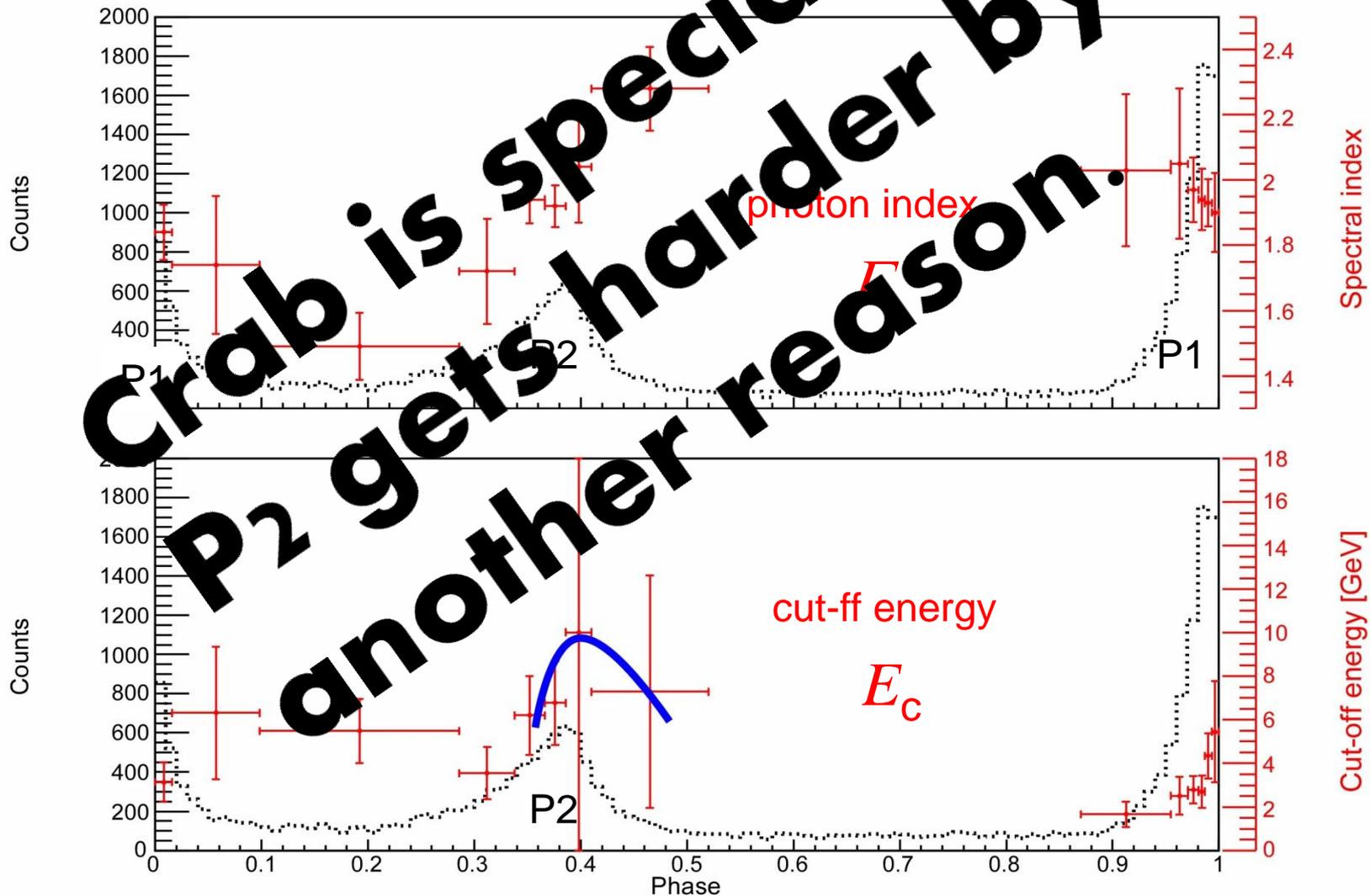
$$\frac{dN}{dE} = KE^{-\Gamma} \exp\left(-\frac{E}{E_c}\right),$$

they find  $E_c$  becomes greater in P2 than in P1.



# §6 Evolution of Spectral Hardness

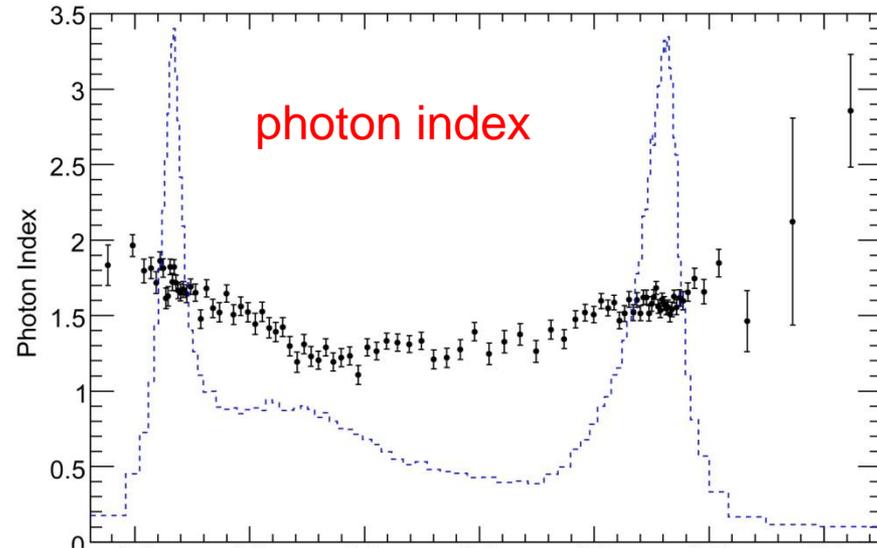
Crab pulsar ( $\tau \sim 1$  kyr)



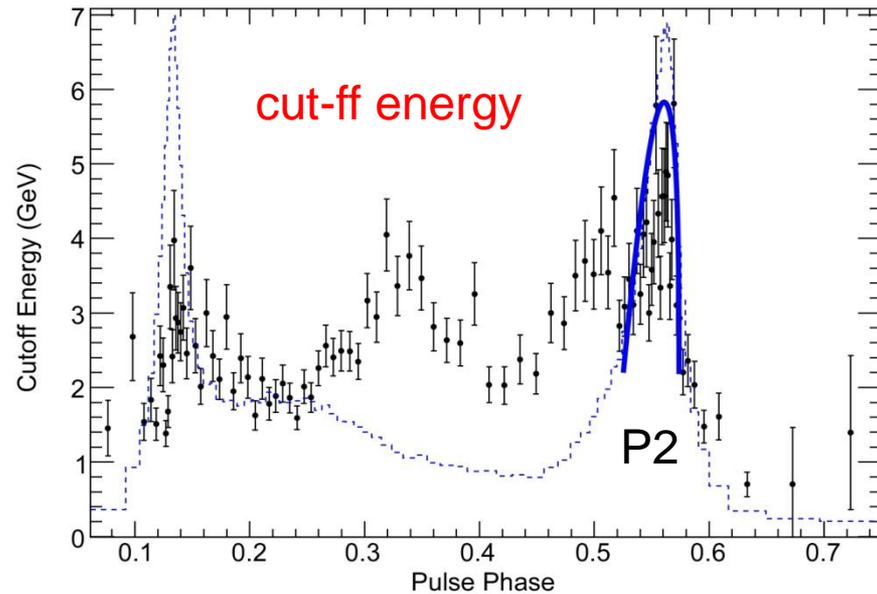
# §6 Evolution of Spectral Hardness

Vela pulsar  
( $\tau \sim 11$  kyr)

$\Gamma$



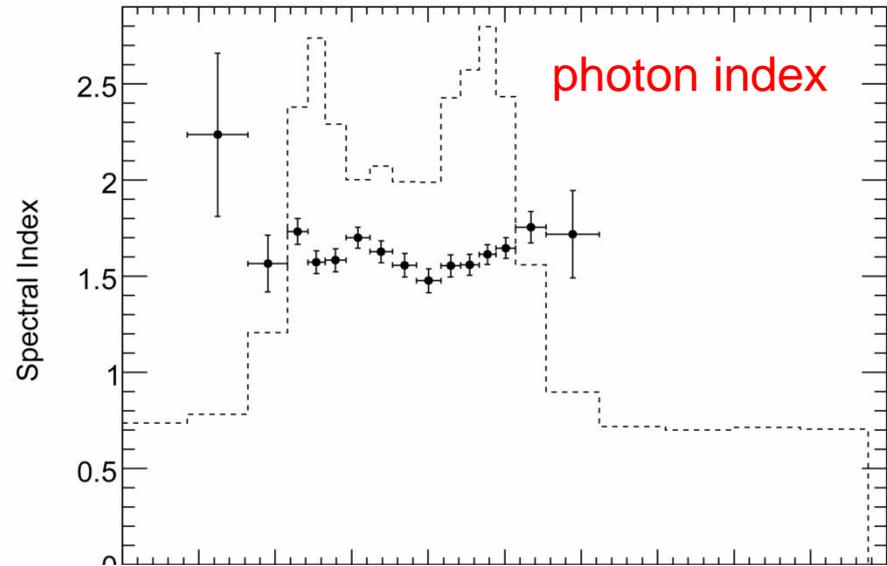
$E_c$



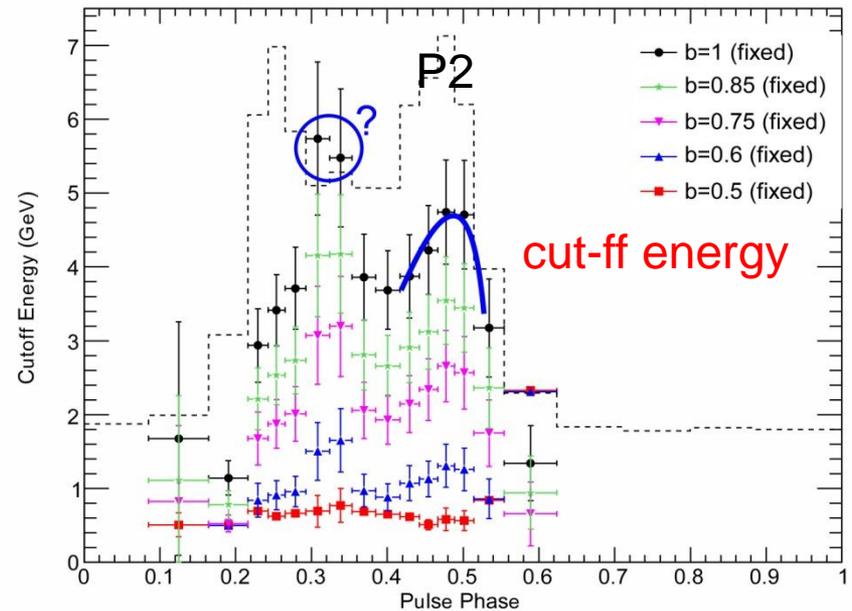
# §6 Evolution of Spectral Hardness

PSR J1709-4429  
( $\tau \sim 18$  kyr)

$\Gamma$



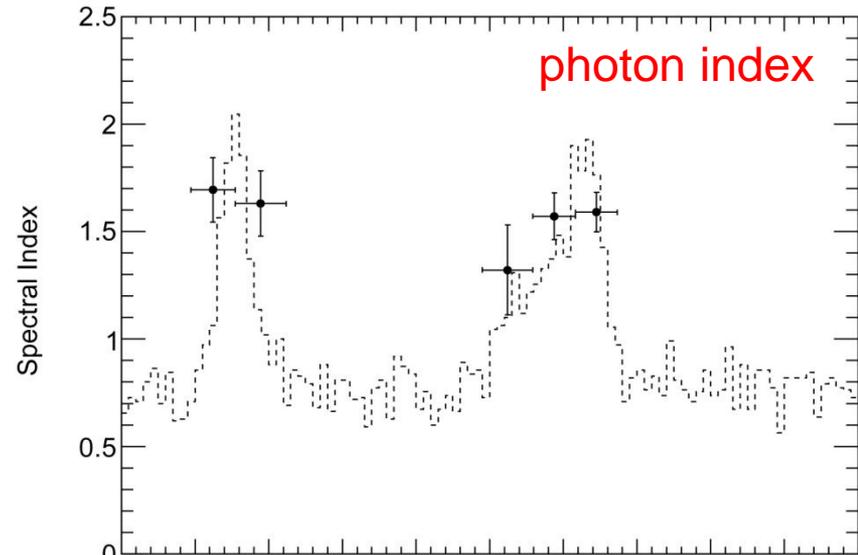
$E_c$



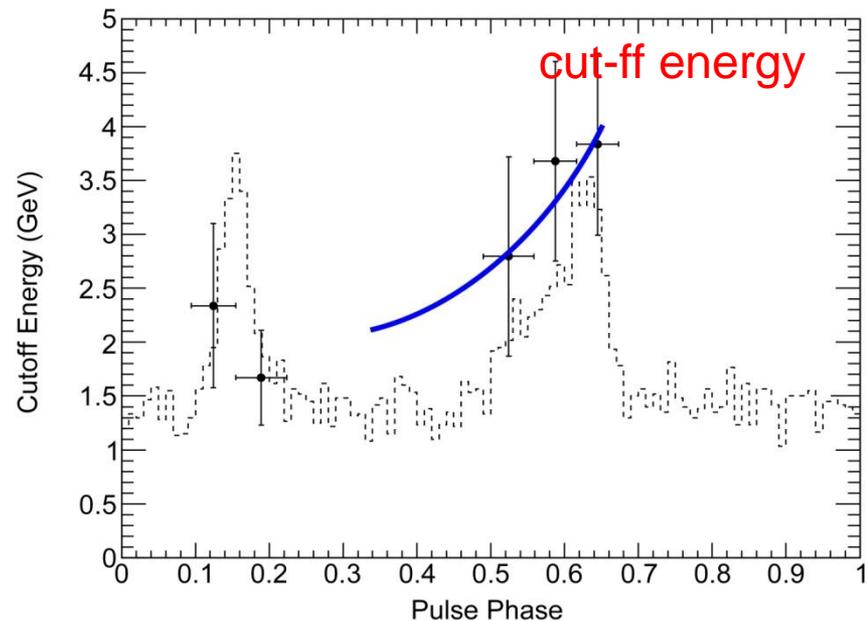
# §6 Evolution of Spectral Hardness

PSR J1952+3252  
( $\tau \sim 110$  kyr)

$\Gamma$



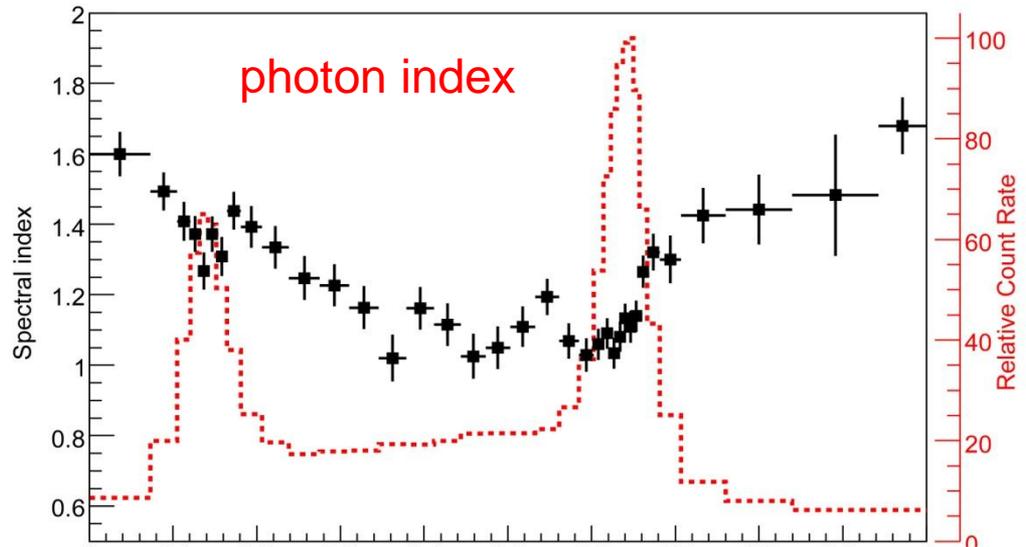
$E_c$



# §6 Evolution of Spectral Hardness

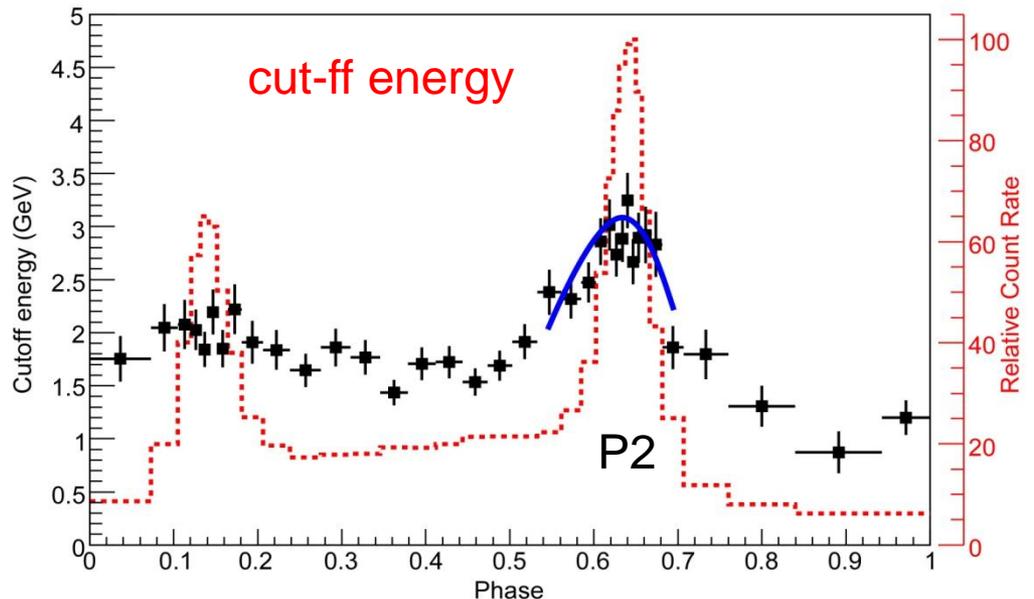
Geminga  
( $\tau \sim 340$  kyr)

$\Gamma$



Spectral hardening  
in the second,  
trailing peak

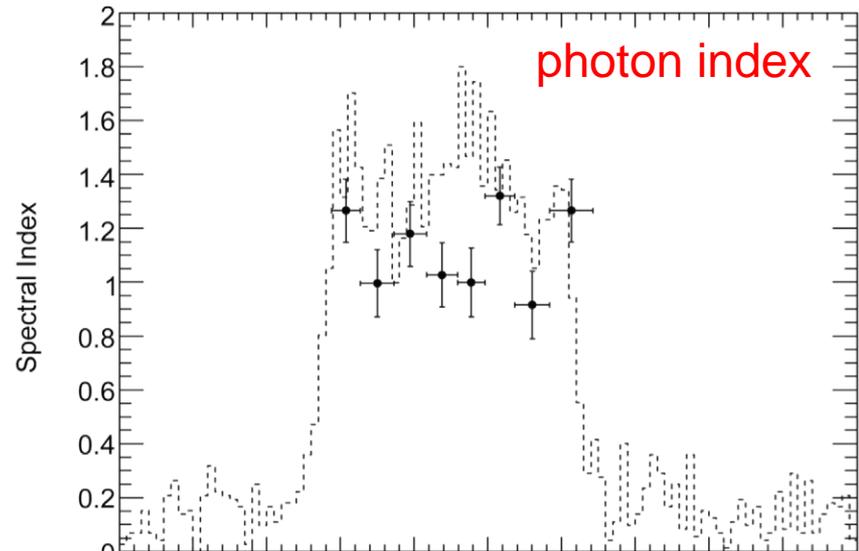
$E_c$



# §6 Evolution of Spectral Hardness

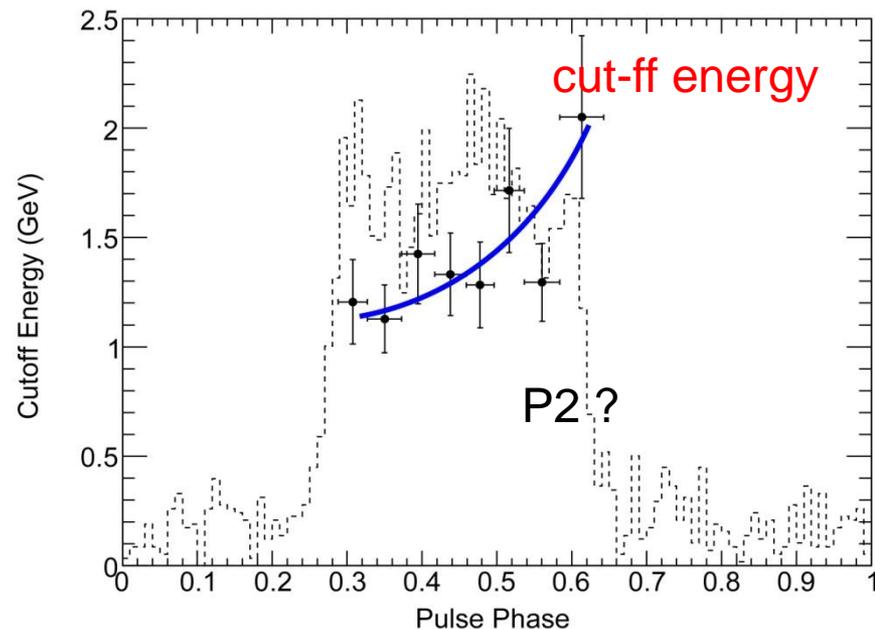
PSR J1057-5226  
( $\tau \sim 540$  kyr)

$\Gamma$



Spectral hardening  
occurs in the  
trailing side  
(maybe P2) of  
light-curve peaks.

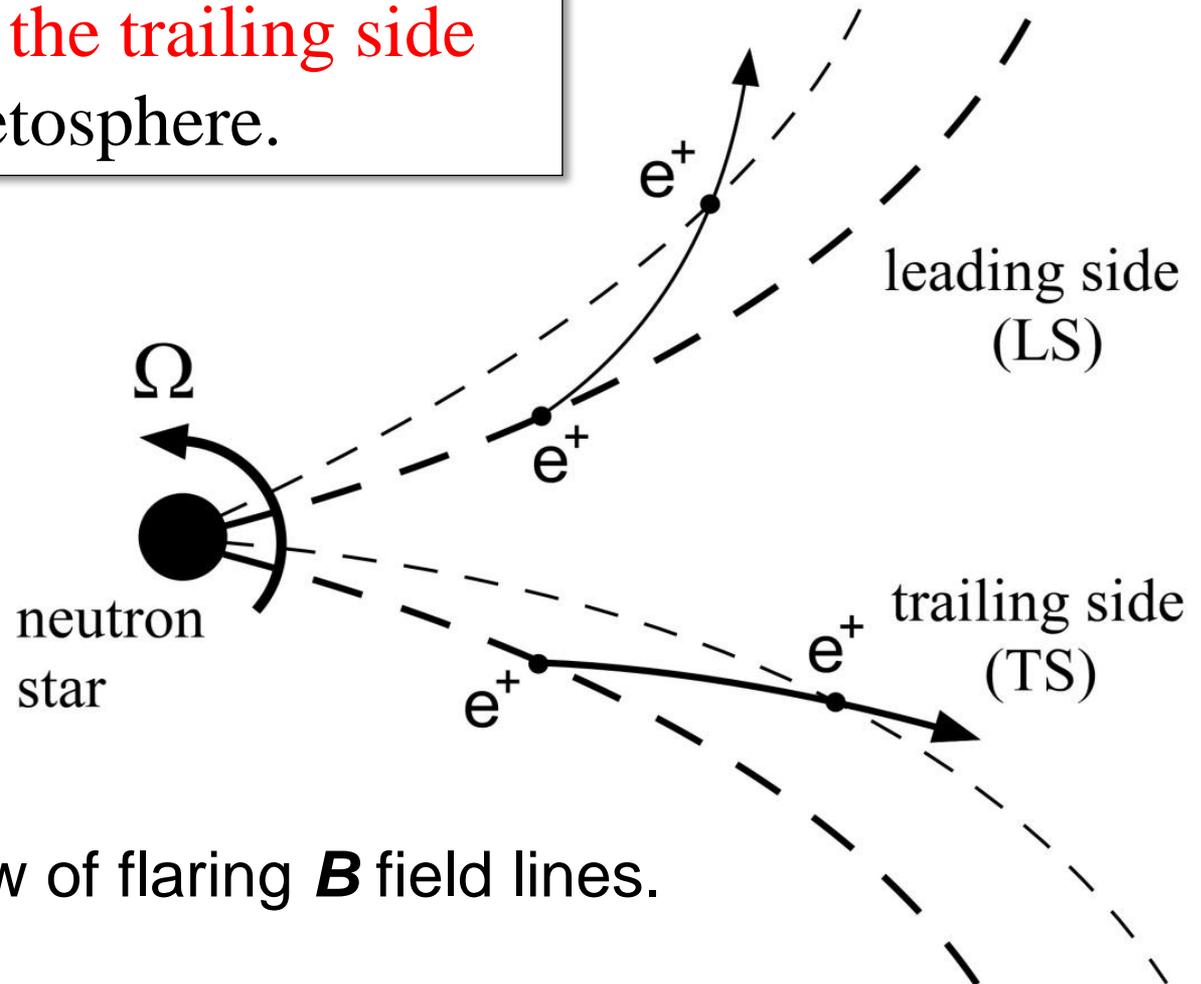
$E_c$



# §6 Evolution of Spectral Hardness

KH (2011), ApJ 733, L49

In short, **hard P2 spectra** can be generally interpreted by the **greater curvature radii in the trailing side** of a pulsar magnetosphere.



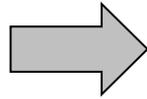
Top view of flaring **B** field lines.

## §7 *ICS spectrum of the Crab pulsar*

Finally, let us numerically examine the Crab pulsar.

Maxwell &

Boltzmann eqs.,

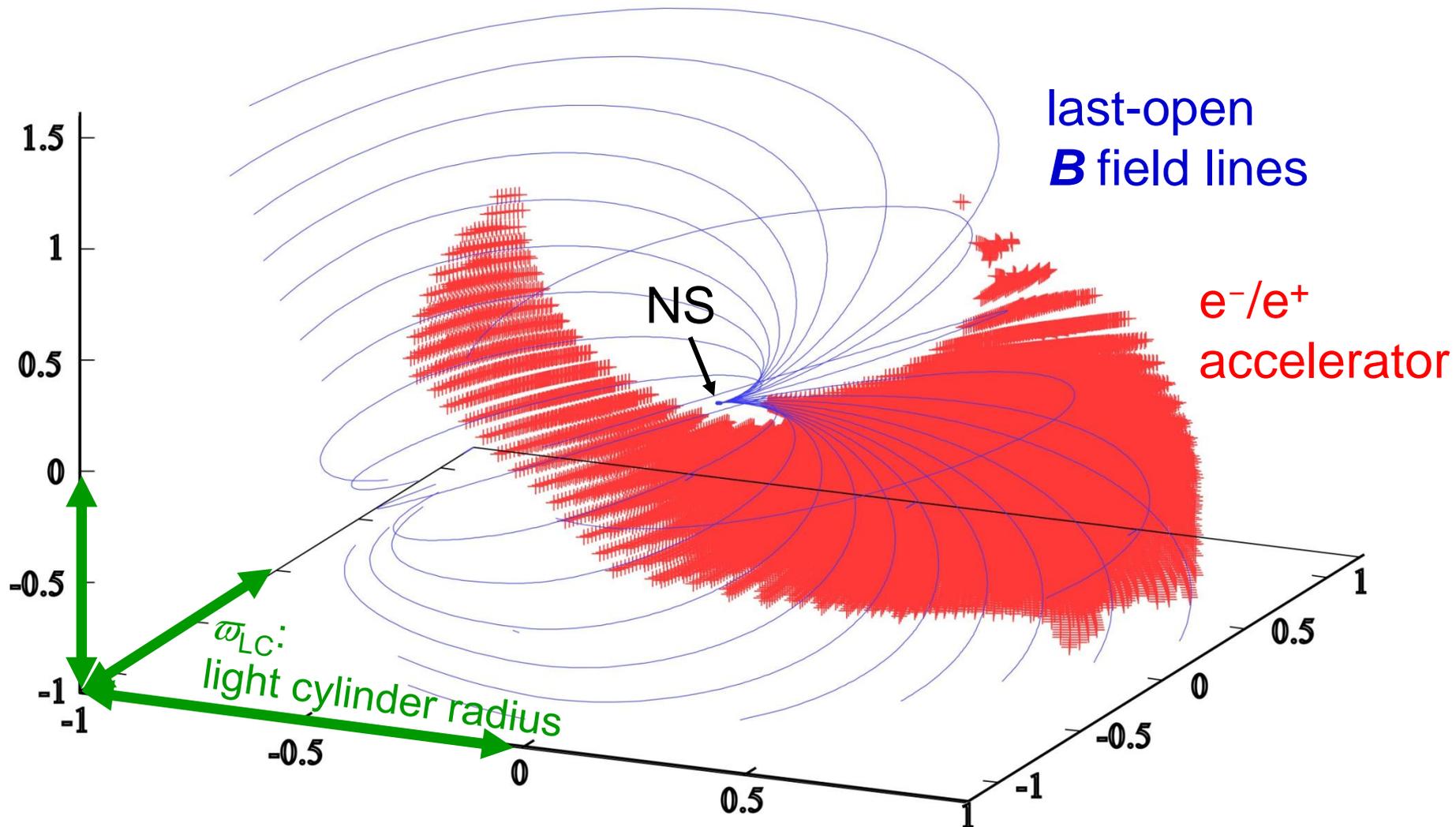


- OG 3-D geometry,
- $E_{\parallel}$  distribution,
- $e^+/e^-$  distribution functions,
- photon specific intensity

Apply this method to the **Crab** pulsar, assuming  
 $\mu=3.8 \times 10^{30}$  G cm<sup>3</sup>,  $\alpha=60^\circ$ ,  $kT=70$  eV.

# §7 *ICS* spectrum of the *Crab* pulsar

3-D distribution of the particle accelerator (i.e., high-energy emission zone) solved from the Poisson eq.:

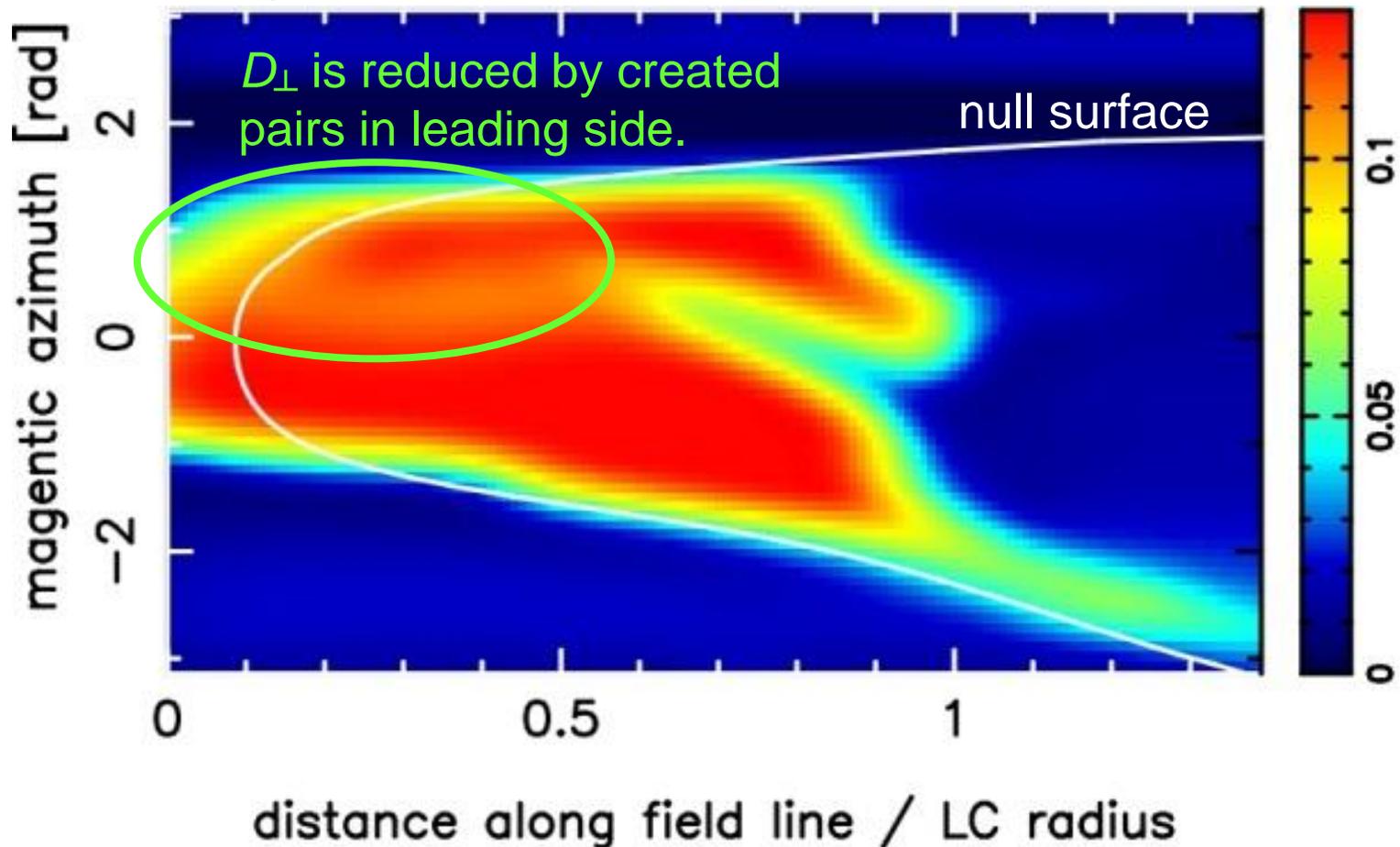


# §7 ICS spectrum of the Crab pulsar

**3-D geometry:** Trans-field gap thickness is self-regulated by pair production.

Crab,  $\alpha=60^\circ$

Fractional gap thickness projection on the last-open **B** line surface

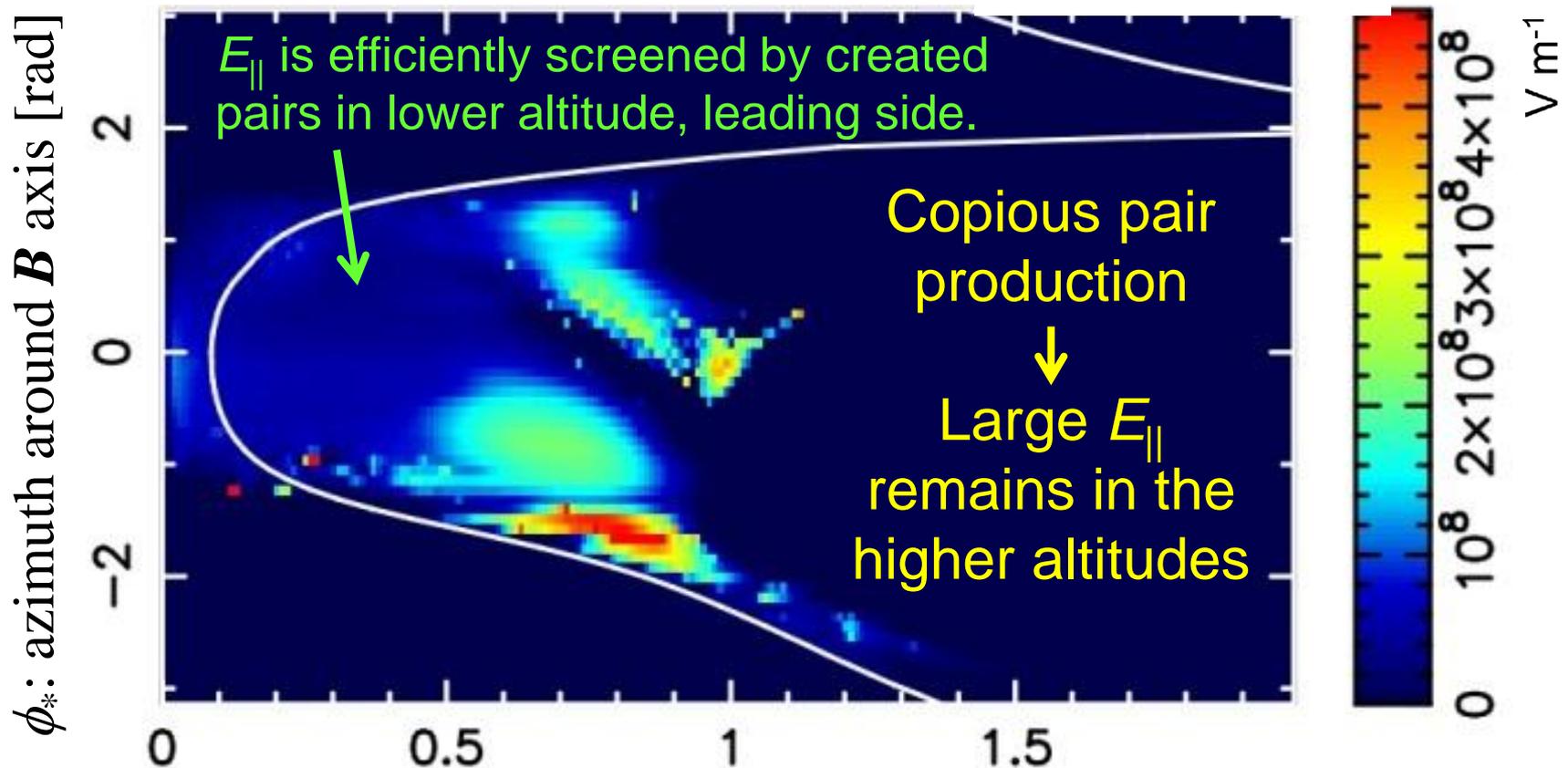


# §7 ICS spectrum of the Crab pulsar

$E_{\parallel}$  is also self-regulated by pair production.

(→ Curvature photon energy changes little for various pulsars.)

Crab,  $\alpha=60^{\circ}$



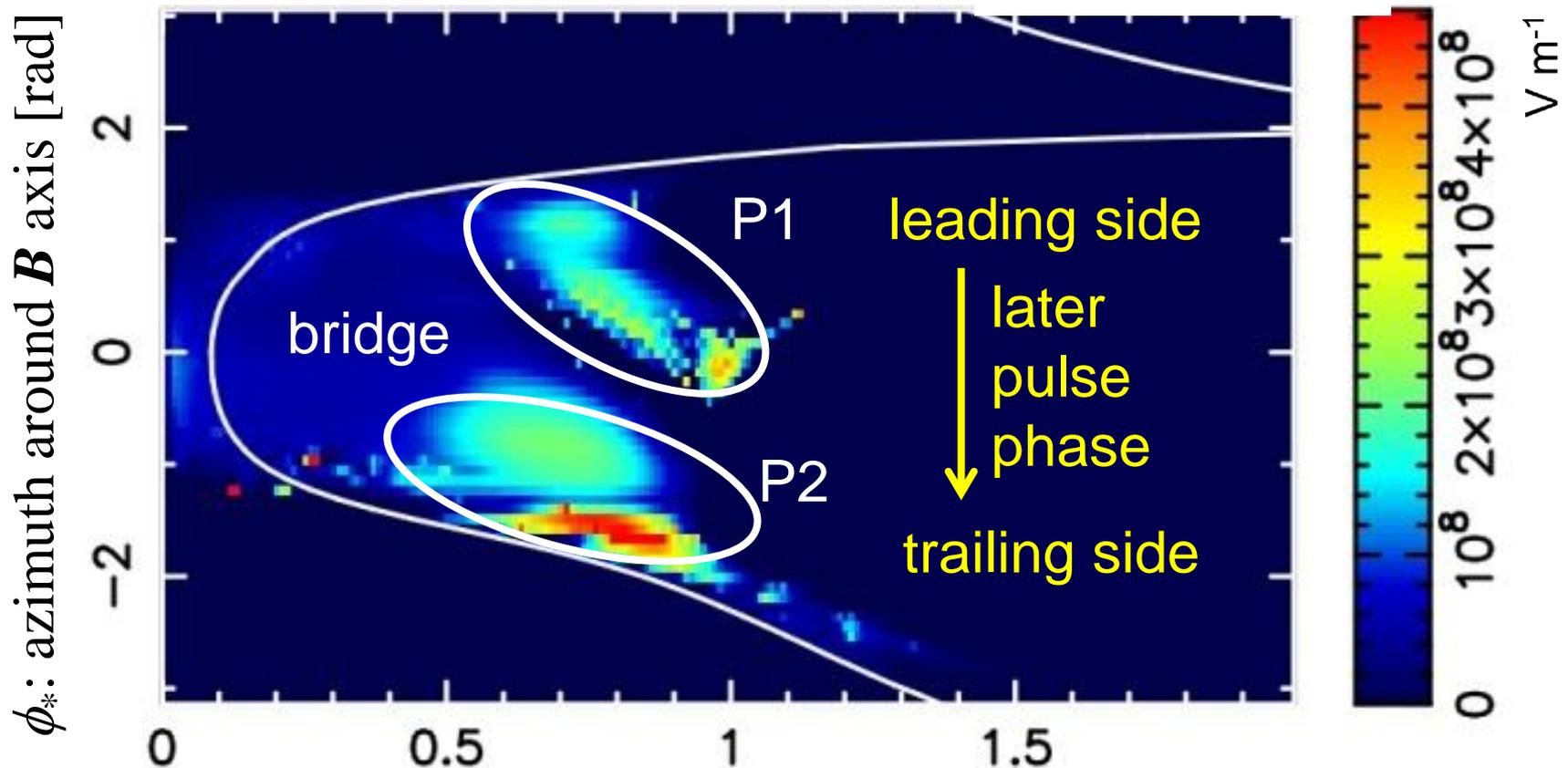
$s$ : distance along  $B$  field lines / light-cylinder radius

# §7 ICS spectrum of the Crab pulsar

$E_{\parallel}$  is also self-regulated by pair production.

(→ Curvature photon energy changes little for various pulsars.)

Crab,  $\alpha=60^{\circ}$



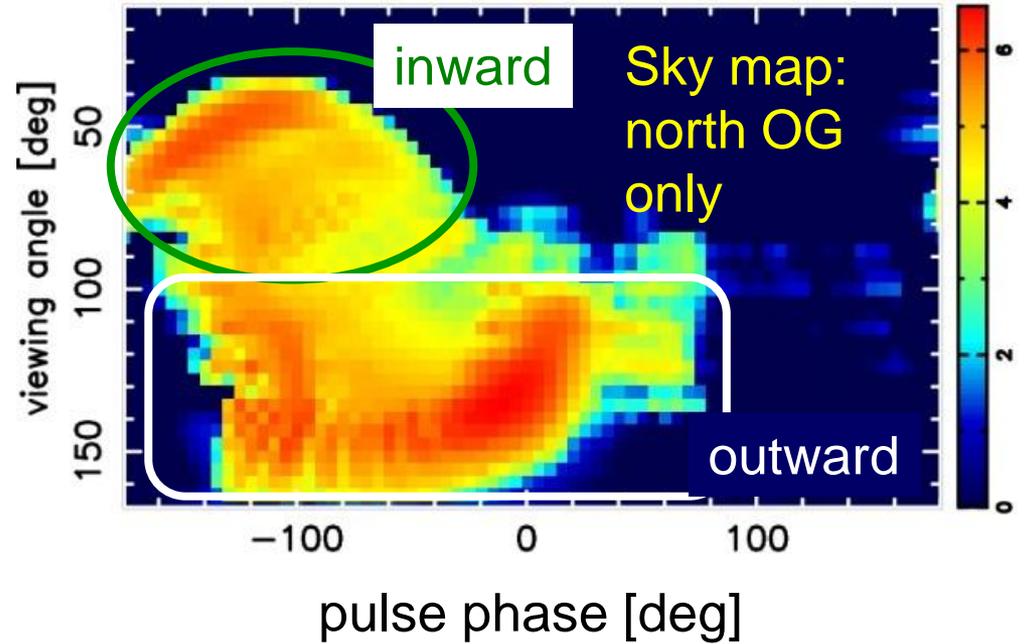
$s$ : distance along  $B$  field lines / light-cylinder radius

# §7 ICS spectrum of the Crab pulsar

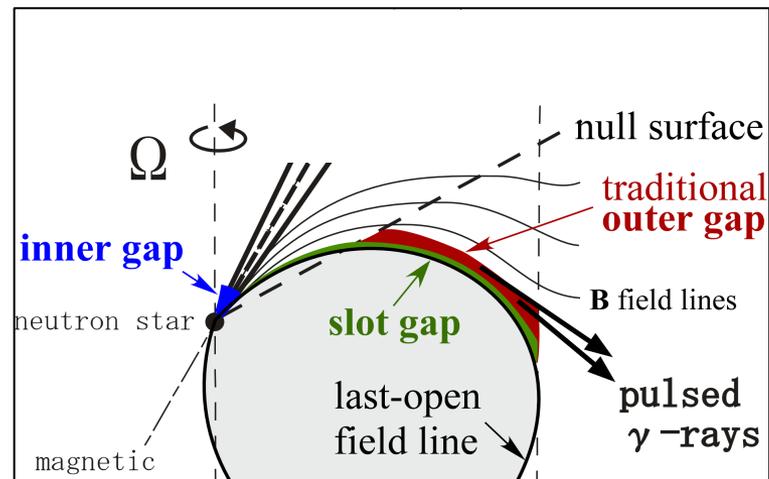
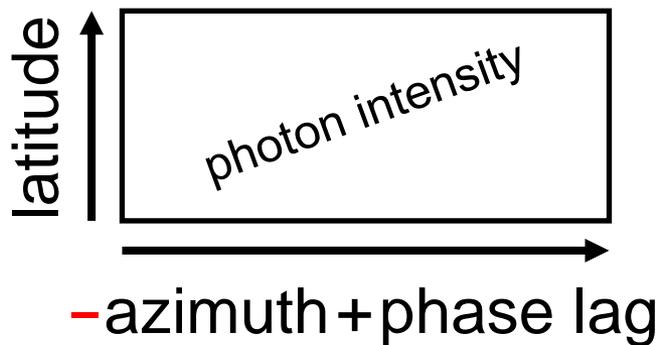
Crab 60°

Using the solved  $E_{\parallel}$ , we can compute emissivity at each position.

Intensity distribution shows a **caustic** pattern in the **sky map**.



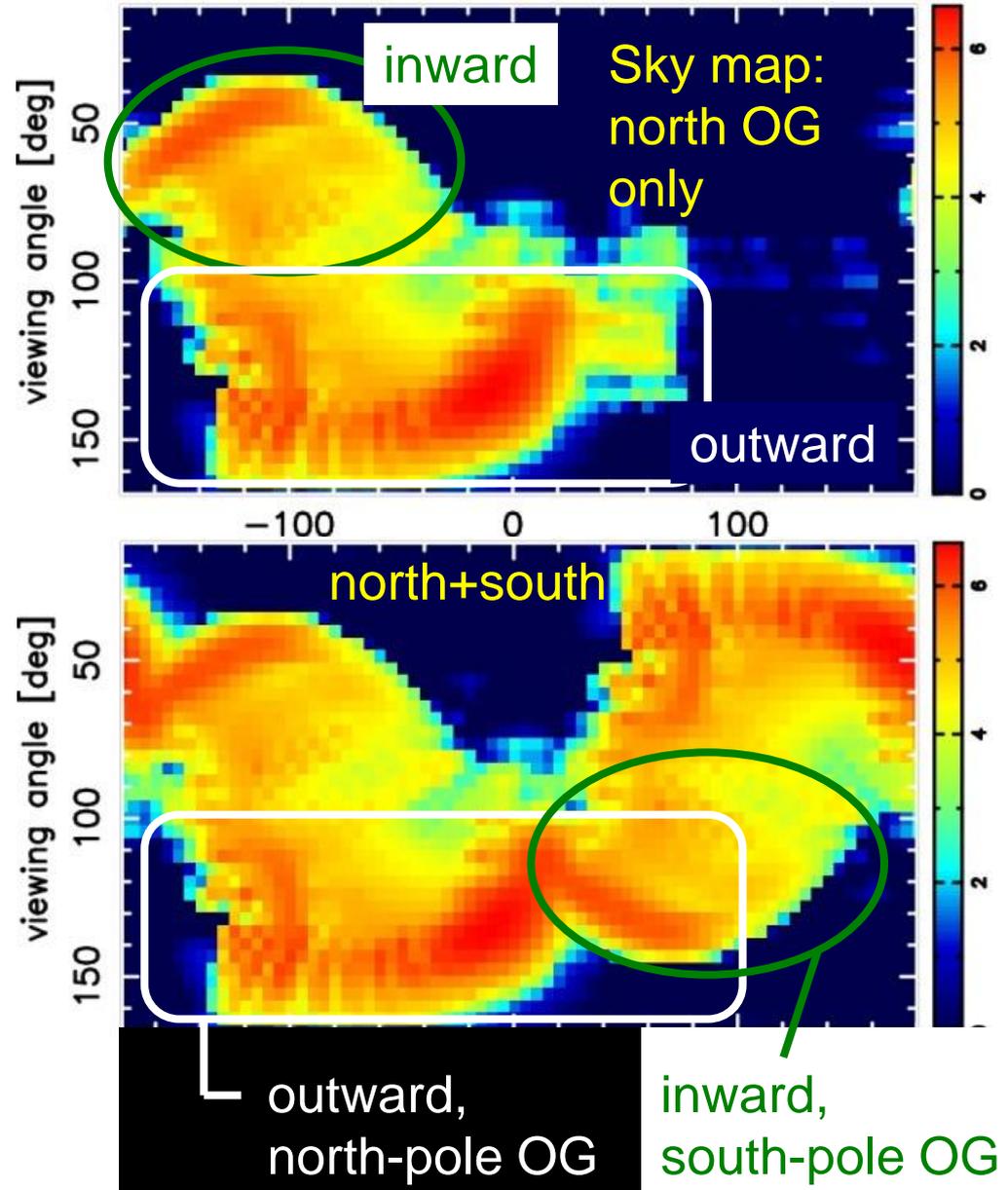
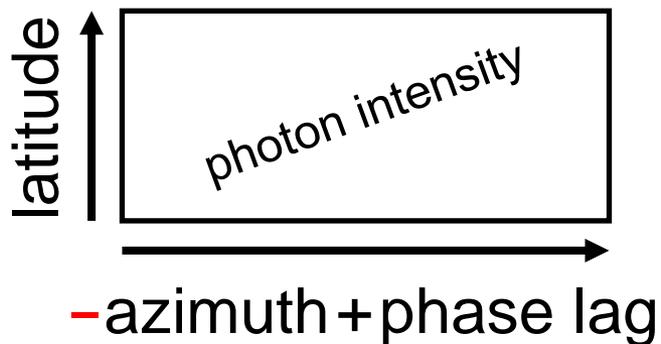
one NS rotation



# §7 ICS spectrum of the Crab pulsar

Crab 60°

To compute pulsar emissions, we must consider the photons emitted from OGs connected to the both poles.

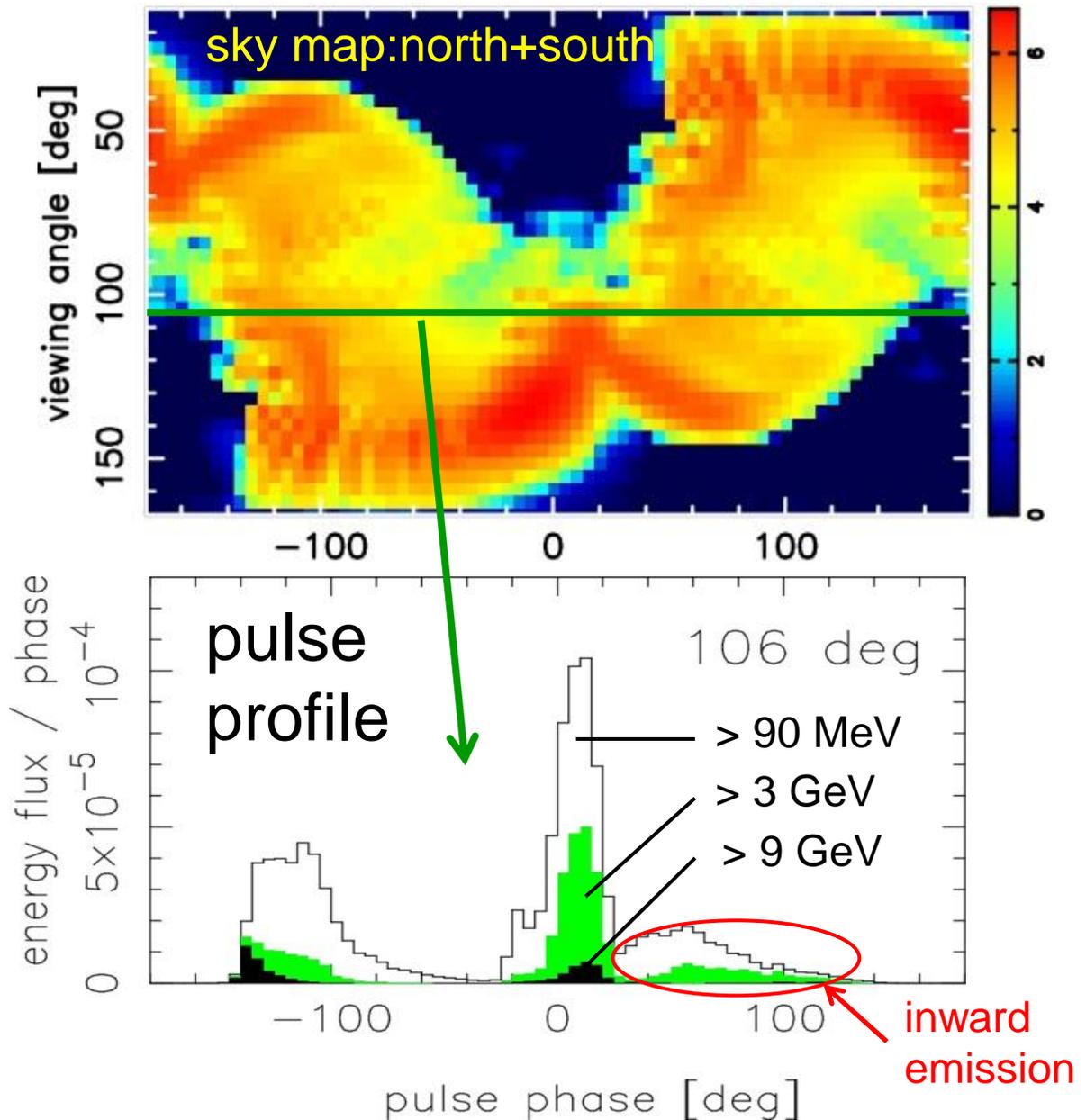


# §7 ICS spectrum of the Crab pulsar

Crab 60°

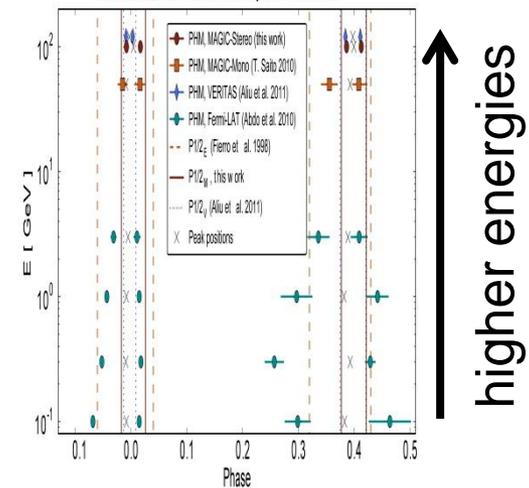
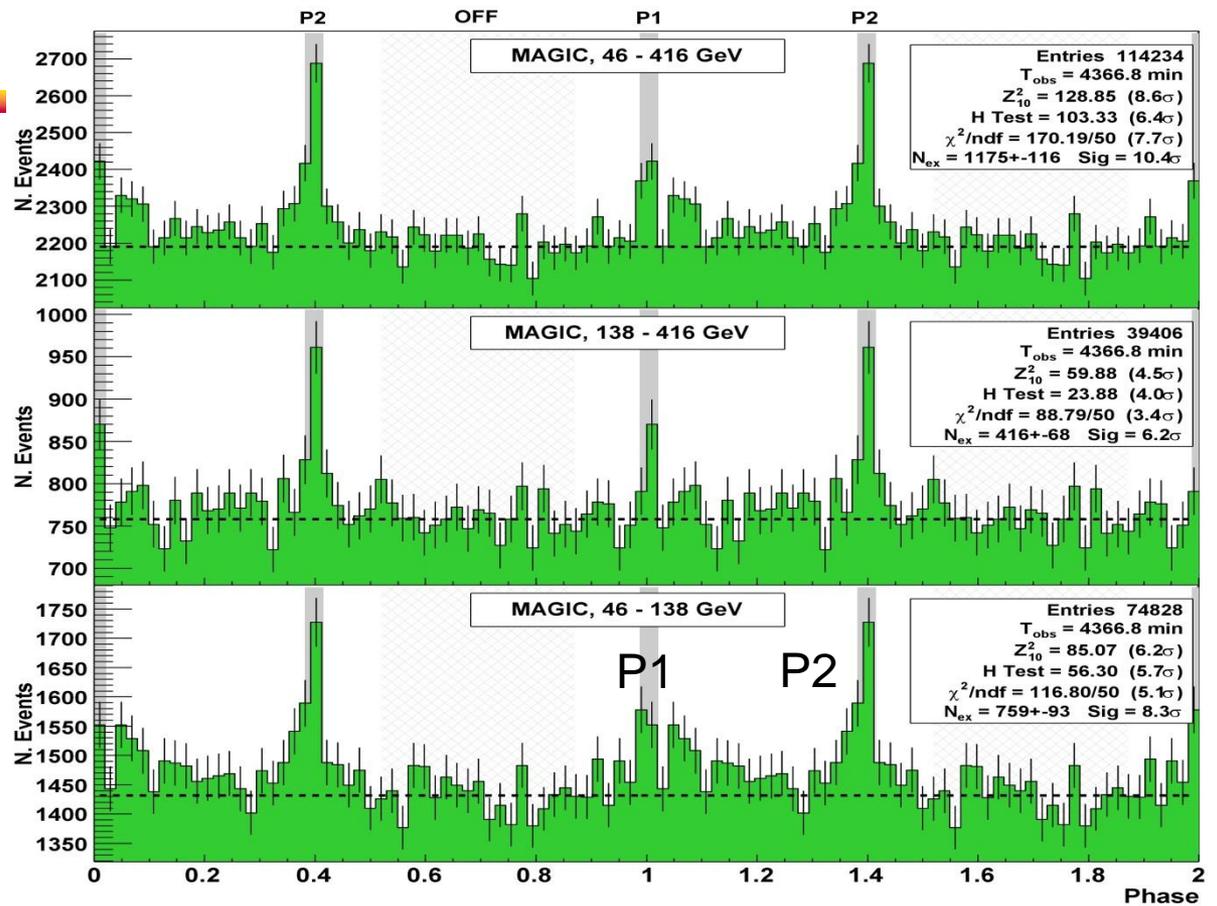
If we cut the sky map at some viewing angle  $\zeta$ , we obtain a pulse profile.

Using an energy-dependent sky map, we obtain pulse profiles at different energy bands.



# §7 ICS

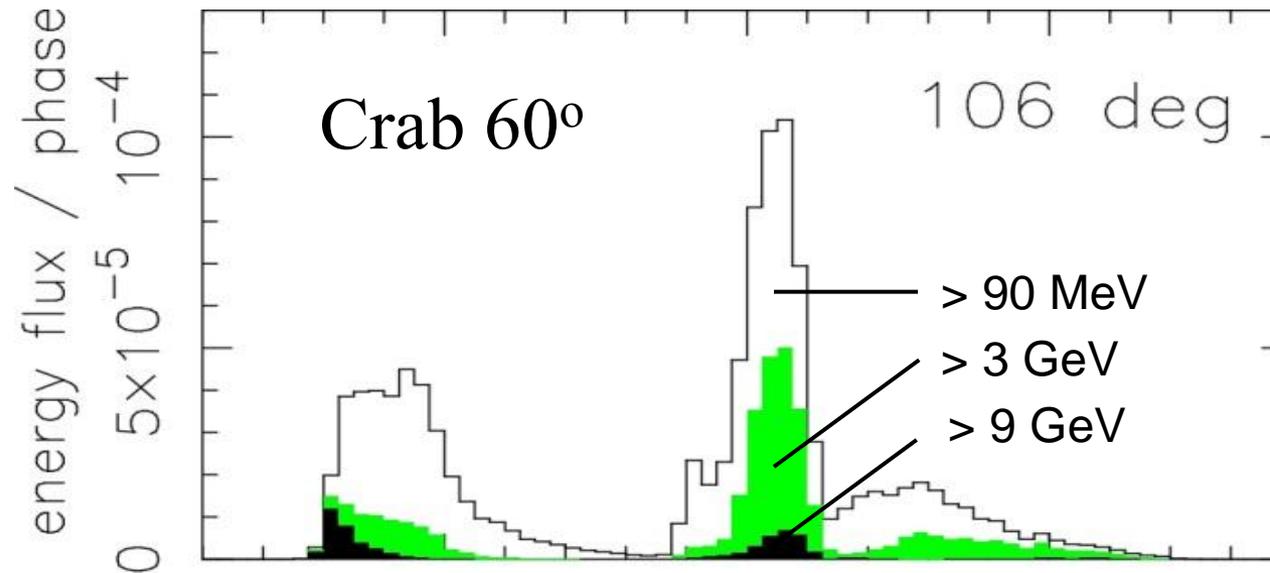
If we look at the details, however, the energy-dependent pulse profile (incl. 30-400 GeV) does not reproduce the MAGIC observations.



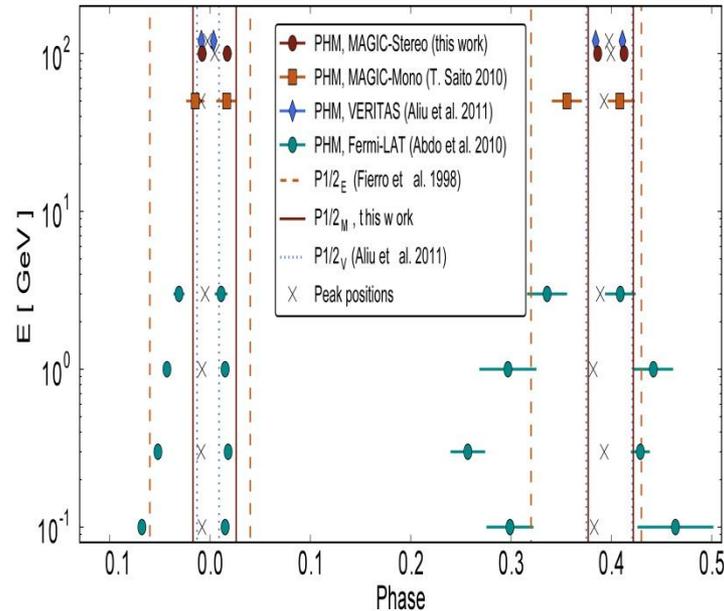
# §7 ICS spectrum of the Crab pulsar

However, my simulation shows that **P1** shifts into **earlier** phase at higher energies !

This is because I adopt the **vacuum rotating dipole  $B$  field** solution, which is **wrong near the light cylinder**.



Crab Pulsar, Pulse extension and phase definitions

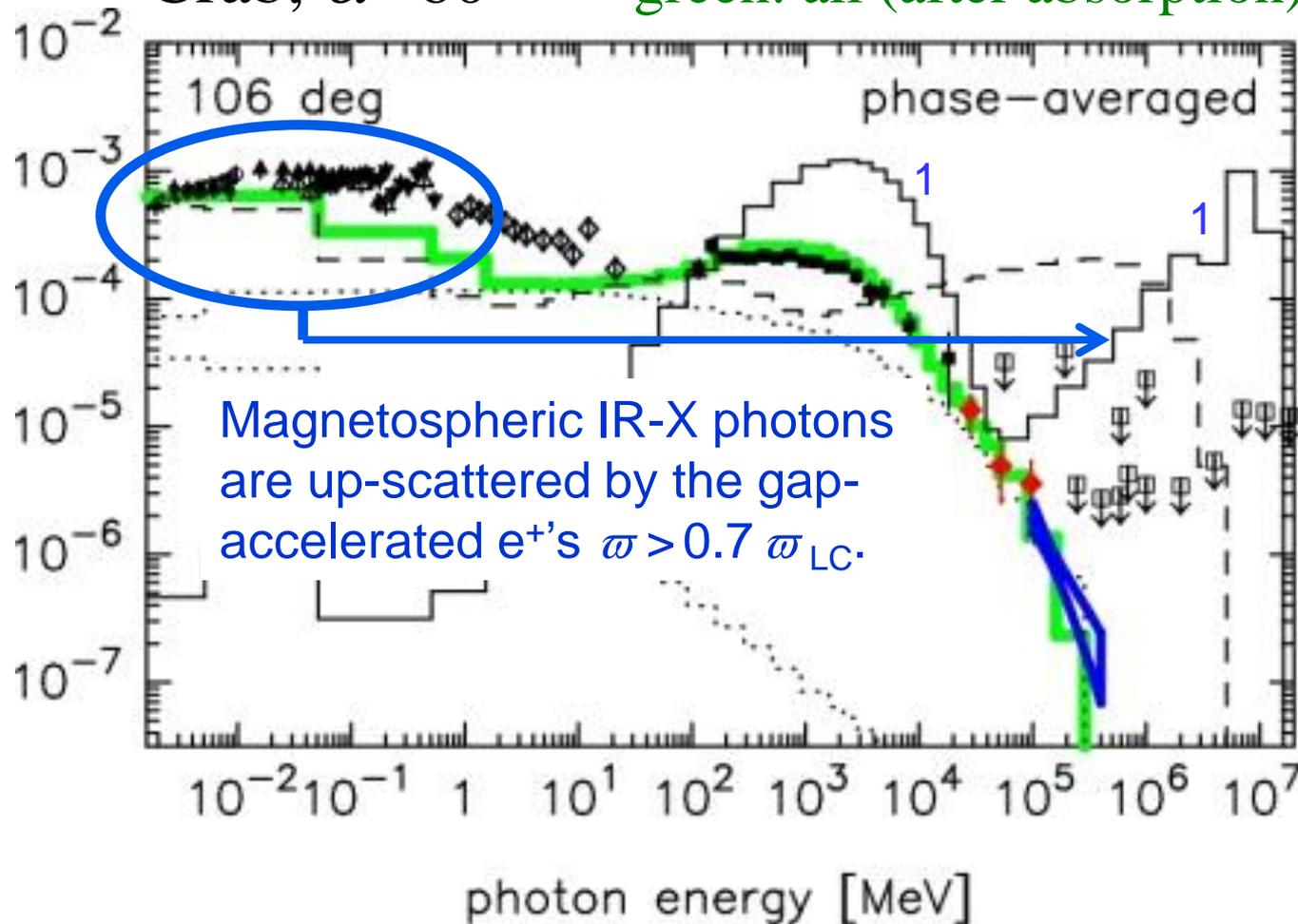


# §7 ICS spectrum of the Crab pulsar

## Phase-averaged spectrum

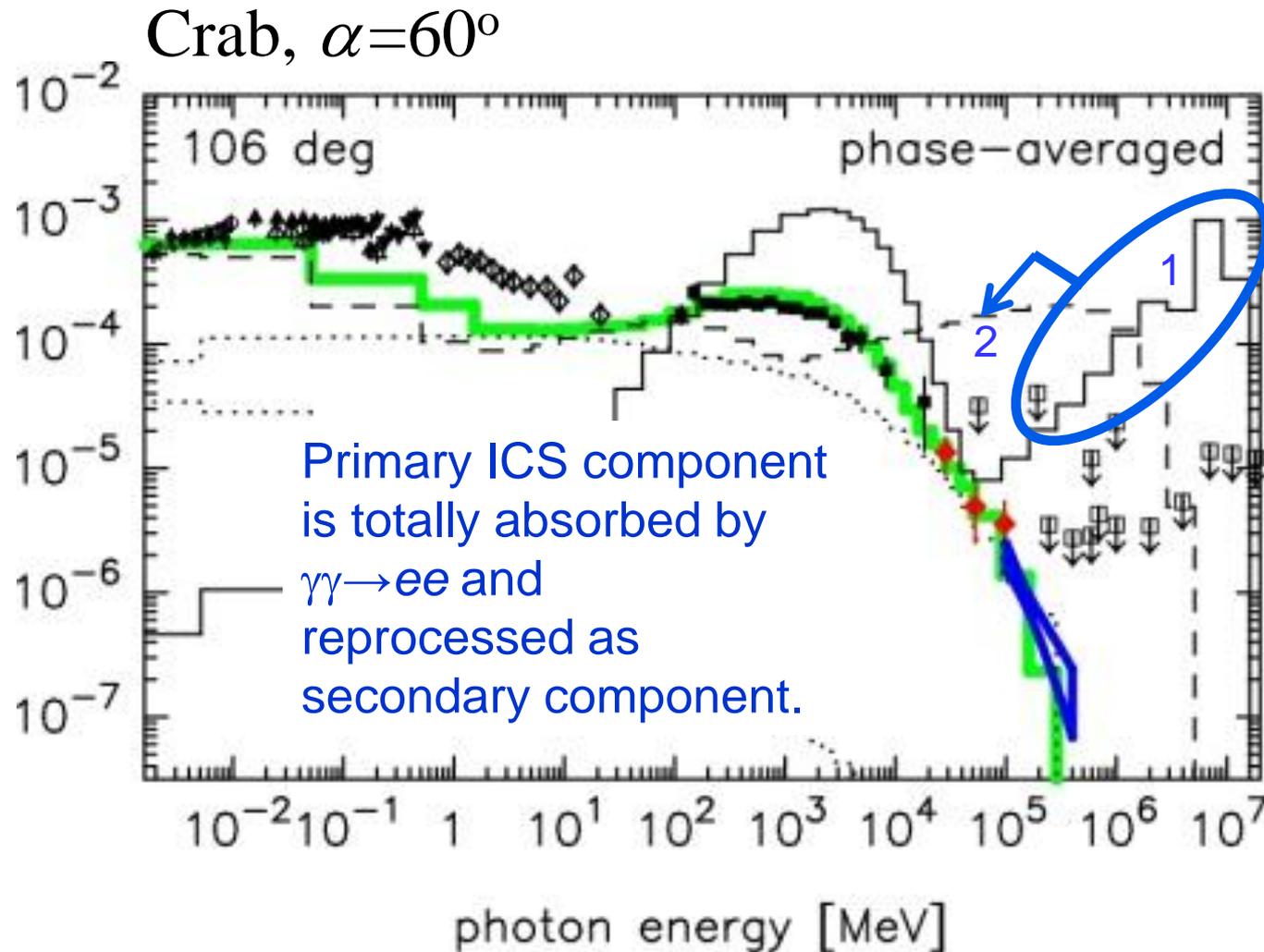
Crab,  $\alpha=60^\circ$

solid: primary (bef. absorption)  
dashed: secondary (bef. absorption)  
dotted: tertiary (bef. absorption)  
green: all (after absorption)



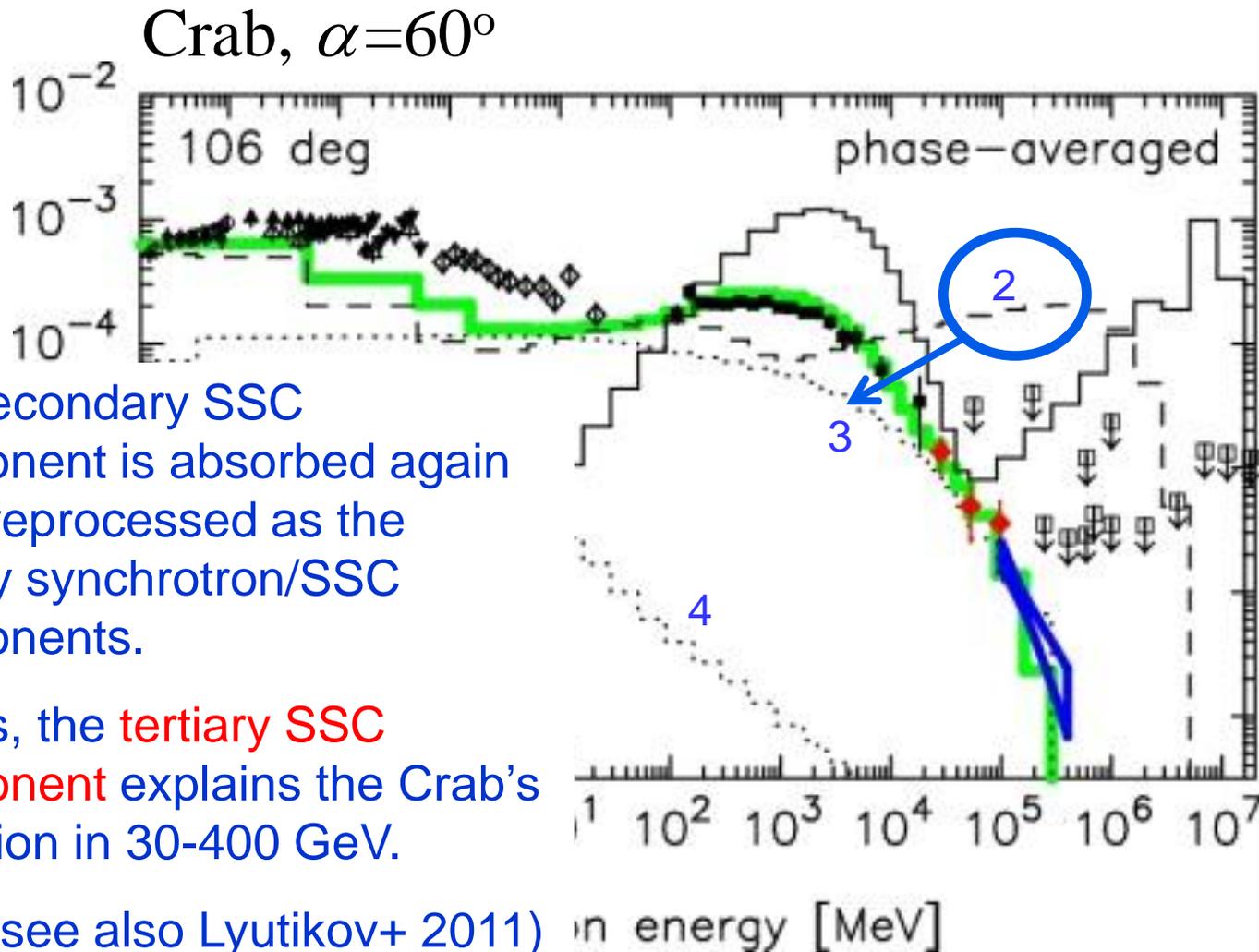
# §7 ICS spectrum of the Crab pulsar

## Phase-averaged spectrum



# §7 ICS spectrum of the Crab pulsar

## Phase-averaged spectrum



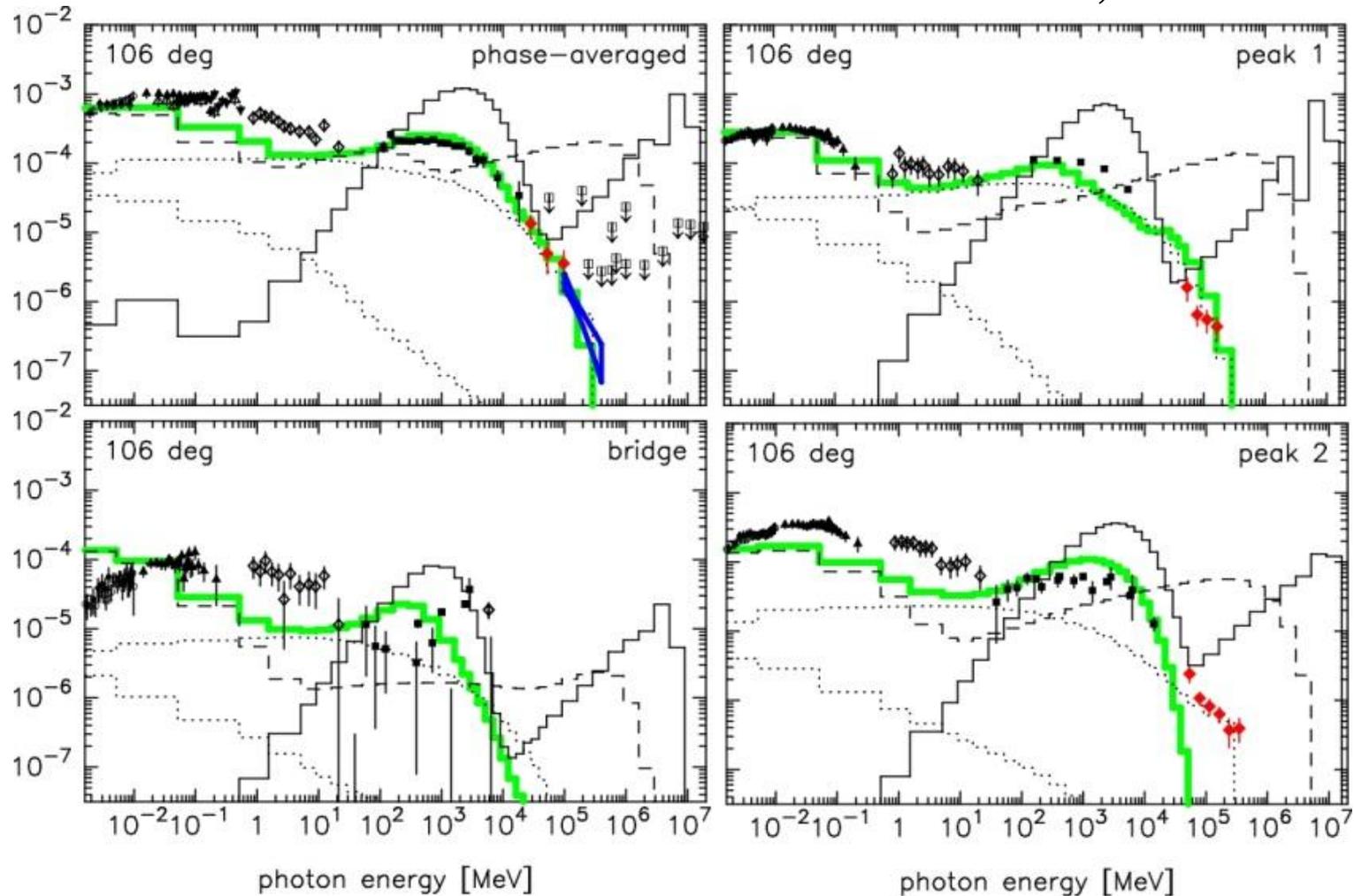
The secondary SSC component is absorbed again to be reprocessed as the tertiary synchrotron/SSC components.

That is, the **tertiary SSC component** explains the Crab's pulsation in 30-400 GeV.

(see also Lyutikov+ 2011)

# §7 ICS spectrum of the Crab pulsar

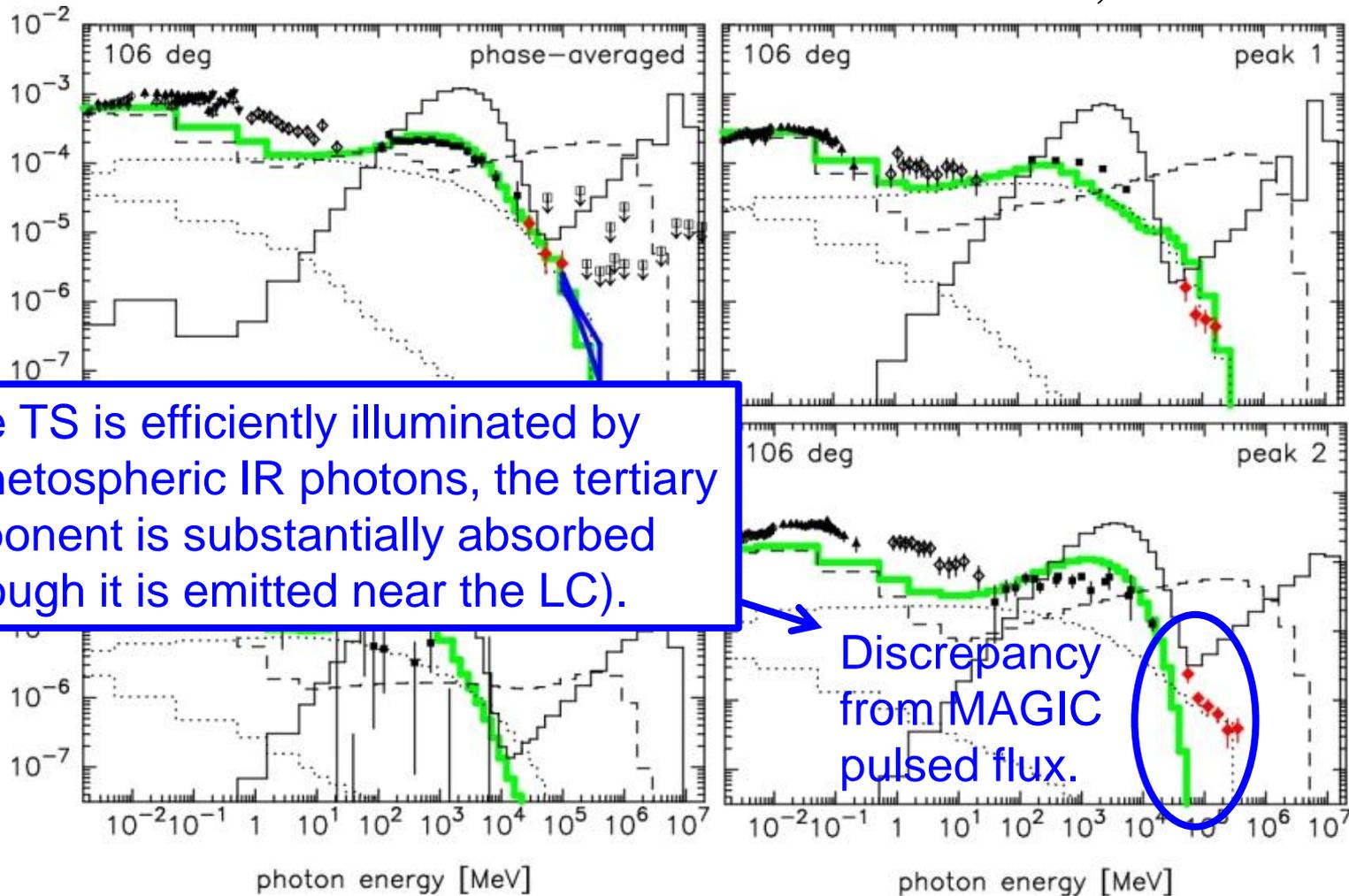
Phase-resolved spectrum (in LAT-defined phase bins):  
Crab,  $\alpha=60^\circ$



# §7 ICS spectrum of the Crab pulsar

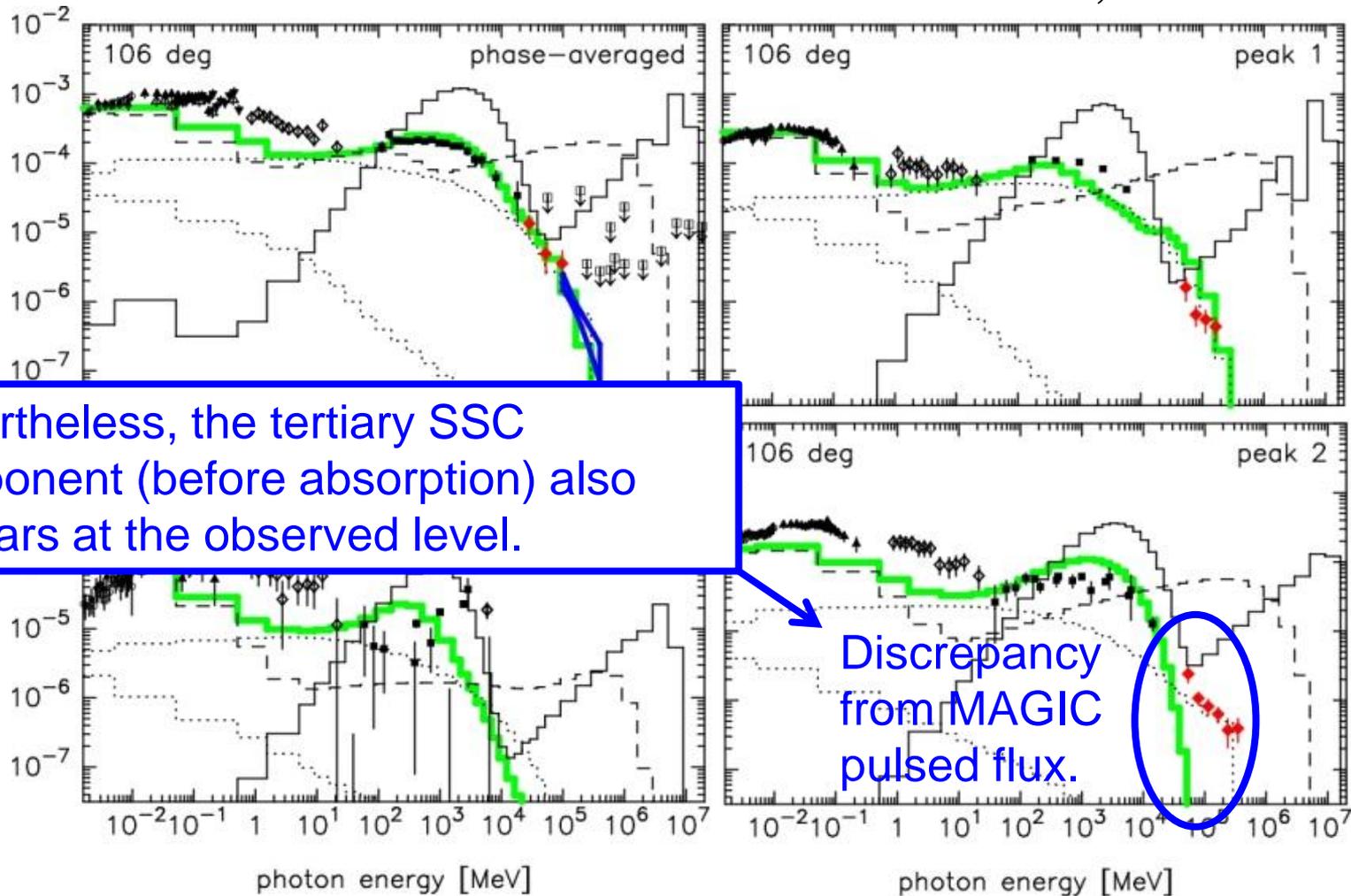
Phase-resolved spectrum (in LAT-defined phase bins):

Crab,  $\alpha=60^\circ$



# §7 ICS spectrum of the Crab pulsar

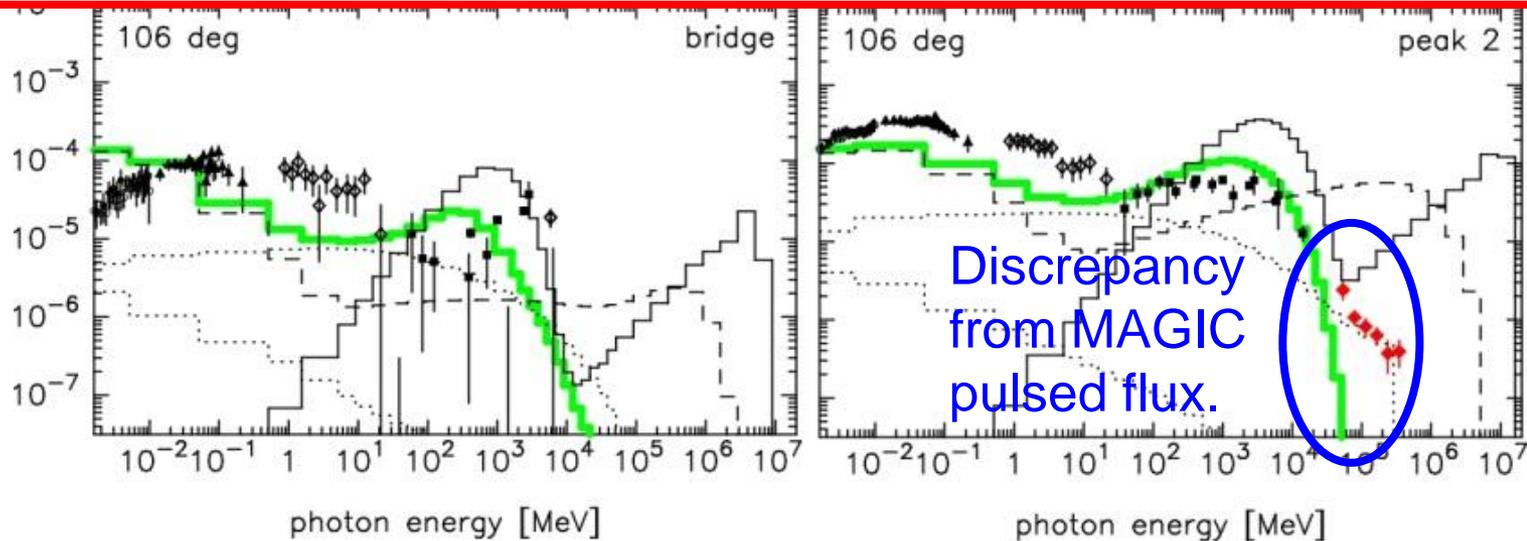
Phase-resolved spectrum (in LAT-defined phase bins):  
Crab,  $\alpha=60^\circ$



# §7 ICS spectrum of the Crab pulsar

Two possibilities to get correct P2 spectrum:

- (1) In TS, photons intensity concentrates in a small solid angle due to special relativistic effects. Thus, if I adopt finer grids for photon propagation directions,  $\gamma\gamma \rightarrow ee$  optical depth may decrease in the TS, which helps the tertiary SSC component appear without absorption.
- (2) The vacuum rotating dipole **B** field structure should be replaced with a more realistic one, e.g., force-free solution.



# Summary

- We can now solve pulsed high-energy emissions from the set of Maxwell ( $\text{div}\mathbf{E}=4\pi\rho$ ) and Boltzmann eqs., if we specify  $P$ ,  $dP/dt$ ,  $\alpha_{\text{incl}}$ ,  $kT_{\text{NS}}$ . We no longer have to assume the gap geometry,  $E_{\parallel}$ ,  $e^{\pm}$  distribution functions.
- By SSC of secondary/tertiary pairs, Crab's total pulse spectrum shows a power-law-like shape.
- P2 exhibits harder curvature spectrum than P1, because actual particle path has greater curvature radius in the TS.
- Millisecond pulsar death line, which was obtained both analytically and numerically, is now being checked by Fermi groups.



Thank you.

# §1 3-D Formalism of Gap Electrodynamics

Assuming  $\partial_t + \Omega \partial_\phi = 0$ , we solve the  $e^\pm$ 's Boltzmann eqs.

$$\frac{\partial N_\pm}{\partial t} + \vec{v} \cdot \nabla N_\pm + \left( e \vec{E}_\parallel + \frac{\vec{v}}{c} \times \vec{B} \right) \cdot \frac{\partial N_\pm}{\partial \vec{p}} = S_{IC} + S_{SC} + \int \alpha_\nu d\nu \int \frac{I_\nu}{h\nu} d\omega$$

together with the radiative transfer equation,

$$\frac{dI_\nu}{dl} = -\alpha_\nu I_\nu + j_\nu$$

$N_\pm$ : positronic/electronic spatial # density,

$E_\parallel$ : magnetic-field-aligned electric field,

$S_{IC}$ : ICS re-distribution function,  $d\omega$ : solid angle element,

$I_\nu$ : specific intensity,  $l$ : path length along the ray

$\alpha_\nu$ : absorption coefficient,  $j_\nu$ : emission coefficient



*§3 Spectral Hardening in the  
Trailing Peak*