Gravitomagnetism and complex orbit dynamics of spinning compact objects around a massive black hole

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Two papers related to this talk

"Fast spinning pulsars a probes of massive black hole's gravity"
Singh, D., Wu, K. & Sarty, G. D., 2013, MNRAS, to be submitted

"Complex orbital dynamics of a double neutron star systems revolving around a massive black hole"
Remmen, G. N. & Wu, K., 2013, MNRAS, in press
(doi:10.1093/mnras/stt023)

This talk deals with finite size spinning objects in a gravitational field

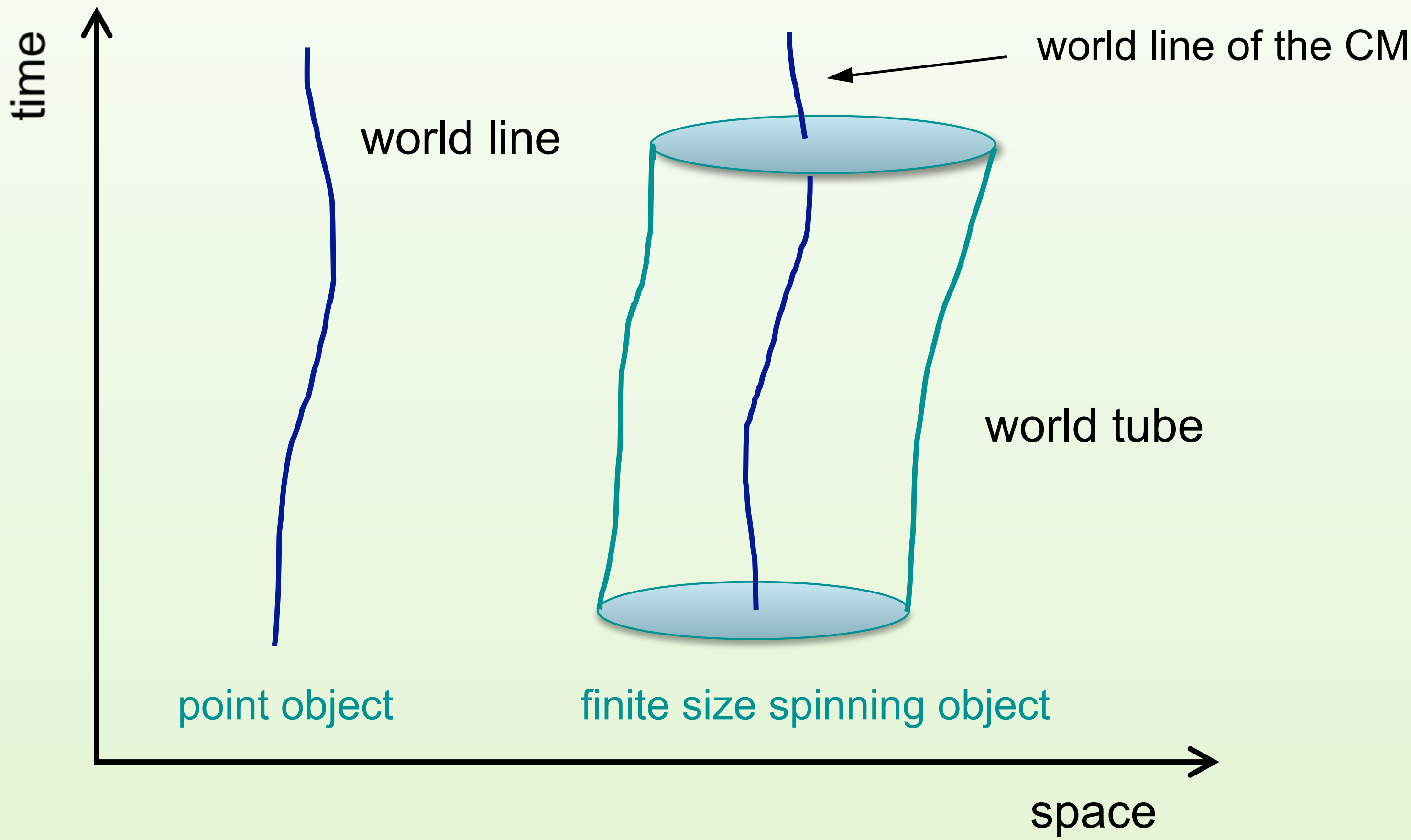
spinning objects: more than a simple geodesic motion

(1) de Sitter precession –
parallel transport of a spin in a non-rotating gravitational field

(2) Lense-Thirring precession –
parallel transport of a spin in a rotating gravitational field (rotational frame dragging)

(3) Thomas precession –
relativistic “addition” of two non-collinear Lorentz boosts (spin-orbit coupling, orbit-orbit coupling)

This talk deals with finite size spinning objects in a gravitational field

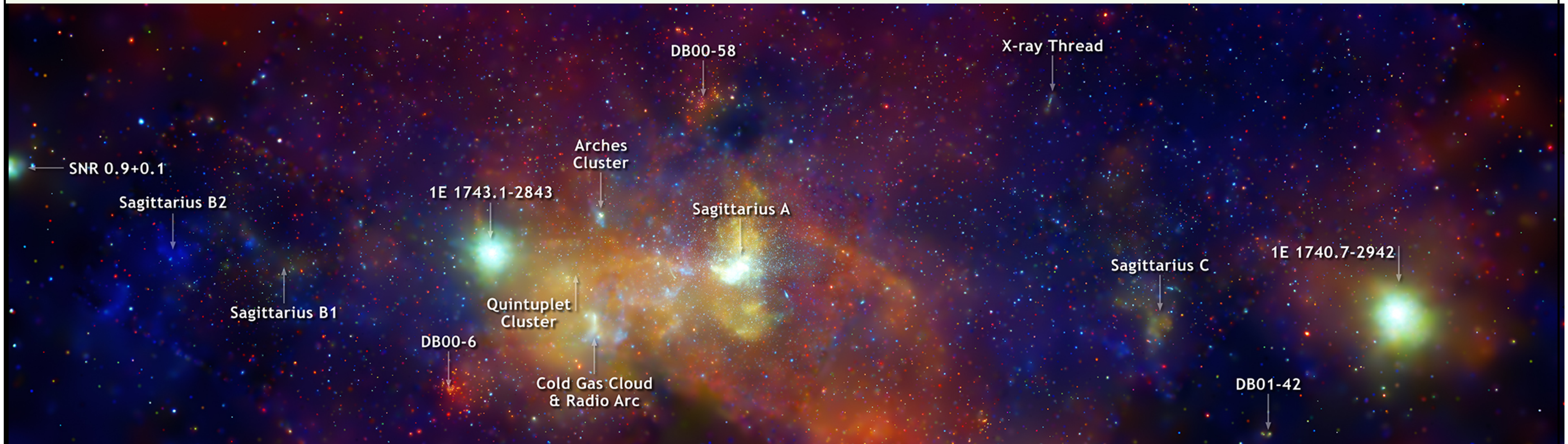


Outline

- Some issues in astrophysics
- Gravitoelectromagnetism (a brief introduction)
- Orbital dynamics of a spinning neutron stars around a massive black hole
- Dynamics of double neutron stars around a supermassive black hole

1. Some issues in astrophysics

Some issues in astrophysics



(credit: NASA Chandra X-ray Observatory)

Some issues in astrophysics

Population of compact objects in the Galactic center

Two estimates:

(1) dynamically relaxed	20,000 - 40,000 stellar mass black holes (Miralda-Escude & Gould 2000)
	~ a few 1,000 neutron stars (Freitag et al. 2006; Wharton et al. 2012)
(2) dynamically un-relaxed	<< 10,000 stellar mass black holes (Merritt 2010; Antonini and Peret 2012)
	>> 1,000 neutron stars (Freitag et al. 2006)

Some issues in astrophysics

Q1: How many neutron stars are there in the Galactic Centre?

Q2: How relaxed or how un-relaxed is the dynamics in the Galactic Centre?

Q3: Are the central regions of the nearby galaxies dynamically relaxed?

Some issues in astrophysics

Mass spectrum of astrophysical black holes

(1) massive and supermassive black holes in the centre of galaxies

$$M \sim 10^5 - 10^9\,\mathrm{M}_\odot$$

(2) stellar-mass black holes as remnants of dead massive stars

$$M \sim 5 - 20\,\mathrm{M}_\odot$$

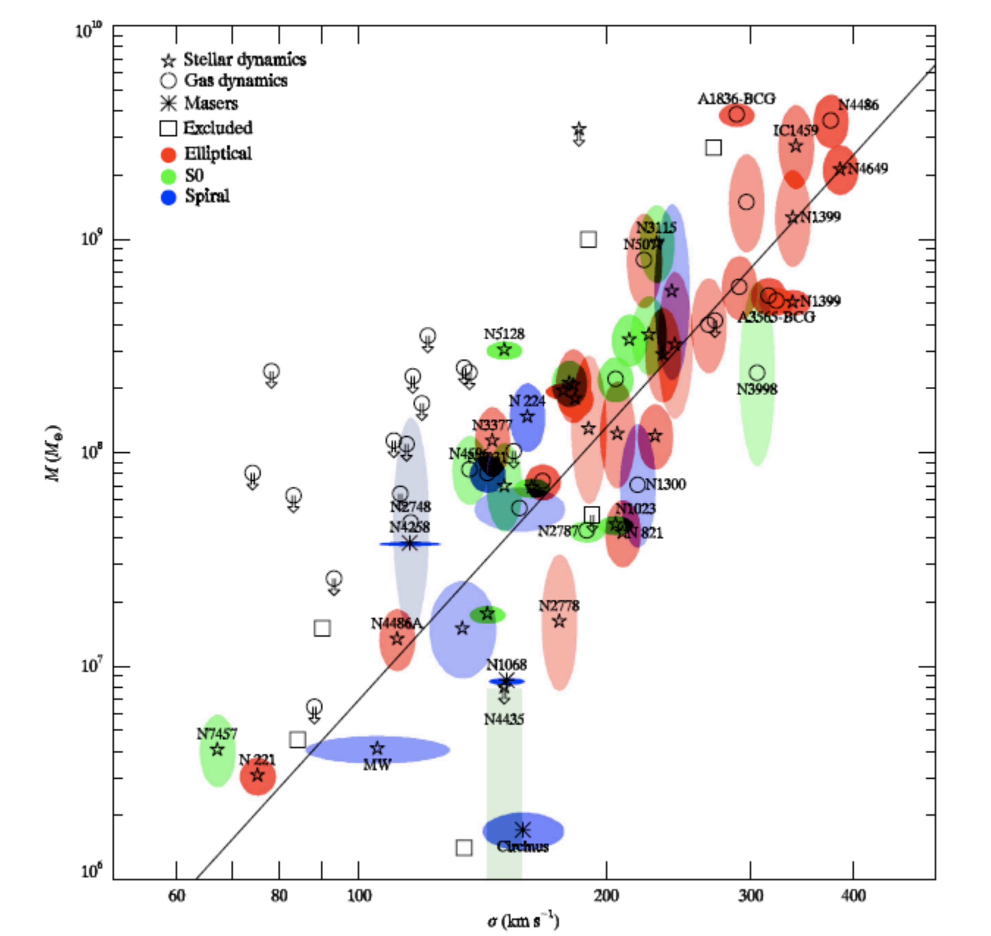
(3) intermediate-mass black holes in galaxies and/or in globular clusters (?)

$$M \sim 10^3 - 10^4\,\mathrm{M}_\odot$$

$$L_{\mathrm{Edd}} = 1.3 \times 10^{38} \left(\frac{M}{\mathrm{M}_\odot}\right) \mathrm{erg\,s}^{-1}$$

$$L_{\mathrm{ULX}} \sim 10^{40} \left(\frac{M}{\mathrm{M}_\odot}\right) \mathrm{erg\,s}^{-1}$$

Some issues in astrophysics

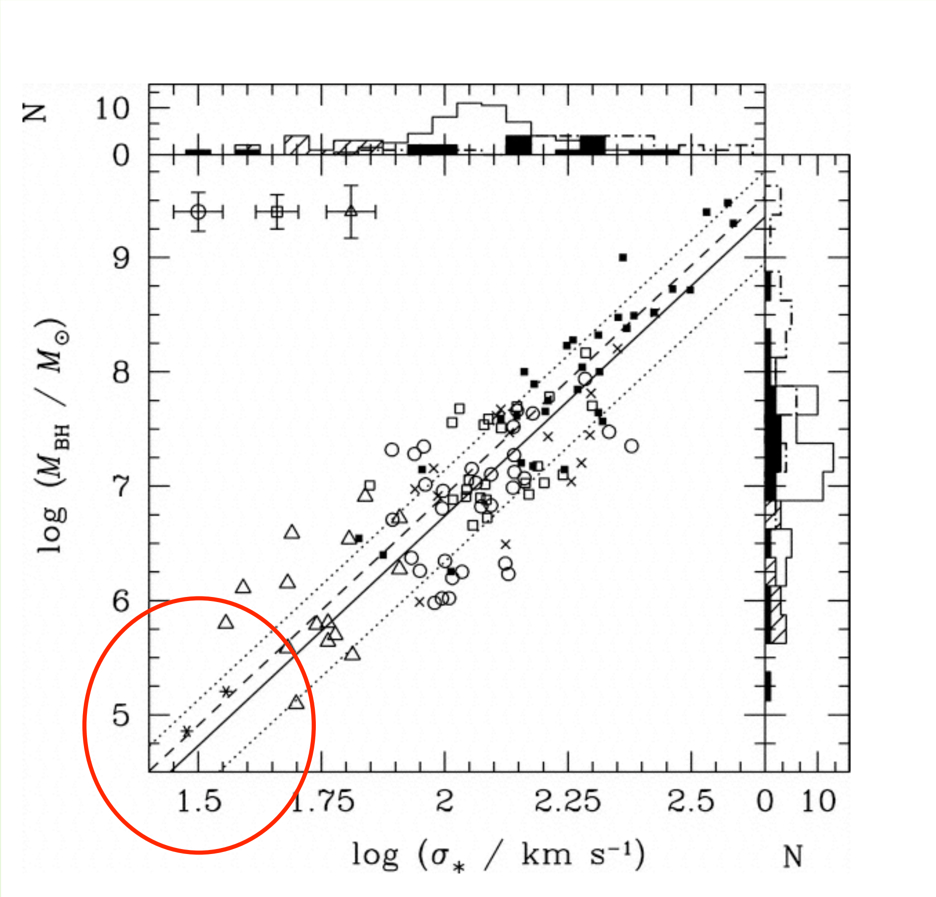


M-σ relation

co-evolution of supermassive black holes and their hosts

(Gueltekin et al. 2009)

Some issues in astrophysics



the M–sigma relation of the nearby galaxies

(Greene and Ho 2006)

Some issues in astrophysics

Issues regarding the low-end of the M-σ relation

- uncertainties in the late-type and small galaxies

  (Greene and Ho 2007, Gueltekin et al. 2011)

- deviation from the M-σ relation and selection bias

  (Graham 2008)

- extrapolation of the low-end of the M-σ relation to include globular clusters

  (Luetzgendorf et al. 2012)

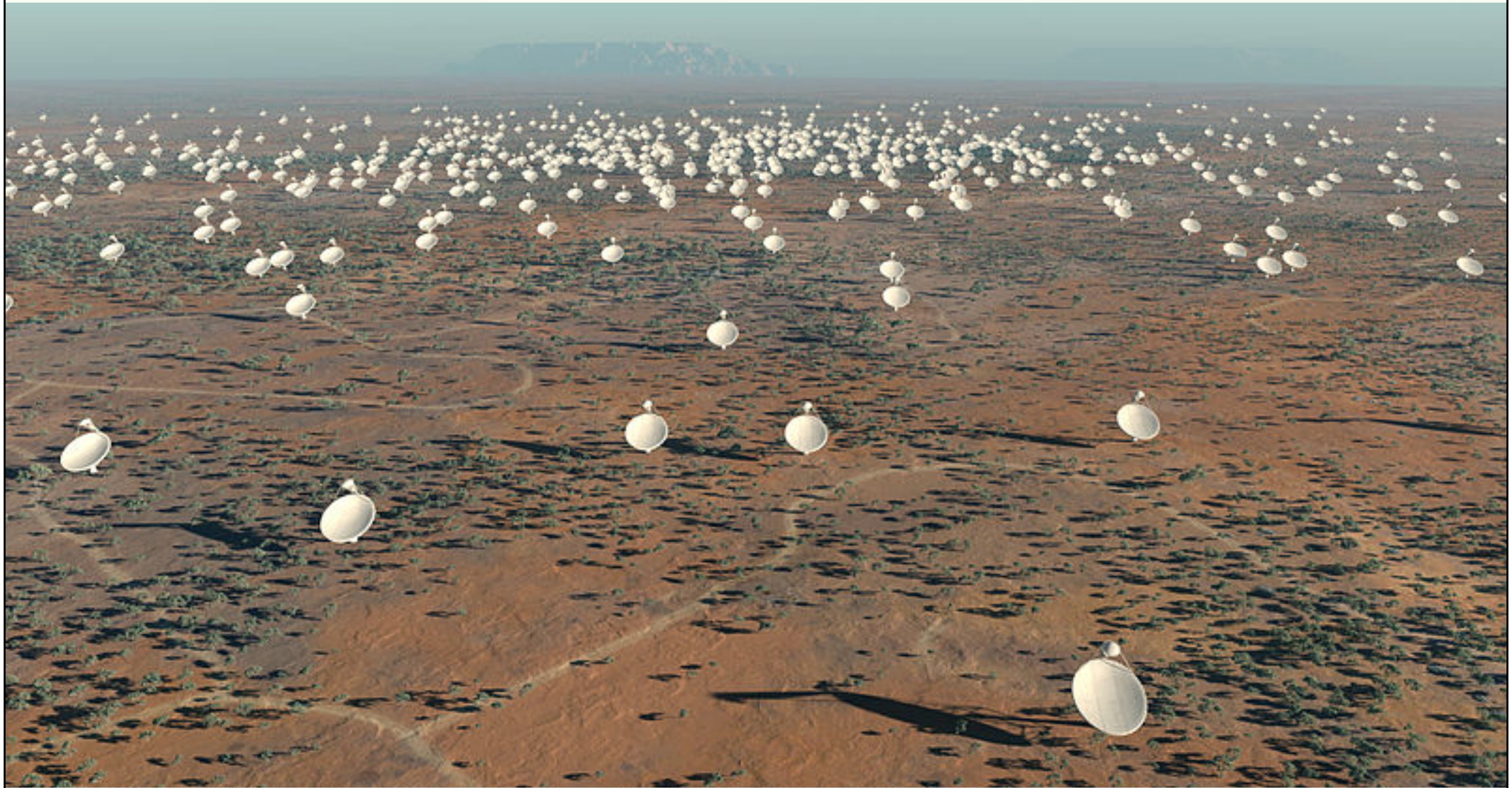
Some issues in astrophysics

Q1: Do dwarf elliptical and dwarf spheroidal galaxies have a massive black hole in their centres?

Q2: Does the M-σ relation still hold for very low mass galaxies?

Q3: Are there any intermediate-mass black holes (residing in the centres of globular clusters)?

Some issues in astrophysics



(credit: SKA Project Development Office and Swinburne Astronomy Production)

2. Gravitoelectromagnetism (GEM) – a very very very …. brief introduction

Gravitoelectromagnetism

A short reminder of classical electrodynamics in the c.g.s. Gaussian units

Maxwell's equation

$$\nabla \cdot \mathbf{E} = 4\pi\rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{j}$$

charge continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0$$

Lorentz force law

$$\mathbf{F} = q\left[\mathbf{E} + (\boldsymbol{\beta} \times \mathbf{B})\right]$$

Gravitoelectromagnetism

Let's start with a perturbation in the Minkowski metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad\qquad x^\mu = (c\,t, x^i)$$

Define $$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h \qquad\qquad h = \mathrm{Tr}\,(h_{\mu\nu})$$

Impose the Lorentz gauge condition $\bar{h}^{\mu\nu}{}_{,\nu} = 0$

The gravitational field equation then takes the form

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

Its retarded solution is

$$\bar{h}^{\mu\nu} = \frac{4G}{c^4} \int d^3\mathbf{x}' \, \frac{T_{\mu\nu}(c\,t - |\mathbf{x} - \mathbf{x}'|, \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}$$

Gravitoelectromagnetism

Insert $T^{00} = c^2 \rho$ ρ matter density

$T^{0i} = c j^i$ $\mathbf{j}\ (= j^i)$ matter current

It follows from the retarded solution that
$$\bar{h}_{00} = \frac{4\Phi}{c^2}$$
$$\bar{h}_{0i} = -\frac{2A_i}{c^2}$$
$$\bar{h}_{ij} \sim \mathcal{O}(c^{-4})$$

We then obtain the GEM scalar and vector potentials

In the far field, they have the asymptotic forms

$$\Phi \longrightarrow \frac{GM}{r} \qquad \mathbf{A}\,(= A^i)\ , \longrightarrow \frac{G}{c}\frac{(\mathbf{J} \times \mathbf{x})}{r^3}$$

Gravitoelectromagnetism

Discard the terms of the order of $\mathcal{O}(c^{-4})$

The Lorentz gauge condition becomes

$$\frac{1}{c}\frac{\partial \Phi}{\partial t} + \nabla \cdot \left(\frac{1}{2}\mathbf{A}\right) = 0$$

The GEM fields are then given by the scalar and vector potentials

$$\mathbf{g} = -\nabla\Phi - \frac{1}{c}\frac{\partial}{\partial t}\left(\frac{1}{2}\mathbf{A}\right)$$

$$\mathbf{h} = \nabla \times \mathbf{A}$$

The factor of $1/2$ is due to the fact gravitational field is spin 2 (whereas electromagnetic field is spin 1)

The GEM charges for test point masses are given by $q_g = \frac{1}{2}q_h = -m$

Gravitoelectromagnetism

It follows that

$$\nabla \cdot \mathbf{g} = 4\pi G\rho$$

$$\nabla \cdot \left(\frac{\mathbf{h}}{2}\right) = 0$$

$$\nabla \times \mathbf{g} = -\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{1}{2}\mathbf{h}\right)$$

$$\nabla \times \left(\frac{\mathbf{h}}{2}\right) = \frac{1}{c}\frac{\partial \mathbf{g}}{\partial t} + \frac{4\pi G}{c}\mathbf{j}$$

These equations are analogous to four Maxwell's equations

They also implicitly imply the charge continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0$$

Gravitoelectromagnetism

The Lagrangian of a test particle is

$$\mathcal{L} = -\gamma m \frac{ds}{dt}$$

In the linear order of the scalar and vector potentials, it is

$$\mathcal{L} = -\gamma m c^2 (1-\beta^2) + \gamma m (1+\beta^2)\Phi - 2\gamma m (\boldsymbol{\beta} \cdot \mathbf{A})$$

Set $\dfrac{\partial \mathbf{A}}{\partial t} = 0$

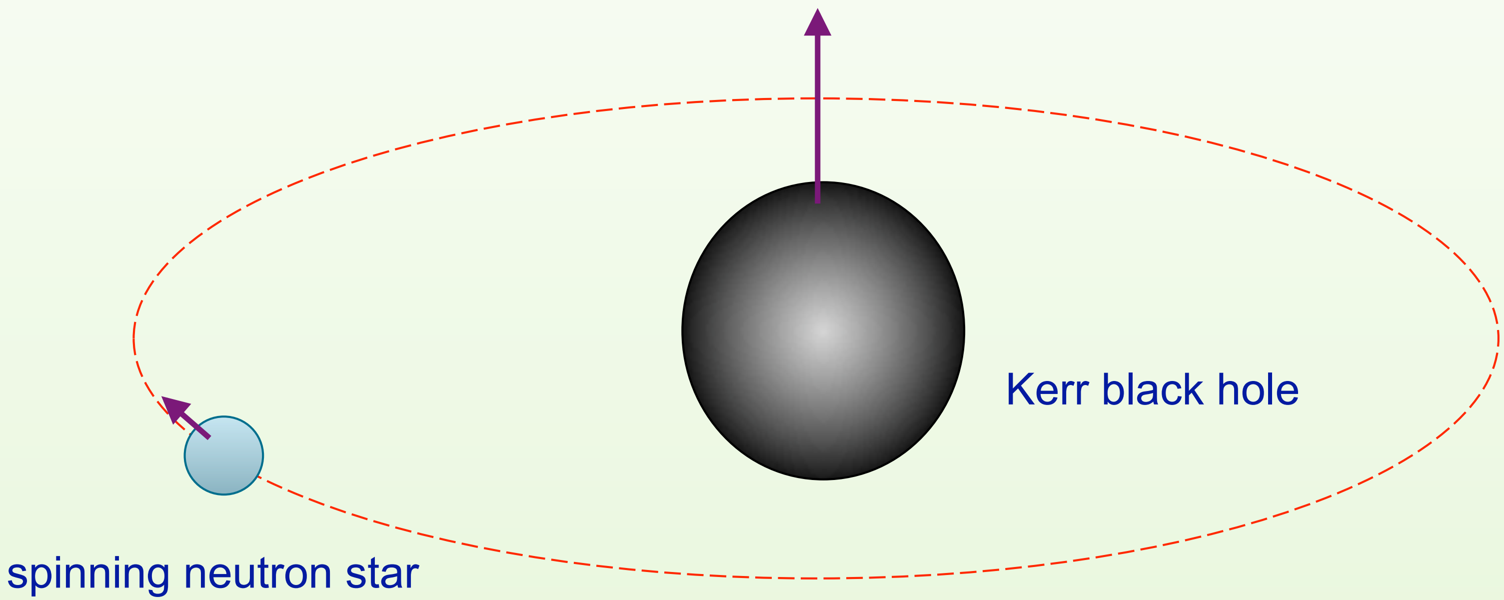
Keep only the lowest order terms in $\boldsymbol{\beta}$

The Euler-Lagrange equation gives the corresponding force equation

$$\begin{aligned}\mathbf{F} &= q_g \mathbf{g} + q_h (\boldsymbol{\beta} \times \mathbf{h}) \\ &= -m\big[\mathbf{g} + 2(\boldsymbol{\beta} \times \mathbf{h})\big]\end{aligned}$$

3. Orbital dynamics of a spinning neutron stars around a massive black hole

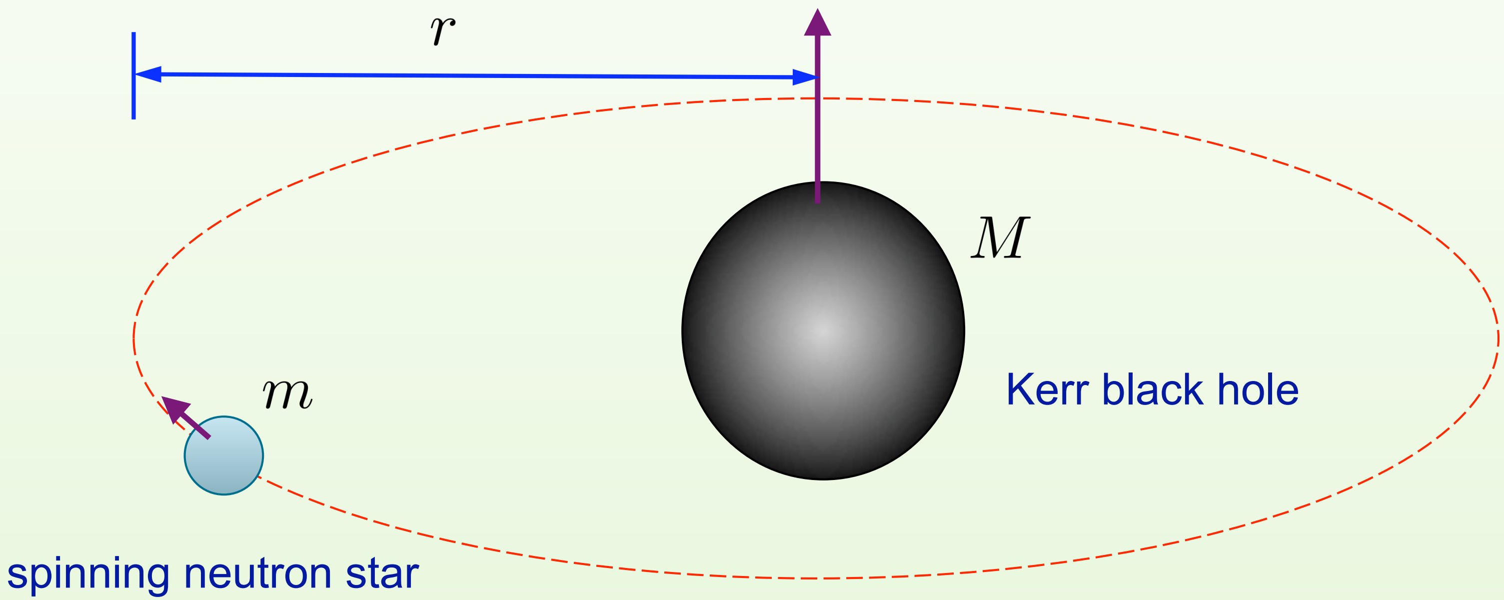
Orbital dynamics of a spinning neutron stars around a massive black hole



$$u^\mu = \frac{dx^\mu}{d\tau} \qquad \text{velocity}$$

$$T^{\mu\nu}{}_{;\nu} = 0 \qquad \text{equation of motion}$$

Orbital dynamics of a spinning neutron stars around a massive black hole

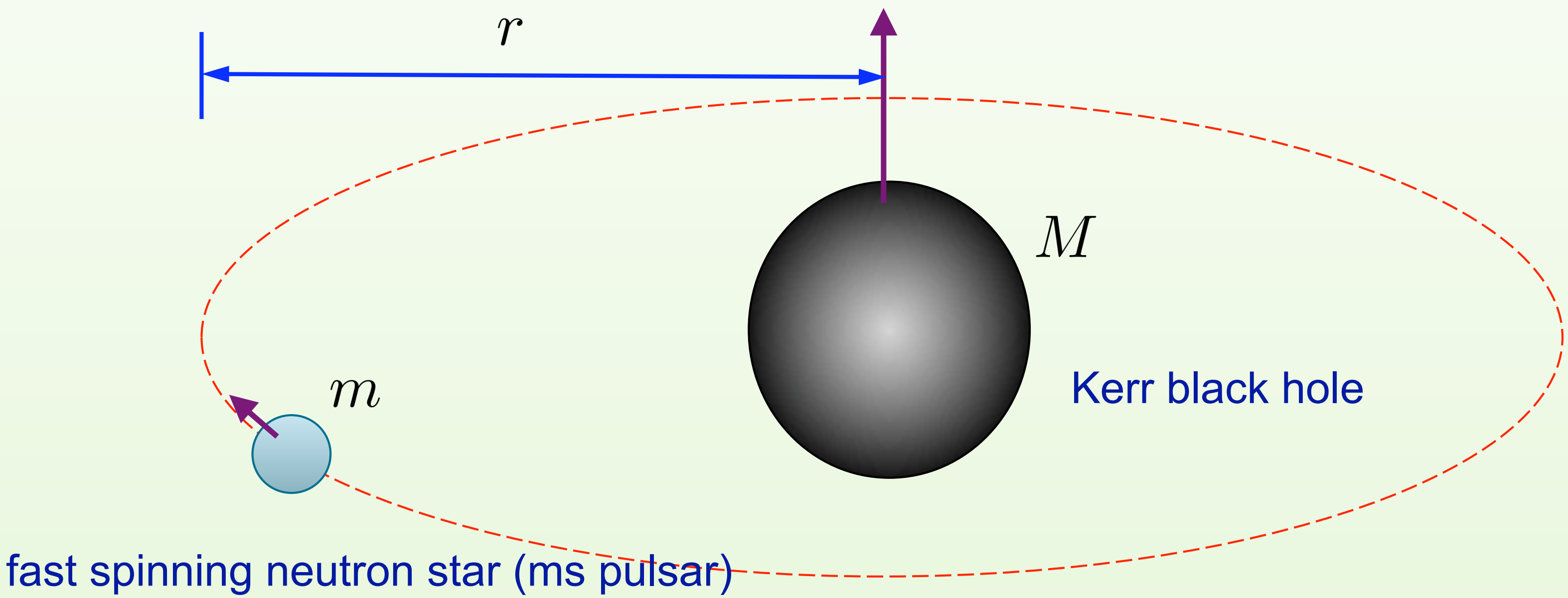


$$r \gg R_{\mathrm{ns}}$$

$$r \gg M \gg m$$

$$-m^2 = p^\mu p_\mu$$

Orbital dynamics of a spinning neutron stars around a massive black hole



$$M = 10^3\,\mathrm{M}_\odot, 10^5\,\mathrm{M}_\odot, 2\times 10^6\,\mathrm{M}_\odot$$
$$m = 1.5\,\mathrm{M}_\odot$$
$$P_{\mathrm{ns}} = 1\,\mathrm{ms}$$

Orbital dynamics of a spinning neutron stars around a massive black hole

Mathisson-Papapetrou-Dixon (MPD) equation

$$\frac{Dp^{\mu}}{d\tau} = -\frac{1}{2} R^{\mu}{}_{\nu\alpha\beta} u^{\nu} s^{\alpha\beta} + \mathcal{F}^{\mu}$$

Dixon force

$$\frac{Ds^{\mu\nu}}{d\tau} = p^{\mu} u^{\nu} - p^{\nu} u^{\mu} + \mathcal{T}^{\mu\nu}$$

Dixon torque

$$s^{\mu\nu} p_{\nu} = 0$$

(Mathisson 1937, Papapetrou 1951, Dixon 1974)

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covariant derivative:

$$\frac{DA^{\beta}{}_{\alpha}}{d\tau} = \frac{dA^{\beta}{}_{\alpha}}{d\tau} + \left(\Gamma^{\beta}_{\mu\gamma} A^{\mu}{}_{\alpha} - \Gamma^{\alpha}_{\mu\gamma} A^{\mu}{}_{\beta}\right) \frac{dx^{\gamma}}{d\tau}$$

Orbital dynamics of a spinning neutron stars around a massive black hole

ignoring quadrupolar or higher-order interactions

$$\mathcal{F}^{\mu} = 0$$

$$\mathcal{T}^{\mu\nu} = 0$$

spin vector and spin tensor of the neutron star (ms pulsar)

$$s_{\mu} = -\frac{1}{2m}\epsilon_{\mu\nu\alpha\beta}p^{\nu}s^{\alpha\beta} \qquad s^{\mu\nu} = \frac{1}{m}\epsilon^{\mu\nu\alpha\beta}p_{\alpha}s_{\beta}$$

$$s^2 = s^{\mu}s_{\mu} = \frac{1}{2}s^{\mu\nu}s_{\mu\nu}$$

the basis of the approximation scheme

$$\left(\frac{p^{\mu}}{m} - u^{\mu}\right) \sim \frac{M}{r}\left[\frac{s}{mr}\right]^2 \ll 1$$

Orbital dynamics of a spinning neutron stars around a massive black hole

Moller radius of the neutron star

$$R_{\mathrm{M}} = \frac{s}{m} \ll r$$

to the first order of $\mathcal{O}(R_{\mathrm{M}}/r)$

$$p^{\nu} \approx m u^{\mu}$$

the equation of motion (a reduced MPD equation) of the system

$$\frac{Du^{\mu}}{d\tau} = -\frac{1}{2m} R^{\mu}{}_{\nu\alpha\beta} u^{\nu} s^{\alpha\beta}$$

$$\frac{Ds^{\mu\nu}}{d\tau} \approx 0$$

$$s_{\mu\nu} u^{\nu} \approx 0$$

Orbital dynamics of a spinning neutron stars around a massive black hole

reduced Mathisson-Papapetrou-Dixon equation

$$\frac{dp^\alpha}{d\tau} = -\Gamma^\alpha_{\mu\nu} p^\mu u^\nu + \lambda \left(\frac{1}{2m} R^\alpha{}_{\beta\rho\sigma} \epsilon^{\rho\sigma}{}_{\mu\nu} s^\mu p^\nu u^\beta \right)$$

$$\frac{ds^\alpha}{d\tau} = -\Gamma^\alpha_{\mu\nu} s^\mu u^\nu + \lambda \left(\frac{1}{2m^3} R_{\gamma\beta\rho\sigma} \epsilon^{\rho\sigma}{}_{\mu\nu} s^\mu p^\nu s^\gamma u^\beta \right) p^\alpha$$

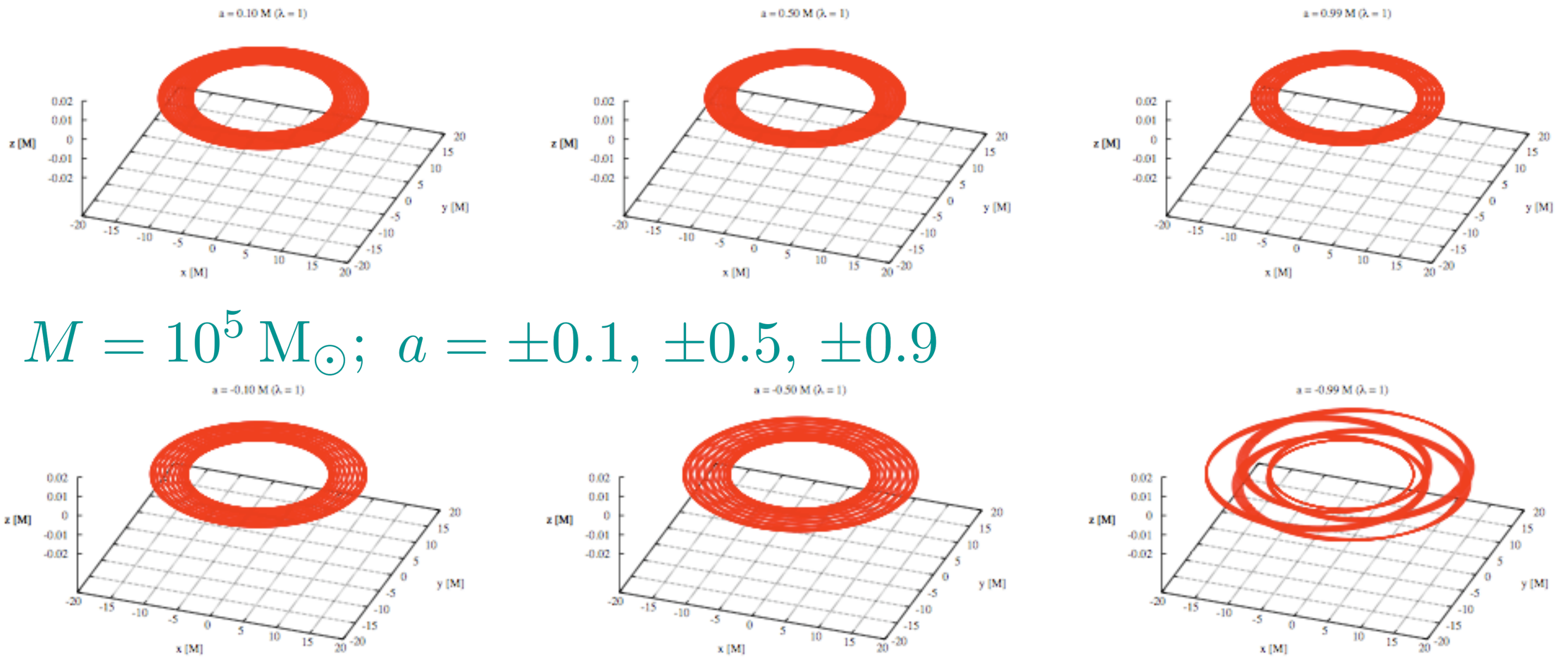
$$\frac{dx^\alpha}{d\tau} = u^\alpha = -\frac{p^\delta u_\delta}{m^2} \left(p^\alpha + \frac{1}{2} \frac{\lambda \left(s^{\alpha\beta} R_{\beta\gamma\mu\nu} p^\gamma s^{\mu\nu} \right)}{m^2 + \lambda \left(R_{\mu\nu\rho\sigma} s^{\mu\nu} s^{\beta\sigma} / 4 \right)} \right)$$

$\lambda = 1$ spin-curvature interaction

$\lambda = 0$ no spin-curvature interaction

numerical scheme for the integration: Runge-Kutter is sufficient

Orbital dynamics of a spinning neutron stars around a massive black hole

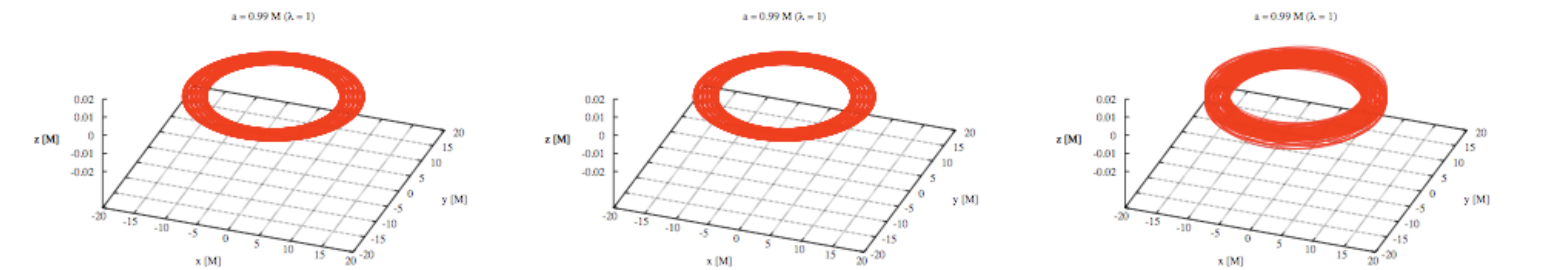


$M = 10^5\,\mathrm{M}_\odot;\ a = \pm 0.1,\ \pm 0.5,\ \pm 0.9$

Figure 3. The orbit of a pulsar around a $M = 10^5$-$\mathrm{M}_\odot$ black hole for spin parameters $a/M = \pm 0.1$, ± 0.5 and ± 0.9 (panels from left to right). The prograde orbits are shown in the top row and the retrograde orbits in the bottom row. The initial orbital separation $r = 10M$. The other parameters are the same as those in Figures 1 and 2. As the strength of spin-curvature interaction increases with the m/M ratio, the orbital variations shown here are more pronounced that those in Figures 1 and 2.

(Singh, Wu and Sarty 2013)

Orbital dynamics of a spinning neutron stars around a massive black hole



$a = \pm 0.9;\ M = 2 \times 10^6\ \mathrm{M}_\odot,\ 10^5\ \mathrm{M}_\odot,\ 10^3\ \mathrm{M}_\odot$

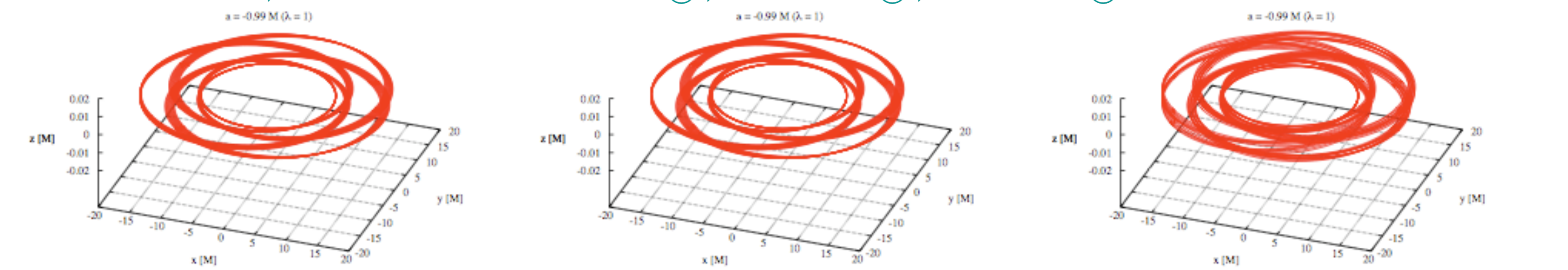
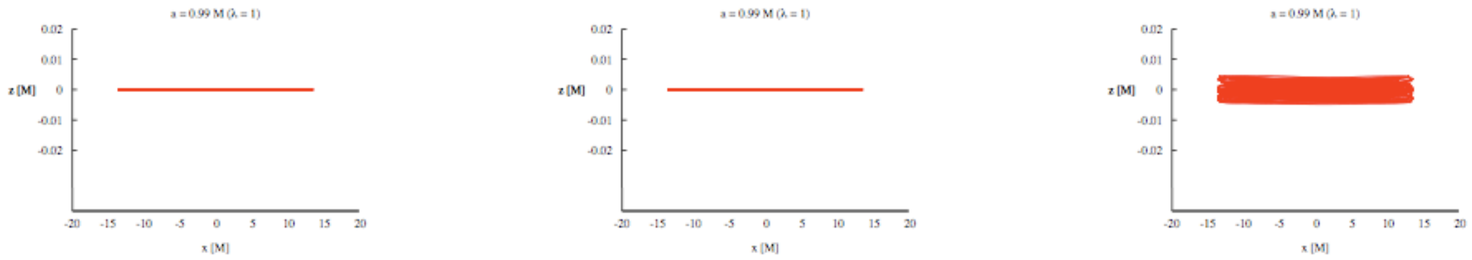


Figure 5. The orbit of a pulsar around a black hole for $M = 2.0 \times 10^6 \mathrm{M}_\odot$ (left panels) $10^5 \mathrm{M}_\odot$ (middle panels) and $10^3 \mathrm{M}_\odot$ (right panels). The black-hole spin parameter $a/M = \pm 0.99$ and the initial orbital separation $r = 10M$. The prograde orbits are shown in the top row and the retrograde orbits in the bottom row. The other parameters are the same as those in Figures 1 and 2. The spin-curvature interaction becomes pronounced when the black hole is maximally rotating, in conjunction with sufficiently small M/m.

(Singh, Wu and Sarty 2013)

4. Dynamics of double neutron stars around a supermassive black hole



$$a = \pm 0.9;\ M = 2 \times 10^6\ \mathrm{M}_\odot,\ 10^5\ \mathrm{M}_\odot,\ 10^3\ \mathrm{M}_\odot$$

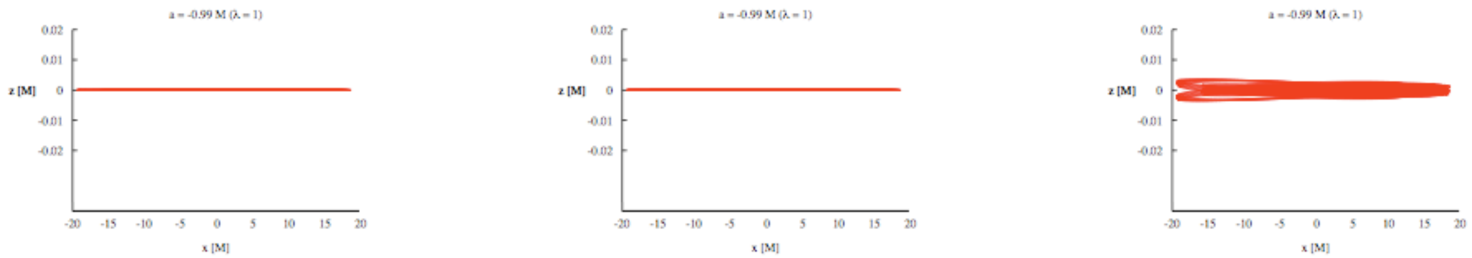


Figure 6. The projection of the orbit of a pulsar around a black hole onto the x-z plane is presented for $M = 2.0 \times 10^6 \mathrm{M}_\odot$ (left panels) $10^5 \mathrm{M}_\odot$ (middle panels) and $10^3 \mathrm{M}_\odot$ (right panels). The black-hole spin parameter is $|a/M| = 0.99$, with prograde orbits in the top row and retrograde orbits in the bottom row. The initial orbital separation is $r = 10M$, and the other parameters are the same as those in Figures 1 and 2. It is clear in the sequence from left to right that the spin-curvature interaction reveals a lifting of the neutron star's orbit off of the orbital plane when M/m is sufficiently small.

(Singh, Wu and Sarty 2013)

4. Dynamics of double neutron stars around a supermassive black hole

$a = +0.1;\ M = 2 \times 10^6\,\mathrm{M}_\odot,\ 10^5\,\mathrm{M}_\odot,\ 10^3\,\mathrm{M}_\odot$

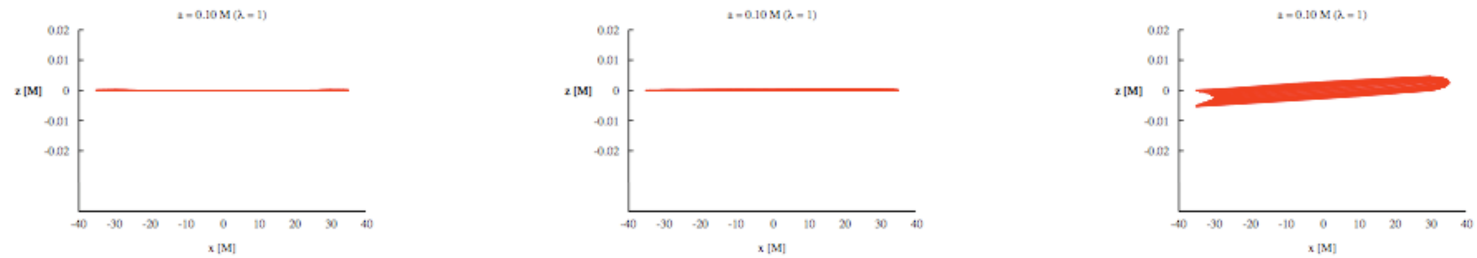


Figure 7. The projection of the orbit of a pulsar around a slowly rotating black hole onto the x-z plane is presented for $M = 2.0 \times 10^6\,\mathrm{M}_\odot$ (left panels) $10^5\,\mathrm{M}_\odot$ (middle panels) and $10^3\,\mathrm{M}_\odot$ (right panels). The black-hole spin parameter $a/M = +0.1$ and the initial orbital separation $r = 30M$. The other parameters are the same as those in Figures 1 and 2. This indicates that the off planar motion is mainly due to the spin-curvature interaction between the neutron star and the black hole rather than the spin-spin between the neutron star and the black hole.

(Singh, Wu and Sarty 2013)

Orbital dynamics of a spinning neutron stars around a massive black hole

Summary of this part:

(1) The spin-curvature coupling terms in the MPD equations depend on the mass ratio between the ms pulsar (neutron star) and the black hole, which modifies the orbital dynamics and the spin of the ms pulsar accordingly.

(2) The coupling is stronger for smaller mass difference between the ms pulsar and the black hole, giving an effective mean to determine the masses of the smaller central black holes in the late-type galaxies, dwarf galaxies and globular clusters

(3) The spin-curvature coupling depends on the spin of the black hole, by such we can measure the black hole spin directly.

4. Dynamics of double neutron stars around a supermassive black hole

Dynamics of double neutron stars around a supermassive black hole

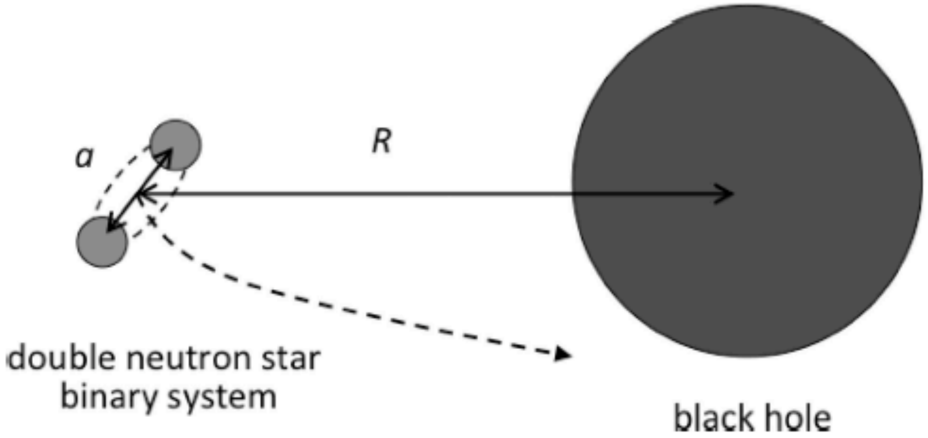


Figure 1. A schematic illustration of the configuration of a tightly bound DNS system orbiting around a massive black hole (not to scale). The orbital separation of the two neutron stars is a and the radius of the orbit of the DNS system around the black hole is R. The orbit of the neutron stars in the DNS system and the orbit of the DNS system around the black hole are not necessarily co-planar.

neutron stars	$m = 1.5\,\mathrm{M}_{\odot}$
Scwarzschild black hole	$M \gg m$

Dynamics of double neutron stars around a supermassive black hole

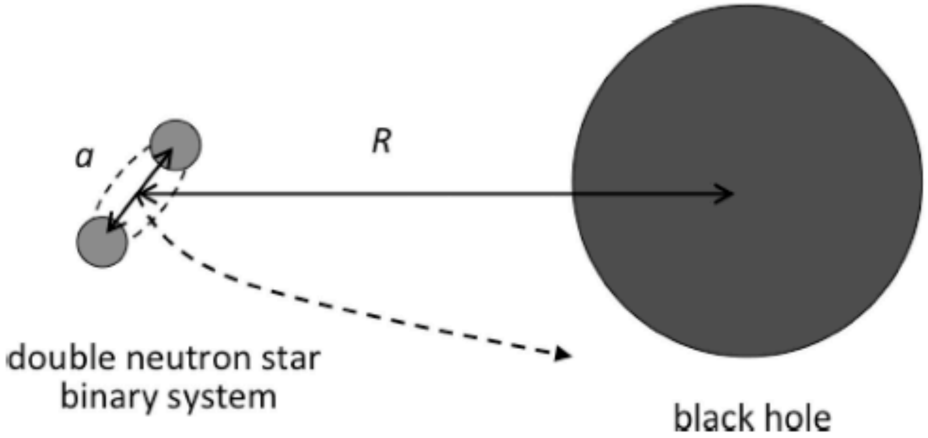


Figure 1. A schematic illustration of the configuration of a tightly bound DNS system orbiting around a massive black hole (not to scale). The orbital separation of the two neutron stars is a and the radius of the orbit of the DNS system around the black hole is R. The orbit of the neutron stars in the DNS system and the orbit of the DNS system around the black hole are not necessarily co-planar.

orbital separations $R \gg a$

angular frequencies $\Omega_{\rm bh} = \sqrt{GM/R^3} \ll \Omega_{\rm ns} = \sqrt{GM/2a^3}$

Dynamics of double neutron stars around a supermassive black hole

tidal force between freely falling objects moving along two adjacent geodesics

$$\frac{D^2\eta^j}{d\tau^2} = -R_{\tau j \tau k}\eta^k$$

in Schwarzschild space time

$$\frac{D^2\eta^r}{d\tau^2} = \frac{2GM}{r^3}\eta^r$$
$$\frac{D^2\eta^\theta}{d\tau^2} = -\frac{GM}{r^3}\eta^\theta$$
$$\frac{D^2\eta^\phi}{d\tau^2} = -\frac{GM}{r^3}\eta^\phi$$

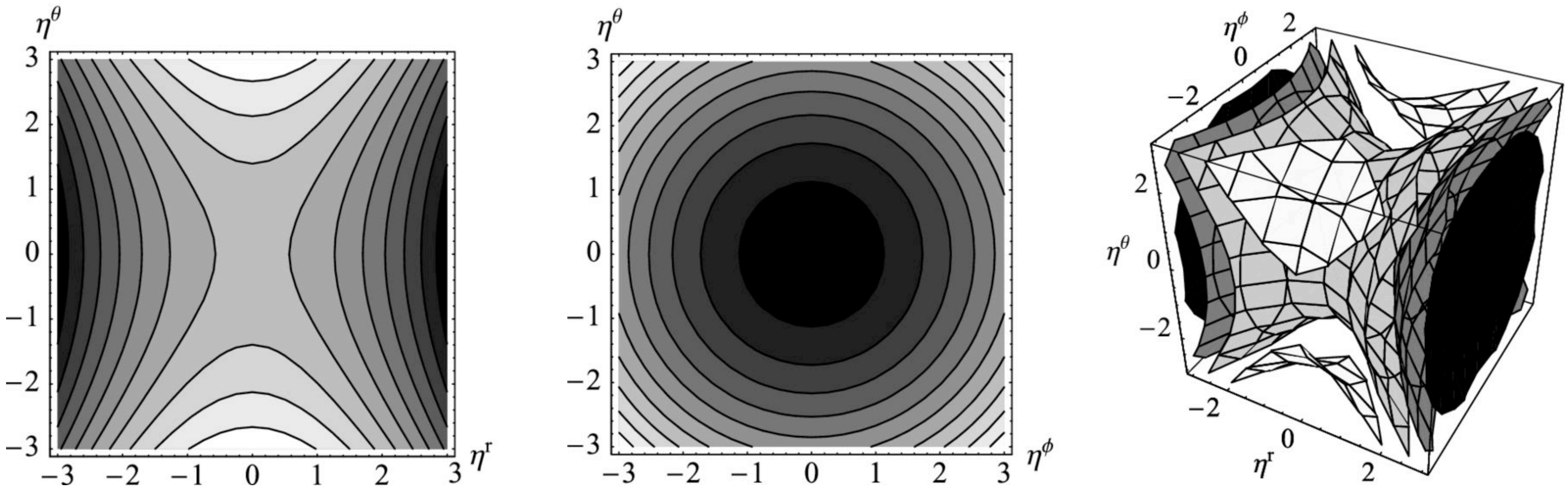
(Remmen and Wu 2013)

the rapid revolution of the neutron stars around each other allows us to use the continuous approximation that the system is a mass ring current

Dynamics of double neutron stars around a supermassive black hole

effective potential on the double neutron star (DNS) system

$$\Phi = \frac{1}{2}\frac{GM}{R^3}\left[\left(\eta^\theta\right)^2 + \left(\eta^\phi\right)^2 - 2\left(\eta^r\right)^2\right] = \frac{1}{2}\frac{GM}{R^3}\left[\left(\frac{a}{2}\right)^2 - 3\left(\eta^r\right)^2\right]$$



(Remmen and Wu 2013)

Dynamics of double neutron stars around a supermassive black hole

the interaction between mass current loop and the orbital interaction can be derived in the gravitoelectromagnetic (GEM) frame work

$$V_{SL} = \frac{3}{2} \frac{GM}{(2m)\, c^2 R^3} (\mathbf{S} \cdot \mathbf{L})$$

GEM spin-orbit coupling

the gravitational interaction between the two neutron stars in the DNS system forms a holonomic constraint which does not participate in the Lagrangian of the orbital dynamics of the DNS system

$$\mathcal{L} = \frac{2GMm}{R} - V_{SL} - \oint_{\mathrm{ring}} dm\, \Phi(\mathbf{r}') + \oint_{\mathrm{ring}} \frac{dm}{2} \left\{ \left|\dot{\mathbf{r}}'\right|^2 + 2\boldsymbol{\Omega} \cdot \left[(\mathbf{r}' + \mathbf{R}) \times \dot{\mathbf{r}}' \right] + \left[\Omega^2 r^2 - (\boldsymbol{\Omega} \cdot \mathbf{r})^2 \right] \right\}$$

(Remmen and Wu 2013)

Dynamics of double neutron stars around a supermassive black hole

The Lagrangian in terms of the three Euler angles for the normal orientation of the DNS mass ring in the reference frame co-rotating with the orbit around the black hole

$$
\begin{aligned}
\mathcal{L} \quad = \; & \tfrac{1}{8} m a^2 \left[\dot{\beta}'^2 \left(1+\cos^2\alpha\right) + \dot{\alpha}^2 + 2\dot{\gamma}^2 + 4\dot{\beta}'\dot{\gamma}\cos\alpha\right] && [T_{\rm rot}] \\
& + \tfrac{1}{2} m a^2 \Omega \left[\dot{\beta}' \left(1 - \tfrac{1}{2}\sin^2\alpha\right) + \dot{\gamma}\cos\alpha\right] && [T_{\rm Cor}] \\
& + \tfrac{1}{4} m a^2 \Omega^2 \left(1 - \tfrac{1}{2}\sin^2\alpha\right) && [V_{\rm cen}] \\
& - \tfrac{1}{4} m a^2 \Omega^2 \left[1 - \tfrac{3}{2}\left(\cos^2\alpha\sin^2\beta' + \cos^2\beta'\right)\right] && [V_{\rm tidal}] \\
& - \tfrac{3}{8} m \Omega^3 \tfrac{a^2 R^2}{c^2} \left[\left(\dot{\beta}' + \Omega\right)\left(1+\cos^2\alpha\right) + 2\dot{\gamma}\cos\alpha\right] && [V_{SL}] \\
& + 3 m \Omega^2 R^2 \, . && [\text{constants}]
\end{aligned}
$$

(Remmen and Wu 2013)

Dynamics of double neutron stars around a supermassive black hole

the Euler-Lagrange equation gives the equations of motion

$$
\begin{aligned}
0 &= \ddot{\alpha} + \left(\dot{\xi} - \dot{\beta}\right)\dot{\beta}\sin\alpha\cos\alpha + \dot{\xi}\dot{\gamma}\sin\alpha \\
&\quad +3\Omega^2\sin^2(\beta - \Omega t)\sin\alpha\cos\alpha \\
0 &= \ddot{\beta}\left(1 + \cos^2\alpha\right) - \dot{\xi}\dot{\alpha}\sin\alpha\cos\alpha - 2\dot{\alpha}\dot{\gamma}\sin\alpha + 2\ddot{\gamma}\cos\alpha \\
&\quad +3\Omega^2\sin^2\alpha\sin(\beta - \Omega t)\cos(\beta - \Omega t) \\
0 &= 2\ddot{\gamma} + 2\ddot{\beta}\cos\alpha - \dot{\xi}\dot{\alpha}\sin\alpha
\end{aligned}
$$

$$\dot{\xi} \equiv 2\dot{\beta} - 3\Omega^3 R^2 c^{-2}$$

(Remmen and Wu 2013)

Dynamics of double neutron stars around a supermassive black hole

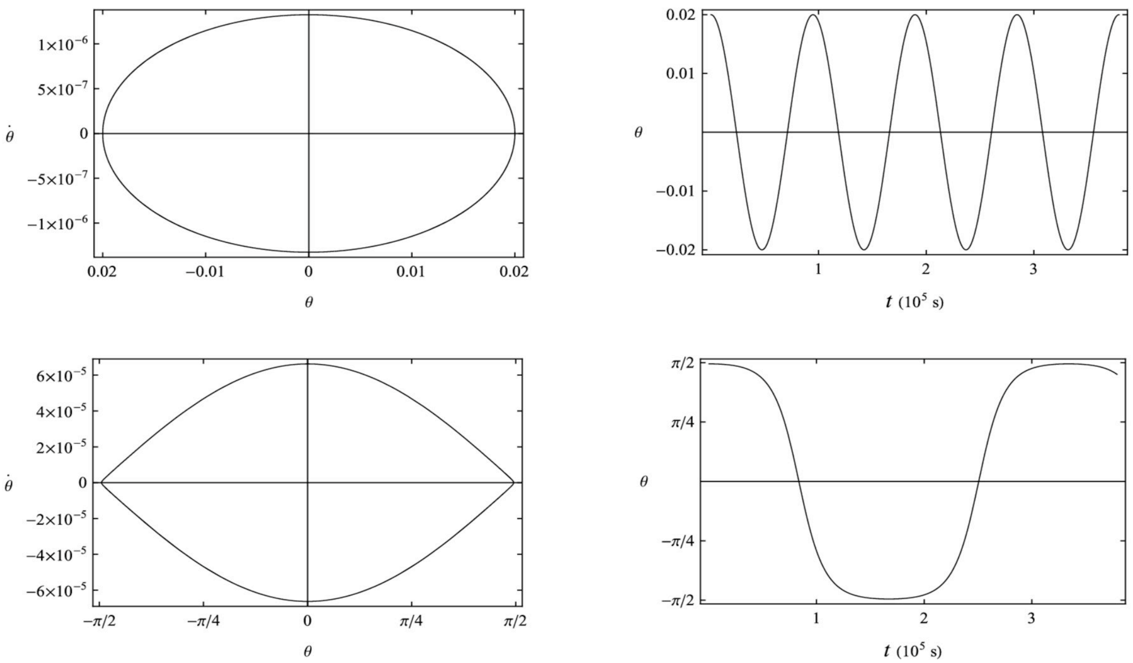


Figure 3. Phase space and time-dependent behaviour in the zenith angle θ for oscillation of the fixed, ϕ-restricted, non-rotating ring. For small oscillations, the behaviour is simply harmonic, while for oscillations with amplitude approaching $\pi/2$, lingering behaviour occurs as a result of the unstable equilibrium. In the calculation we use $M = 1.5 \times 10^7\ \mathrm{M}_{\odot}$ (black hole), $m = 1.5\ \mathrm{M}_{\odot}$ (neutron stars), $R = 25r_s$, where $r_s = 2GM/c^2 = 4.4 \times 10^{12}$ cm, and $\Omega = \sqrt{GM/R^3} = 3.8 \times 10^{-5}\ \mathrm{rad\ s^{-1}}$. The units of $\dot{\theta}$ are rad s^{-1}.

(Remmen and Wu 2013)

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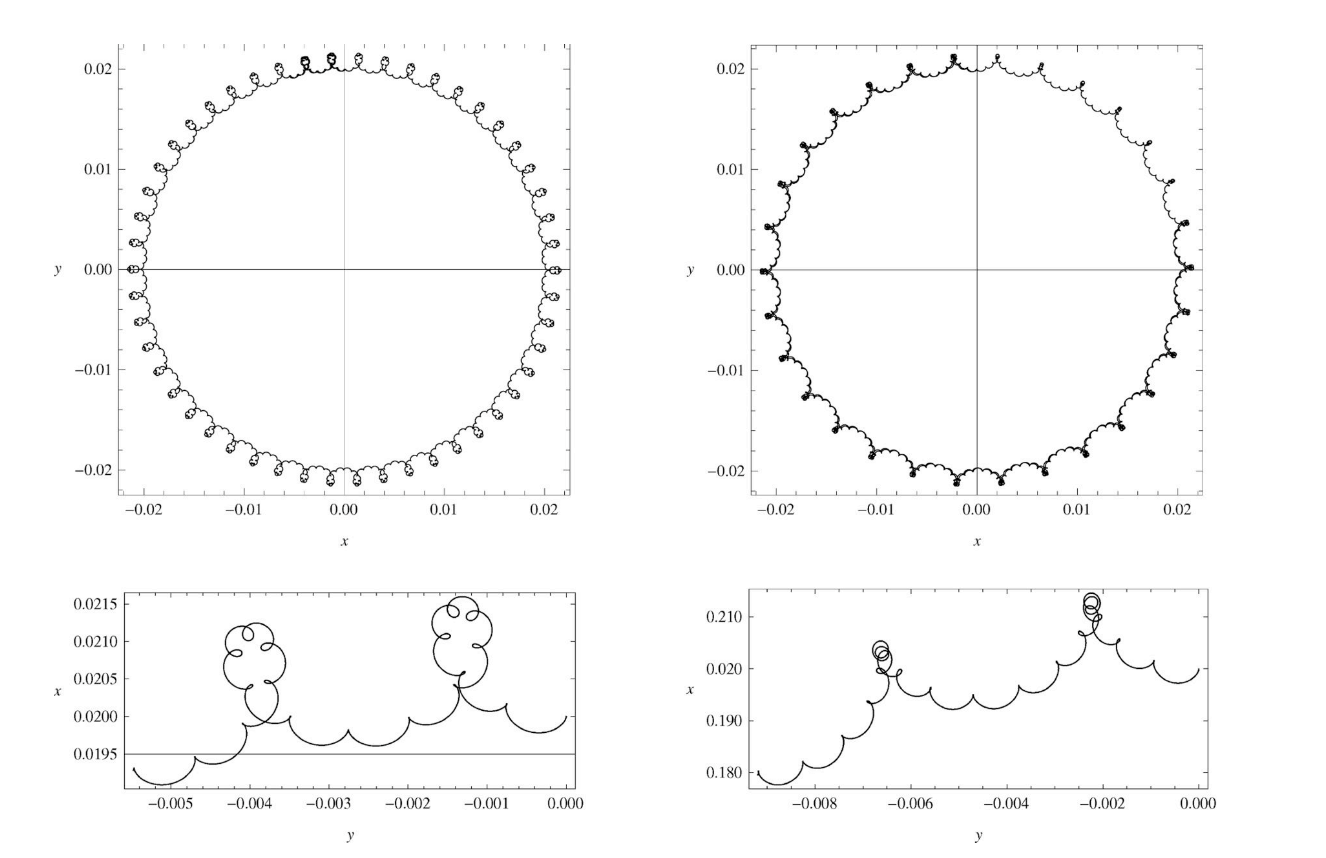

(Remmen and Wu 2013)

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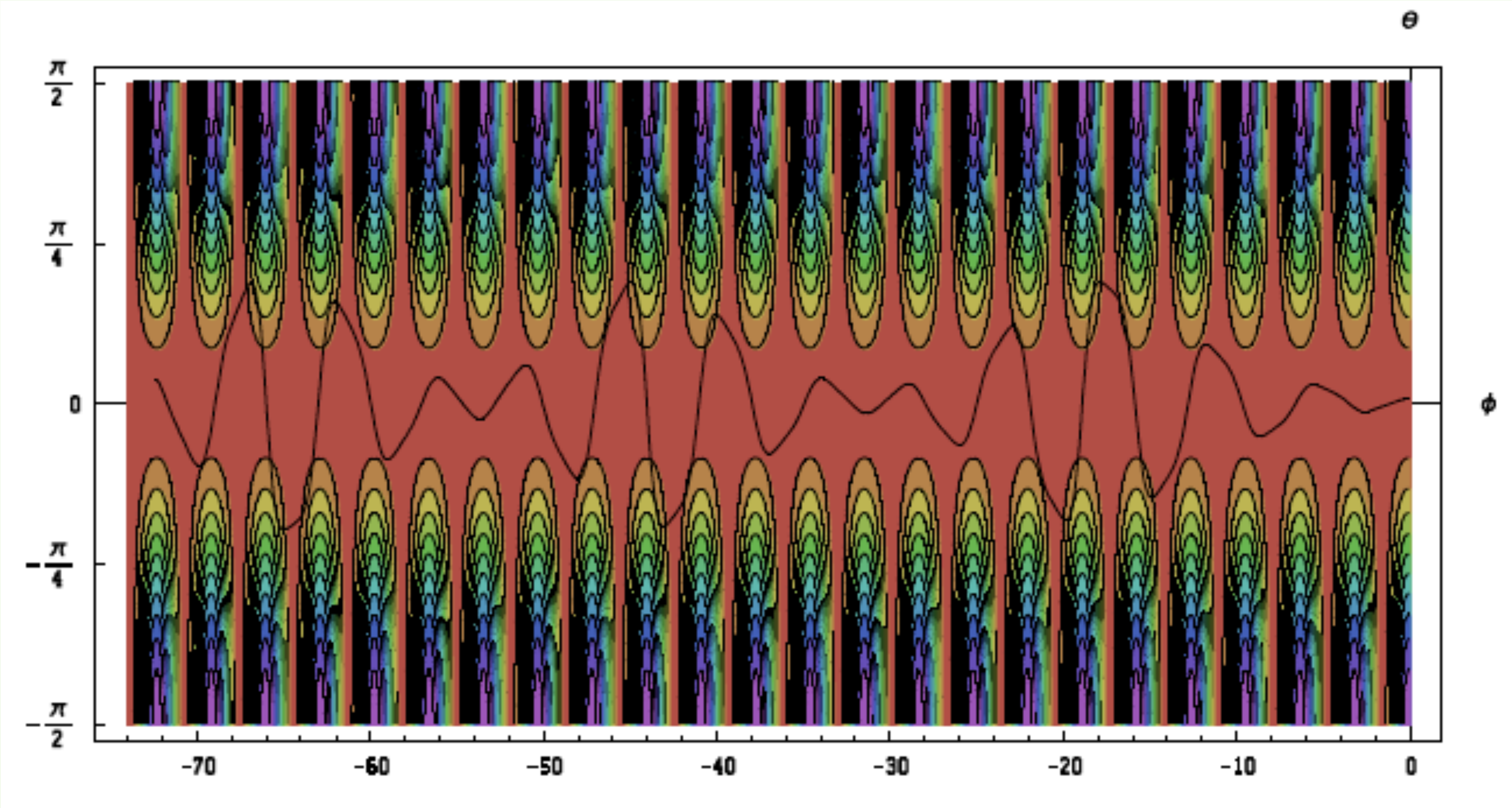

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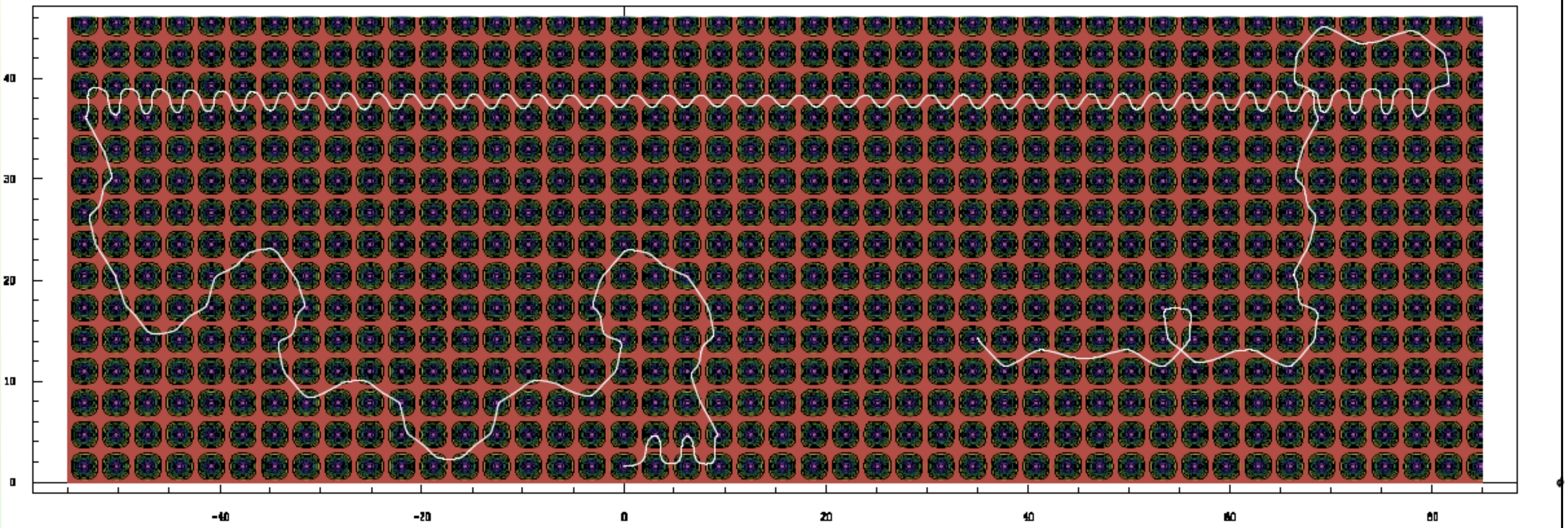

Dynamics of double neutron stars around a supermassive black hole

Summary of this part:

(1) A tightly bound double neutron star (DNS) system orbiting around a massive black hole shows induced gravitoelectromagnetism (GEM).

(2) The GEM interaction gives rise to very complex orbital dynamics, depending on the initial system parameters.

(3) DNS systems orbiting around a supermassive black hole are special cases where GEM does not require a rotating black hole.

(4) Such DNS systems containing a ms pulsar are useful for testing GEM predictions in general relativity as well as to constrain the population of double compact object systems in the centres of galaxies