Basic Emission mechanisms in pulsar and black-hole magnetospheres

March 4, 2013 National Center for Theoretical Sciences, NTHU

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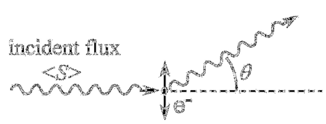
§1 Inverse-Compton Scatterings

From Rybicki & Lightman (1979), "Radiative Processes in Astrophysics"

§1 Inverse-Compton Scatterings

Emission processes can be interpreted as the Compton scatterings of real or virtual photons. Thus, let us begin by considering its classical limit, Thomson scatterings,

A free e^- oscillates in the $\boldsymbol{E}$ field to radiate (by dipole formula)



$$\frac{dP}{d\Omega} = \langle S \rangle \frac{d\sigma}{d\Omega} \cdot \qquad d\Omega = 2\pi \sin\theta d\theta$$

For unpolarized radiation field, the differential cross becomes

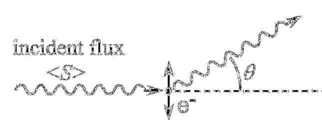
$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2}\left(1+\cos^2\theta\right) \text{, where } r_0 \equiv \frac{e^2}{m_e c^2}$$

§1 Inverse-Compton Scatterings

The total cross section becomes the Thomson cross section,

$$\sigma_T = \int \frac{d\sigma}{d\Omega} d\Omega = 2\pi r_0^2 \int_{-1}^{1} (1-\mu^2) d\mu = \frac{8\pi}{3} r_0^2$$

Note that the scattering can also occur if the incident photon is a virtual photon (§ 3).



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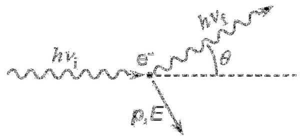
§1 *Inverse-Compton Scatterings*

If incident photon energy becomes comparable or greater than $m_e c^2$, quantum effects appear in two ways:

electron rest frame



- recoil of e^-,
- reduction of cross section.

Consider the scattering of a photon off an electron in the electron rest frame.

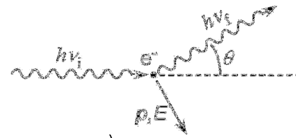
Energy and momentum conservation gives

$$h\nu_f = \frac{h\nu_i}{1 + \frac{h\nu_i}{m_e c^2}(1-\cos\theta)}$$

§1 *Inverse-Compton Scatterings*

QED gives the differential cross section for unpolarized radiation, the Klein-Nishina formula,

electron rest frame



$$\frac{d\sigma}{d\Omega} = \frac{3}{16\pi}\sigma_T \left(\frac{\nu_f}{\nu_i}\right)^2 \left(\frac{\nu_i}{\nu_f} + \frac{\nu_f}{\nu_i} - \sin^2\theta\right),$$

where

$$\sigma_T \equiv \frac{8\pi}{3} r_0^2 = 6.652462\times10^{-25}\ \mathrm{cm}^2$$

§1 *Inverse-Compton Scatterings*

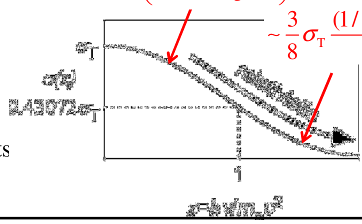
Integrating $d\sigma/d\Omega$ over the solid angle, we obtain the total cross section

$$\sigma = \sigma_T \cdot \frac{3}{4}\left\{\frac{1+x}{x^3}\left[\frac{2x(1+x)}{1+2x} - \ln(1+2x)\right] + \frac{\ln(1+2x)}{2x} - \frac{1+3x}{(1+2x)^2}\right\}$$

where $x \equiv h\nu / m_e c^2$. $\sim \sigma_T\left(1 - 2x + \frac{26}{5}x^2\right)$

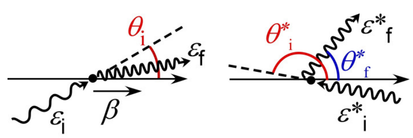
$\sim \frac{3}{8}\sigma_T \frac{(1/2) + \ln 2x}{x}$

Cross section reduces due to quantum effects



§1 *Inverse-Compton Scatterings*

So far, we have considered in the e^- rest frame. However, we must convert quantities into the observer's frame, in which e^- is moving relativistically (w/ Lorentz factor γ).



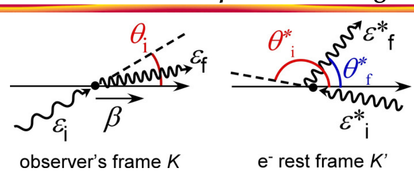
observer's frame K e^- rest frame K'

The Doppler shift formula gives

$$\varepsilon^*_i = \varepsilon_i \gamma(1 - \beta\cos\theta_i)$$

$$\varepsilon_f = \varepsilon_f^* \gamma(1 + \beta\cos\theta_f^*)$$

§1 Inverse-Compton Scatterings



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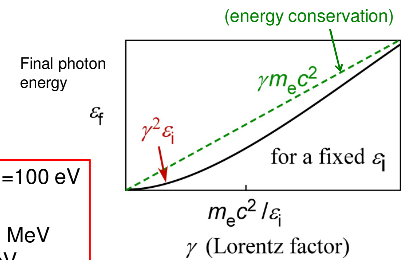
$$\varepsilon^*_i = \varepsilon_i \gamma (1 - \beta \cos\theta_i)$$

$$\varepsilon_f = \varepsilon_f^* \gamma (1 + \beta \cos\theta_f^*)$$

If elastic ($\varepsilon^*_f \approx \varepsilon^*_i$) in the e^--rest frame, we obtain

$$\varepsilon_i : \varepsilon^*_i : \varepsilon_f \approx 1 : \gamma : \gamma^2$$

§1 Inverse-Compton Scatterings



For example, if ε_i =100 eV and γ=10³,
$\varepsilon_i^* \sim \varepsilon_f^* \sim$0.1 MeV
$\varepsilon_f \sim$100 MeV

If elastic ($\varepsilon^*_f \approx \varepsilon^*_i$) in the e^--rest frame, we obtain

$$\varepsilon_i : \varepsilon^*_i : \varepsilon_f \approx 1 : \gamma : \gamma^2$$

§1 Inverse-Compton Scatterings

For isotropic distribution of photons, an e^- emits the inverse-Compton radiation at a rate [ergs s⁻¹],

$$P_{IC} = \frac{4}{3} \sigma_T c \gamma^2 U_{ph} \ ,$$

where U_{ph} denotes the energy density of the photons.

cU_{ph} : incident photon flux [erg s⁻¹ cm⁻²]
$\sigma_T c U_{ph}$: collision rate [erg s⁻¹]
γ^2 : energy amplification factor by a single IC.

§1 Inverse-Compton Scatterings

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$$P_{IC} = \frac{4}{3} \sigma_T c \gamma^2 U_{ph} \ ,$$

where U_{ph} denotes the energy density of the photons.
The factor 4/3 comes from the angle average of

$$\left\langle (1-\beta\cos\theta)^2 \right\rangle = 1 + \frac{\beta^2}{3} = \frac{4}{3} ,$$

which comes from the Lorentz transformation,

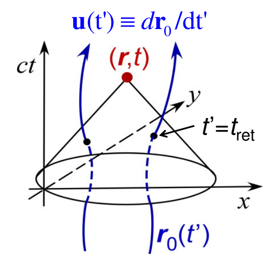
$$\varepsilon^*_i = \varepsilon_i \gamma (1 - \beta \cos\theta_i) \ .$$

§2 Radiation from Relativistic Charges

From Rybicki & Lightman (1979),
"Radiative Processes in Astrophysics"

§2 Radiation from Relativistic Charges

Consider a charge q moving along a world line, $\boldsymbol{r}'=\boldsymbol{r}_0(t')$. It produces an electro-magnetic field at point $(\boldsymbol{r},t)$, if it is causality connected to $\boldsymbol{r}'$.



§2 Radiation from Relativistic Charges

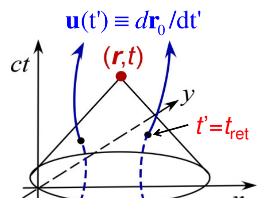
Maxwell eq. $\nabla^2\phi-(1/c^2)\partial^2\phi/\partial^2 t=-4\pi\rho$ gives the Lienard-Wiechart scalar potential,

$$\phi(\mathbf{r},t)=\int_{-\infty}^{t}dt'\iiint d^3r'\ 4\pi\rho(\mathbf{r}',t')\frac{\delta\left(t-t'-|\mathbf{r}-\mathbf{r}'|/c\right)}{4\pi|\mathbf{r}-\mathbf{r}'|}$$

$$=\left[\frac{q}{\kappa R}\right]_{t'=t_{\rm ret}}$$

where [ ] denotes time t' is evaluated at retarded time, $t_{\rm ret}$,

$$t-t_{\rm ret}-\frac{|\mathbf{r}-\mathbf{r}_0(t_{\rm ret})|}{c}=0$$



$\mathbf{R}(t')=\mathbf{r}-\mathbf{r}_0(t'),\ \ R(t')=|\mathbf{R}(t')|,$

$\kappa(t')=1-\mathbf{n}(t')\cdot\mathbf{u}(t')/c\ \ \ \mathbf{n}=\mathbf{R}/R,\ \ \rho(\mathbf{r}',t')=q\delta(\mathbf{r}'-\mathbf{r}_0(t'))$

§2 Radiation from Relativistic Charges

The vector potential, $\boldsymbol{A}(\boldsymbol{r},t)$, is given in the same way,

$$\phi(\mathbf{r},t)=\left[\frac{q}{\kappa R}\right]_{t'=t_{\rm ret}},\ \ \mathbf{A}(\mathbf{r},t)=\left[\frac{q\mathbf{u}}{c\kappa R}\right]_{t'=t_{\rm ret}}.$$

Thus, a moving charge produces an EM field,

$$\mathbf{E}(\mathbf{r},t)=q\left[\frac{(\mathbf{n}-\boldsymbol{\beta})(1-\beta^2)}{\kappa^3R^2}\right]_{t'=t_{\rm ret}}+\frac{q}{c}\left[\frac{\mathbf{n}}{\kappa^3R}\times\left\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right\}\right]_{t'=t_{\rm ret}}$$

$$=\mathbf{E}_{\rm vel}+\mathbf{E}_{\rm acc}$$

$$\mathbf{B}(\mathbf{r},t)=\left[\mathbf{n}\times\mathbf{E}(\mathbf{r},t)\right]_{t'=t_{\rm ret}}$$

§2 Radiation from Relativistic Charges

$$\mathbf{E}(\mathbf{r},t) = q\left[\frac{(\mathbf{n}-\boldsymbol{\beta})(1-\beta^2)}{\kappa^3R^2}\right]_{t'=t_{\rm ret}} + \frac{q}{c}\left[\frac{\mathbf{n}}{\kappa^3 R}\times\left\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right\}\right]_{t'=t_{\rm ret}}$$
$$= \mathbf{E}_{\rm vel}+\mathbf{E}_{\rm acc}$$

Noting $E_{\rm vel} \propto 1/R^2, \quad E_{\rm rad} \propto 1/R$

$$\frac{E_{\rm rad}}{E_{\rm vel}} \sim \frac{R\dot{u}}{c^2} \sim \frac{Ru\nu}{c^2} \sim \frac{u}{c}\frac{R}{\lambda} \quad (\dot{u} \sim u\nu),$$

we find that the $\boldsymbol{E}_{\rm vel}$ dominates in the near zone, $R<\lambda$, while the $\boldsymbol{E}_{\rm acc}$ dominates in the far zone, $R \gg \lambda$.

The velocity field does not carry energy to large distances, while the radiation field does. $(\mathbf{S} = \mathbf{E}\times\mathbf{B}/4\pi)$

§2 Radiation from Relativistic Charges

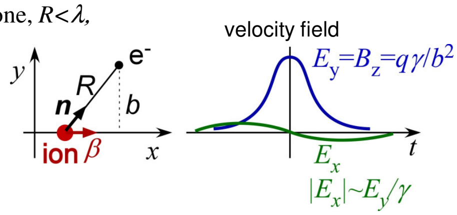
$$\mathbf{E}(\mathbf{r},t) = q\left[\frac{(\mathbf{n}-\boldsymbol{\beta})(1-\beta^2)}{\kappa^3R^2}\right]_{t'=t_{\rm ret}} + \frac{q}{c}\left[\frac{\mathbf{n}}{\kappa^3 R}\times\left\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right\}\right]_{t'=t_{\rm ret}}$$
$$= \mathbf{E}_{\rm vel}+\mathbf{E}_{\rm acc}$$

Using the velocity field, we can now understand the radiation from moving charges in an external EM field, in terms of the IC scatterings of virtual quanta.

§2 Radiation from Relativistic Charges

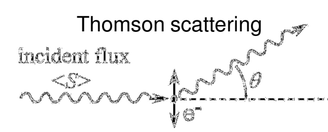
$$\mathbf{E}(\mathbf{r},t) = q\left[\frac{(\mathbf{n}-\boldsymbol{\beta})(1-\beta^2)}{\kappa^3R^2}\right]_{t'=t_{\rm ret}} + \frac{q}{c}\left[\frac{\mathbf{n}}{\kappa^3 R}\times\left\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right\}\right]_{t'=t_{\rm ret}}$$
$$= \mathbf{E}_{\rm vel}+\mathbf{E}_{\rm acc}$$

Consider e^--ion Bremsstrahlung (free-free emission). In the e^- rest frame, relativistic ion produces a pulse of $\mathbf{E}_{\rm vel}$ in the near zone, $R<\lambda$,

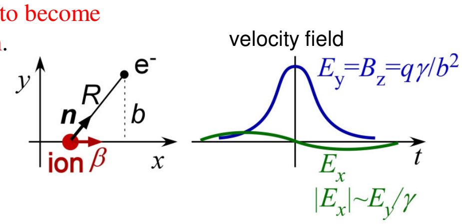


§2 Radiation from Relativistic Charges

Because of this pulsed $E_{\rm vel}$, e^- oscillates to radiate by the dipole formula. That is, a virtual photon, which does not carry energy to infinity, is up-scattered to become a real photon.



= Relativistic bremsstrahlung



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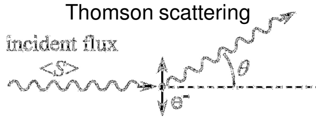
Because of this pulsed E_{vel}, e^- oscillates to radiate by the dipole formula. That is, a virtual photon, which does not carry energy to infinity, is up-scattered to become a real photon.

Thomson scattering



= Relativistic bremsstrahlung

The same idea can be applied when a charge is moving in an external magnetic field.

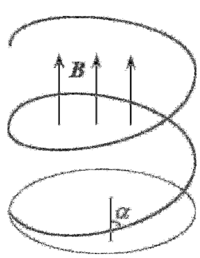
→ Synchrotron radiation

§3 Synchrotron Radiation

From Rybicki & Lightman (1979), "Radiative Processes in Astrophysics"

§3 Synchrotron Radiation

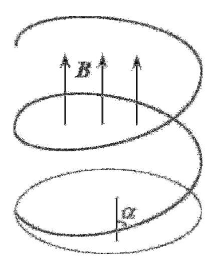
Consider a charge q moving in an external magnetic field, $\boldsymbol{B}$.



§3 Synchrotron Radiation

In e^--rest frame, time-varying $\boldsymbol{E}_{vel}$ arises.

Then, e^- oscillates to emit synchrotron photons.



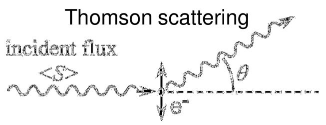
§3 Synchrotron Radiation

In e^--rest frame, time-varying $\boldsymbol{E}_{\mathrm{vel}}$ arises.

Then, e^- oscillates to emit synchrotron photons.

Virtual photons have energy, $\sim \hbar\omega_{\mathrm{cyclotron}} \sin\alpha = \hbar \dfrac{qB}{m_e c} \sin\alpha$

Up-scattered photon has energy $\sim \gamma^2 \hbar\omega_{\mathrm{cyclotron}} \sin\alpha$



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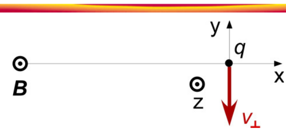
By Lorentz transformation, we obtain $\mathbf{E}_{\mathrm{vel}} = -\gamma B \sin\alpha \cdot \mathbf{e}_r$ in e^- rest frame.

§3 Synchrotron Radiation

Lorentz transformation



In observer's frame,

$$\mathbf{E} = 0,\ \mathbf{B} = B\mathbf{e}_z$$

In e^--rest frame,

$$E_y^{'} = 0$$

$$E_x^{'} = \gamma\left(E_x - \frac{v_\perp}{c} B\right) = -\gamma B \sin\alpha$$

$$E_z^{'} = 0$$

Acceleration:

$$ma^{'} = q(-\gamma B \sin\alpha) \ \Rightarrow\ \left|a^{'}\right| = \frac{q\gamma B \sin\alpha}{m}$$

§3 Synchrotron Radiation

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§3 Synchrotron Radiation

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Acceleration: $ma' = q(-\gamma B \sin\alpha) \Rightarrow |a'| = \frac{q\gamma B \sin\alpha}{m}$

Larmor's formula:

Synchrotron power $P' = \frac{2}{3}\frac{q^2}{c^3}|a'|^2 = \frac{2}{3} r_0^2 c B^2 \gamma^2 \sin^2\alpha$

§3 Synchrotron Radiation

e^- rest frame: Sync. power $P' = \frac{2}{3}\frac{q^2}{c^3} a^2 = \frac{2}{3} r_0^2 c B^2 \gamma^2 \sin^2\alpha$

Total radiation power is Lorentz invariant for any emission with front-back symmetry.

Consider instantaneous rest frame K' (e^- motion is NR).

Larmor's formula, $\left(\frac{dW}{dt d\Omega}\right)' = \frac{q^2 a^2}{4\pi c^3} \sin^2\Theta'$ gives

$$\frac{dp_z'}{dt'} = \frac{1}{c}\int \left(\frac{dW}{dt d\Omega}\right)' \cos\Theta' d\Omega' = \frac{q^2 a^2}{2c^3}\int_{-1}^{1} (1-\mu^2)\mu \, d\mu = 0,$$

$$\frac{dp_x'}{dt'} = \frac{1}{c}\int \left(\frac{dW}{dt d\Omega}\right)' \sin\Theta' \cos\varphi' d\Omega' = 0, \qquad \frac{dp_y'}{dt'} = 0.$$

§3 Synchrotron Radiation

e^- rest frame: Sync. power $P' = \frac{2}{3}\frac{q^2}{c^3} a^2 = \frac{2}{3} r_0^2 c B^2 \gamma^2 \sin^2\alpha$

Total radiation power is Lorentz invariant for any emission with front-back symmetry.

That is, in K', radiation momentum vanishes, $\frac{d\mathbf{p}'}{dt'} = 0.$

In observer's frame K, we obtain $dW = \gamma dW'$.

Also, we obtain $dt = \gamma dt'$, because dt' is the proper time.

Therefore, the total power becomes

$$P = \frac{dW}{dt}, \quad P' = \frac{dW'}{dt'}, \quad \text{that is, } P = P'$$

§3 Synchrotron Radiation

In e^--rest frame, time-varying $\boldsymbol{E}_{\text{vel}}$ arises.

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Up-scattered photon has energy $\sim \gamma^2 \hbar\omega_{\text{cyclotron}} \sin\alpha$

In e^- rest frame, we obtain $\mathbf{E}_{\text{vel}} = -\gamma B \sin\alpha \cdot \mathbf{e}_r$

Acceleration: $ma' = q(-\gamma B \sin\alpha) \Rightarrow |a'| = \frac{q\gamma B \sin\alpha}{m}$

In observer's frame,

Synchrotron power $P = \frac{2}{3}\frac{q^2}{c^3} a^2 = \frac{2}{3} r_0^2 c B^2 \gamma^2 \sin^2\alpha$

§3 Synchrotron Radiation

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Next, let us consider the synchrotron photon energy.

§3 Synchrotron Radiation

For photon energy, a detailed argument shows an additional factor, 3/2, arises. Thus, the synchrotron characteristic energy becomes,

$$\hbar\omega_c = \frac{3}{2}\gamma^2\hbar\omega_{\rm cyclotron}\sin\alpha = \frac{3}{2}\gamma^2\hbar\frac{qB}{m_e c}\sin\alpha = \frac{3}{2}\hbar\gamma^3\frac{c}{r_g},$$

where the gyro radius is defined by $r_g \equiv \dfrac{\gamma m_e c^2}{qB\sin\alpha}$.

Since $\hbar\omega_{\rm cyclotron} = 11.576765\left(\dfrac{B}{10^{12}\,{\rm G}}\right)$ keV , synchrotron photons have the typical energy,

$$\hbar\omega_c = 17.365148\left(\frac{B}{10^{12}\,{\rm G}}\right)\gamma^2\sin\alpha \quad {\rm keV}$$

§3 Synchrotron Radiation

The synchrotron radiation power can be estimated by

$$P_{\rm synch} = N_{\rm virtual}\sigma_T c\gamma^2\hbar\omega_{\rm cyclotron}$$
$$\approx \sigma_T c\gamma^2 U_B ,$$

where $U_B = B^2/8\pi \approx N_{\rm virtual}\hbar\omega_{\rm cyclotron}$.

For an isotropic distribution of α, a detailed argument shows $P_{\rm synch} = \dfrac{4}{3}\sigma_T c\gamma^2 U_B$.

Reminding $P_{\rm IC} = \dfrac{4}{3}\sigma_T c\gamma^2 U_{\rm ph}$, we find $\dfrac{P_{\rm synch}}{P_{\rm IC}} = \dfrac{U_B}{U_{\rm ph}}$.

§3 Synchrotron Radiation

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For an isotropic distribution of α, a detailed argument shows $P_{\rm synch} = \dfrac{4}{3}\sigma_T c\gamma^2 U_B$

Same ICS factor appears for virtual & real photons.

Reminding $P_{\rm IC} = \dfrac{4}{3}\sigma_T c\gamma^2 U_{\rm ph}$, we find $\dfrac{P_{\rm synch}}{P_{\rm IC}} = \dfrac{U_B}{U_{\rm ph}}$.

Coefficients weakly depend on anisotropy of α or photon field.

§3 Synchrotron Radiation

The trans-field motion is quantized with discrete energy values, called Landau levels,

$$E = \hbar\omega_{\rm cyclotron}\left(n + \frac{-m+|m|}{2} + \frac{1}{2}\right),$$

$$n + \frac{-m+|m|}{2} = 0,\ 1,\ 2,\ 3,\ \ldots$$

n: radial quantum number
m: magnetic quantum number

The ground state has the "zero-point" energy, $\dfrac{\hbar\omega_{\rm cyclotron}}{2}$.

After falling onto the ground state, an e^- no longer emits synchrotron photons.

§4 Synchro-curvature Radiation

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The longitudinal motion (along $\boldsymbol{B}$) is not quantized, allowing continuous $P_{||}$.

After the particle falls onto the ground Landau state, only the longitudinal motion contributes to an emission.

If the macroscopic particle motion follows a curved trajectory with curvature radius R_c, it emits the pure curvature radiation with characteristic energy,

$$\hbar\omega_{\rm curv} = \frac{3}{2}\hbar\gamma^3\frac{c}{R_c}. \qquad \text{cf.} \quad \hbar\omega_c = \frac{3}{2}\hbar\gamma^3\frac{c}{r_g}$$

(synchrotron case)

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cf. synchrotron case

characteristic energy: $\hbar\omega_{\rm curv} = \frac{3}{2}\hbar\gamma^3\frac{c}{R_c}$ | $\hbar\omega_c = \frac{3}{2}\hbar\gamma^3\frac{c}{r_g}$

radiation power: $P_{\rm curv} = \frac{3e^2}{2c^3}\gamma^4\left(\frac{c^2}{R_c}\right)^2$ | $P_{\rm synch} = \frac{3e^2}{2c^3}\gamma^4\left(\frac{c^2}{r_g}\right)^2$

§4 Synchro-curvature Radiation

If the particle has not fallen onto the ground Landau state, but moves along a curved trajectory with curvature radius R_c, it emits the synchro-curvature radiation.

(Cheng & Zhang 1996, ApJ 463, 271-283)

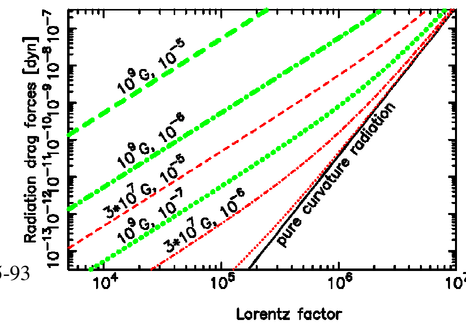
In $\alpha \rightarrow 0$, it reduces to the pure curvature process.
In $R_c \rightarrow \infty$, it reduces to the pure synchrotron process.

General expression of SC radiation is complicated. Thus, we consider typical e^- motion in a pulsar magnetosphere and show how it deviates from the pure curvature process when both ***B*** and α are large.

§4 Synchro-curvature Radiation

If $B > 10^7$G and $\alpha > 10^{-6}$, synchro-curvature process deviates from the pure curvature process

KH 2006,
ApJ 652, 1475-93

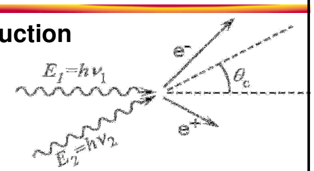


§5 Electron-positron Pair Production

§5 Electron-positron Pair Production

Photon-photon pair production

If two photons collide with angle θ_c, e^--e^+ pair may be produced. The total cross section of $\gamma\gamma \rightarrow ee$ becomes



$$\sigma_p(u) = \frac{3}{16}\sigma_T(1-u^2)\left[(3-u^4)\ln\frac{1+u}{1-u} - 2u(1-u^2)\right],$$

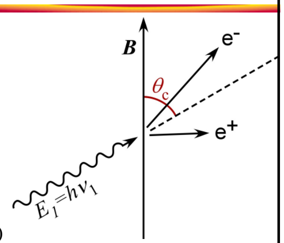
$$\text{where} \quad u(E_1, E_2, \theta_c) \equiv \sqrt{1 - \frac{2}{1-\cos\theta_c}\frac{(m_e c^2)^2}{E_1 E_2}}$$

Note that $\gamma\gamma \rightarrow ee$ takes place only when the threshold is satisfied,

$$E_1 E_2 > \frac{1-\cos\theta_c}{2}(m_e c^2)^2$$

§5 Electron-positron Pair Production

Magnetic pair production



When a photon propagates in a ***B*** field, it may be absorbed to materialize as a pair.

The photon mean-free path to $\gamma B \rightarrow ee$ becomes
(Erber 1966, Rev. mod. Phys. 38, 626)

$$\lambda_B = 600 \frac{c}{\frac{eB}{m_e c} \sin\theta_c} \exp\left[\frac{8}{3}\frac{B_{cr}}{B\sin\theta_c}\frac{m_e c^2}{E_1}\right] \text{ cm},$$

$$\text{where} \quad B_{cr} \equiv \frac{m_e^2 c^3}{e\hbar} = 4.413\times10^{13}\,\text{G}$$