A Review of Primary and Recent GRMHD Simulations

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I will try to cover wide topics of GRMHD simulations from its motivation to recent results.

- Ideal GRMHD: primary results ← our group
  latest results ← other groups at US and UK.
- Resistive GRMHD: on trial.
- Radiative GRMHD: not covered, while it becomes promising.

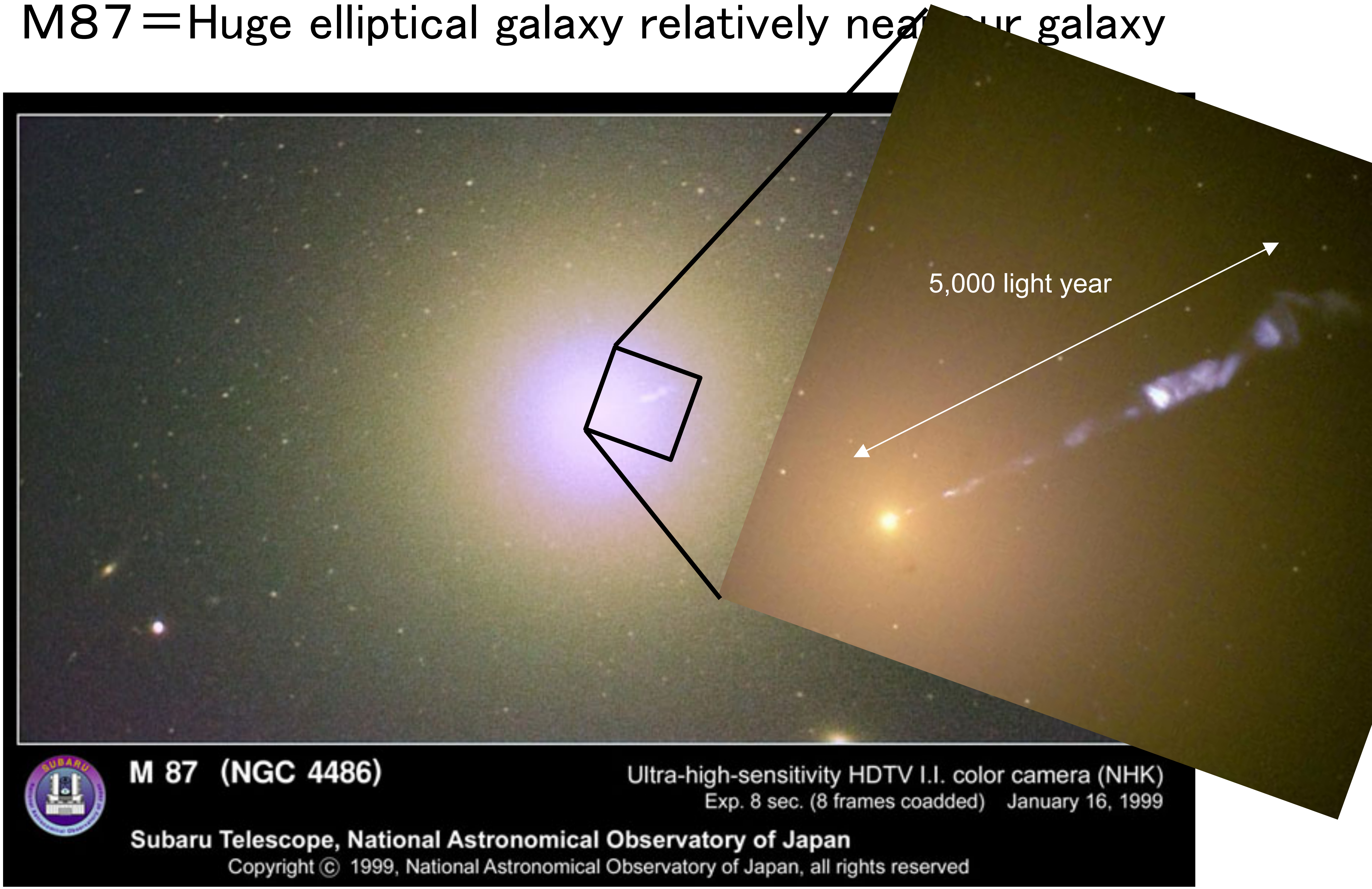
The Sixth Workshop on Black Hole Magnetospheres
March 5, 2013 at National Tsing Hua University, Hsinchu

Outline

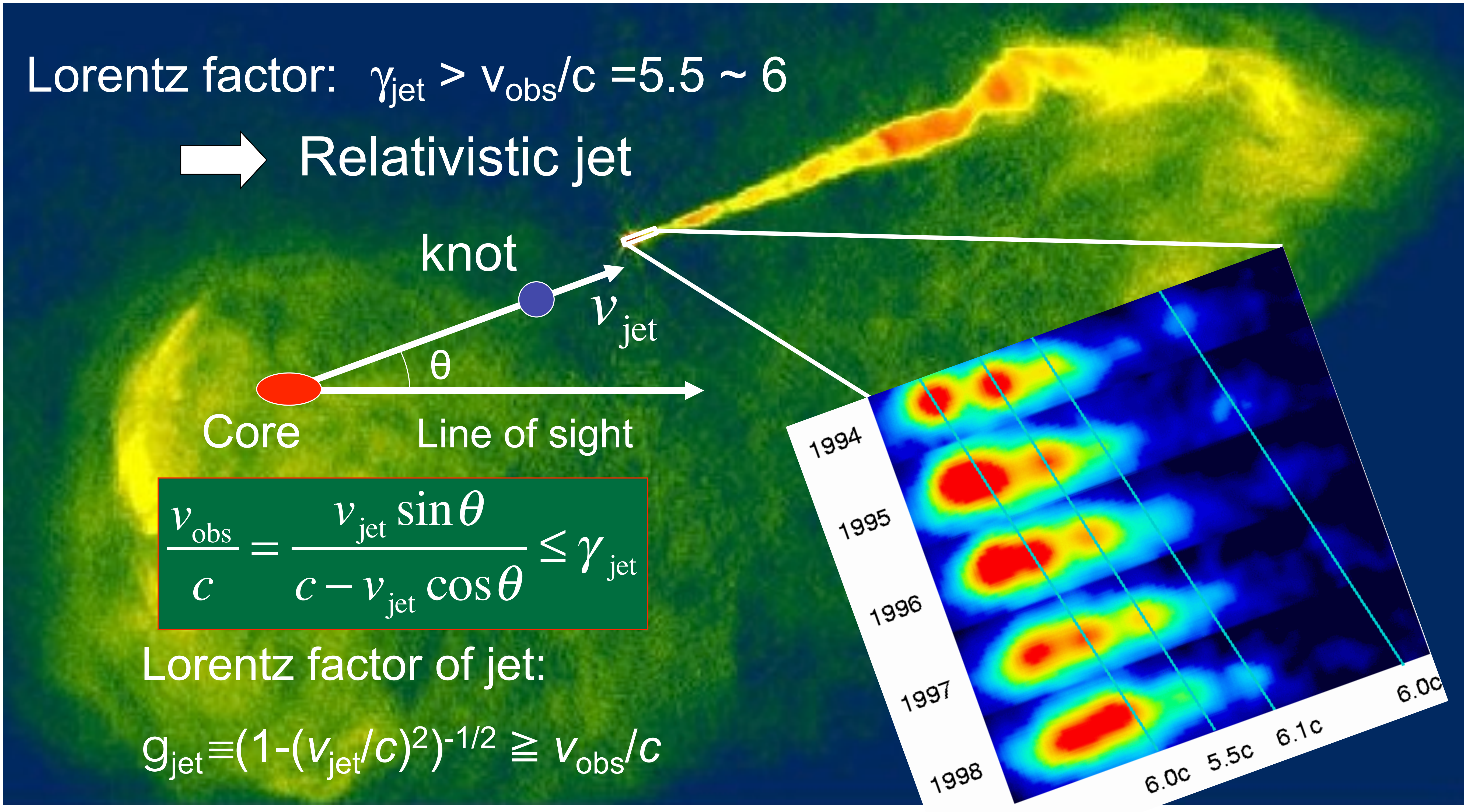
- Observations of relativistic jets in universe.
- Models/theories of relativistic jet formation
- Numerical method of ideal GRMHD
- Review of ideal GRMHD simulations
  - Primary calculations: Magnetically energy extraction of rotating black hole
  - Recent calculations: Relativistic jet formation by magnetic bridge
- Resistive GRMHD simulations on trial
- Summary and future works

Optical observation of huge elliptical galaxy M87

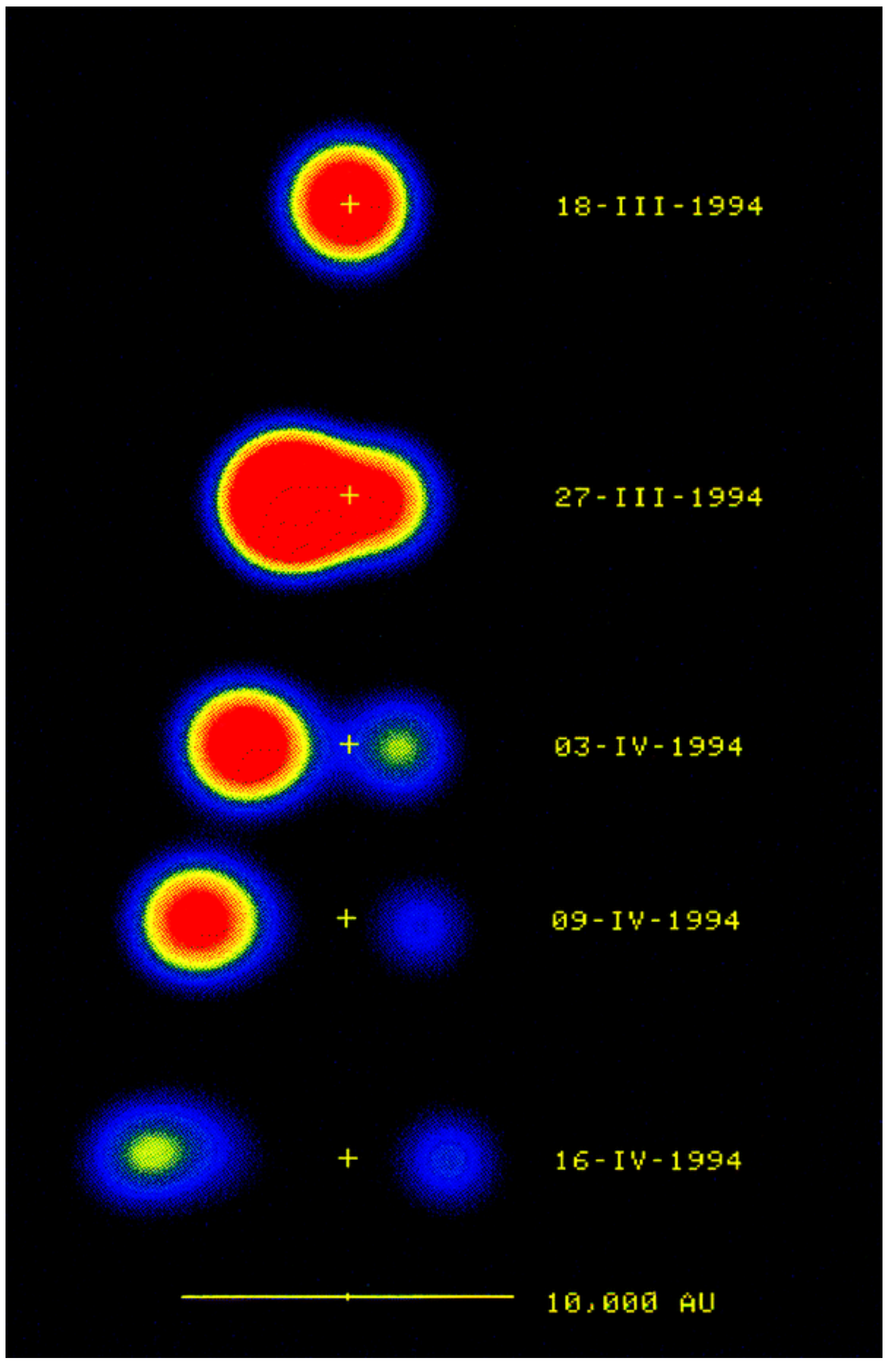
M87＝Huge elliptical galaxy relatively near our galaxy



Radio observation of relativistic jet from core region of M87.



Superluminal motion of jet from object in our Galaxy (Micro-Quasar)



GRS1915+105

V_{obs}(left) =1 .25c ← Superluminal motion

V_{obs}(right) = 0.65c

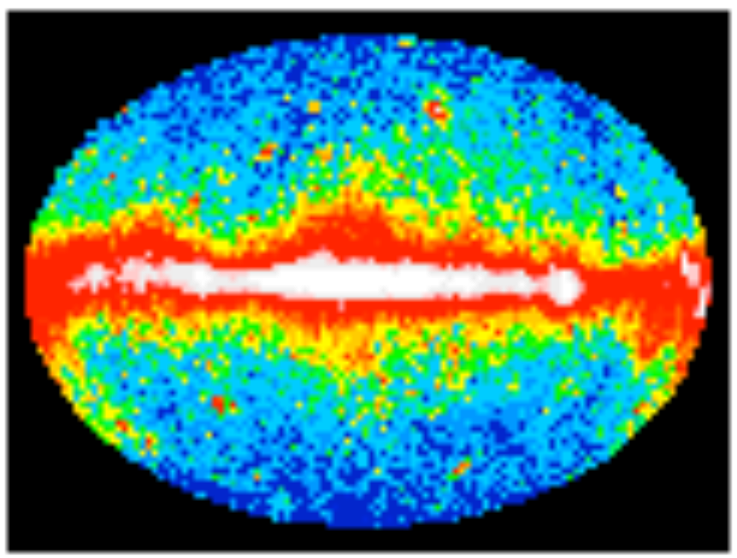
V_{jet}= 0.92c
(Lorentz factor, $\gamma_{jet} \sim 2.6$)

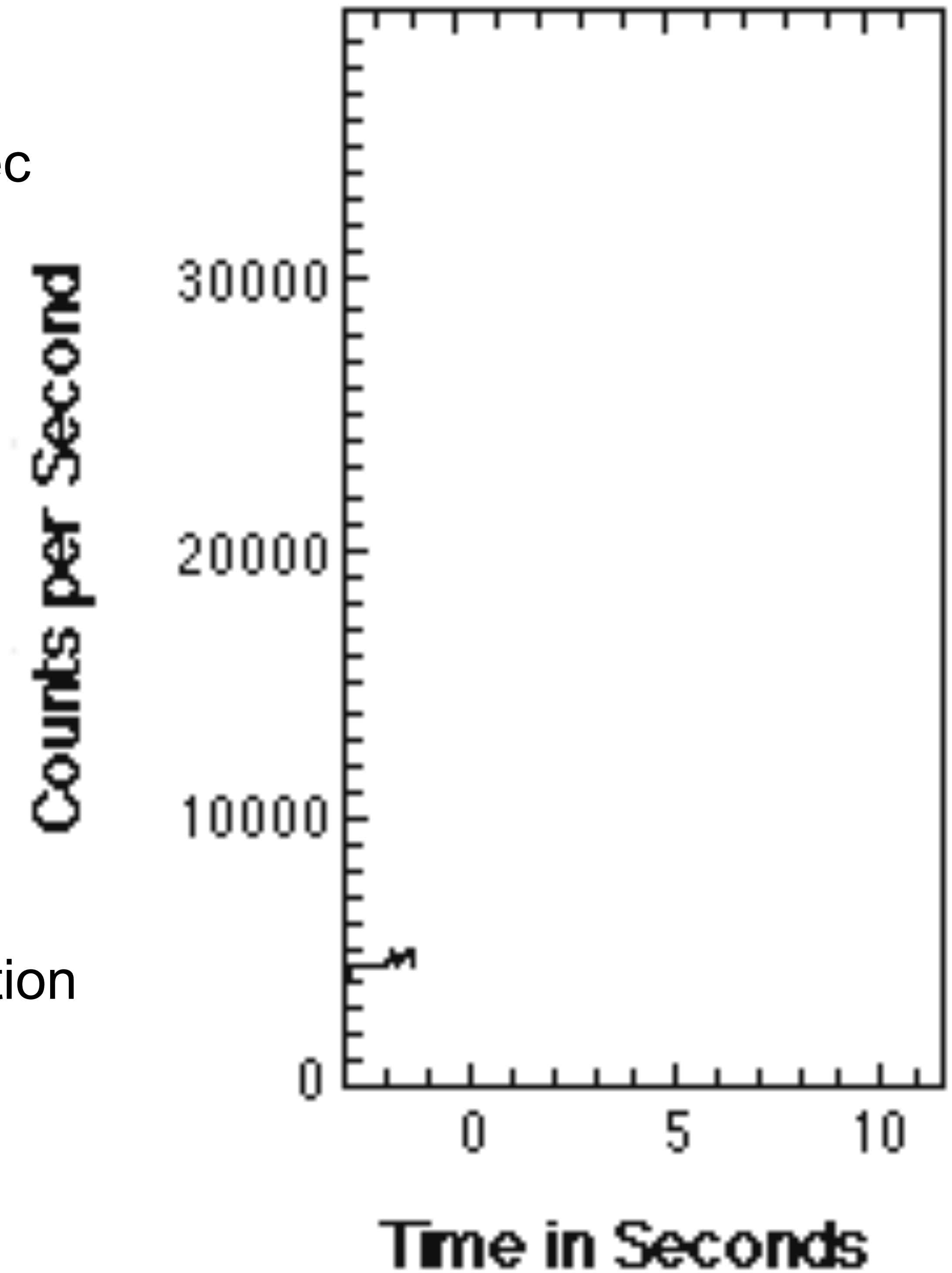
Mirabel, Rodrigus 1995

Gamma-ray burst (animation image)

- Event rate ∼ 1 event/day
- Frequency component X-MeV gamma ray
- Time scale 1msec — 10^3 sec
- Red shift z = 0.1 ∼ > 6
- Luminosity 10^{49} — 10^{52} erg/sec
- Total energy 10^{50} — 10^{53} erg

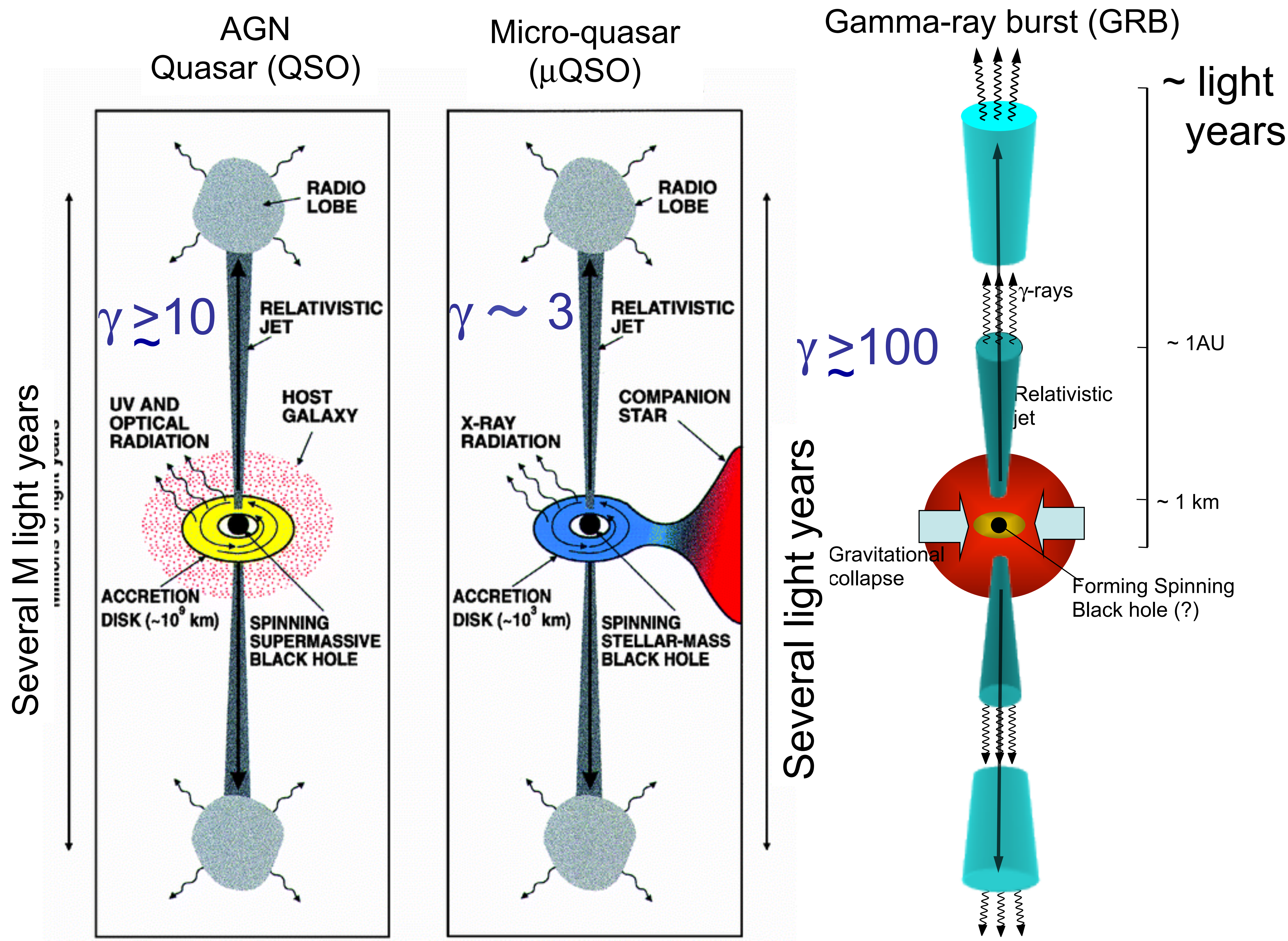
by M. C. Miller





- Identification ⇒ GRB030329: identification as hypernova/collapsar in distant galaxy

Relativistic Jets in the Universe



Mirabel, Rodriguez 1998

Mechanism of relativistic jet formation

In spite of drastic difference of scale and Lorentz factor of the jets, the jet formation mechanism may be common. These relativistic jets are formed by drastic phenomena around black holes. However, distinct mechanism are not shown yet.

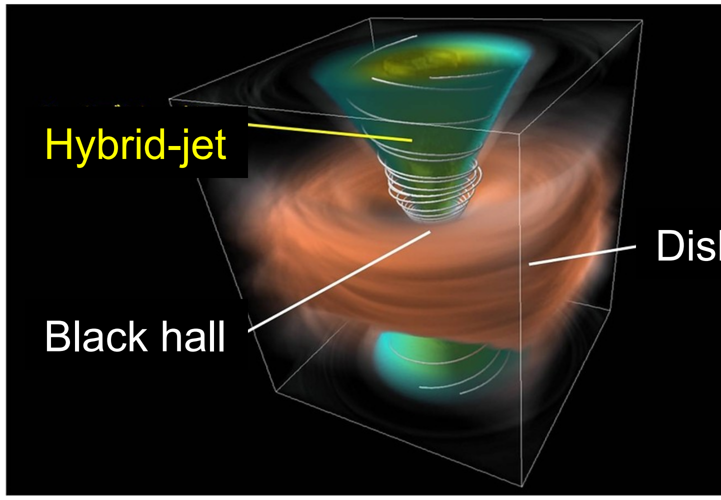
Model should explain:

- Acceleration of plasma/gas
- Collimation of the outflows

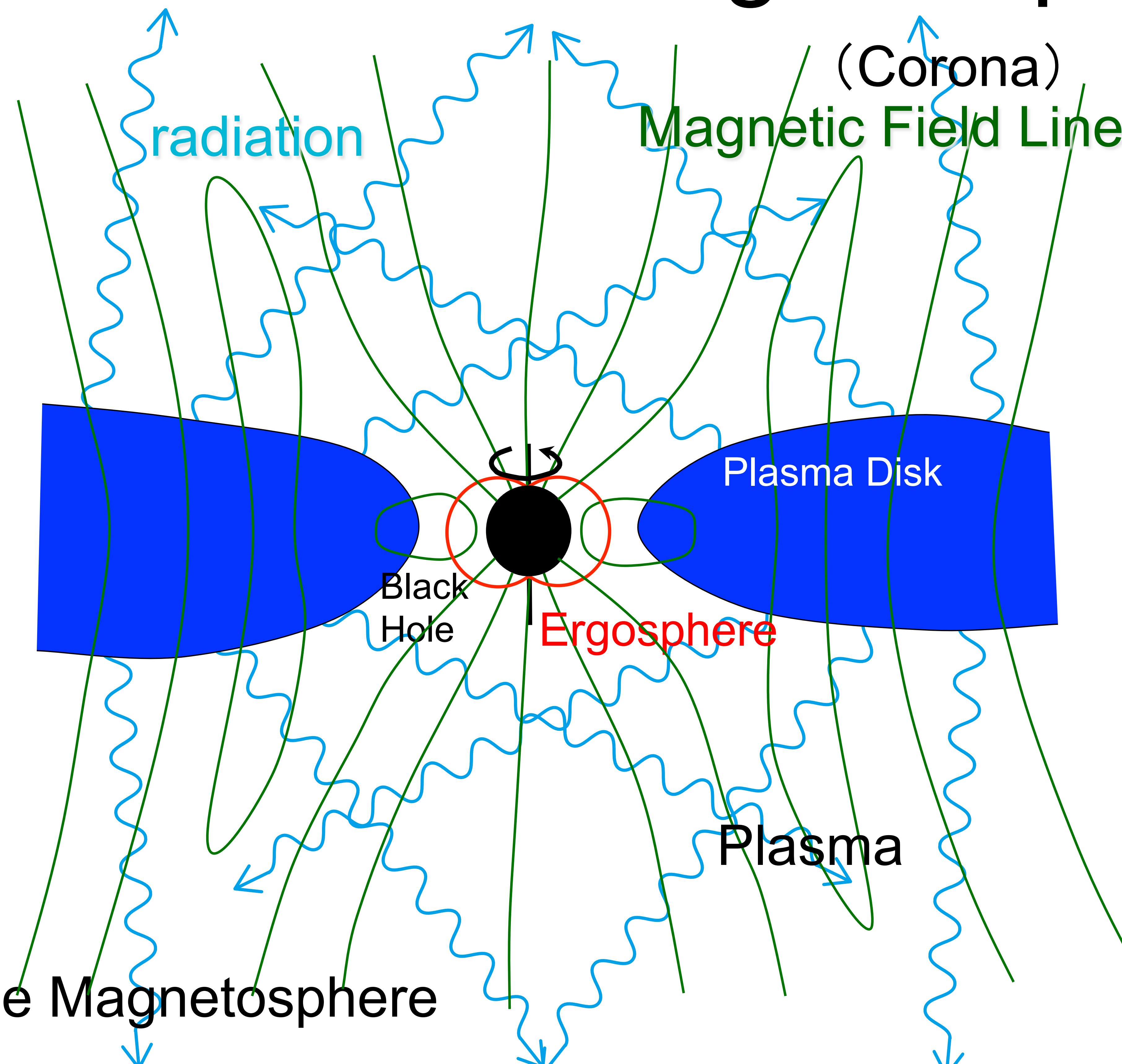
Force of acceleration and collimation:

1) Magnetic field
2) Radiation pressure
3) Gas pressure

Recently, hybrid jet was proposed (Ohsuga, Takeuchi & Mineshige 2010)

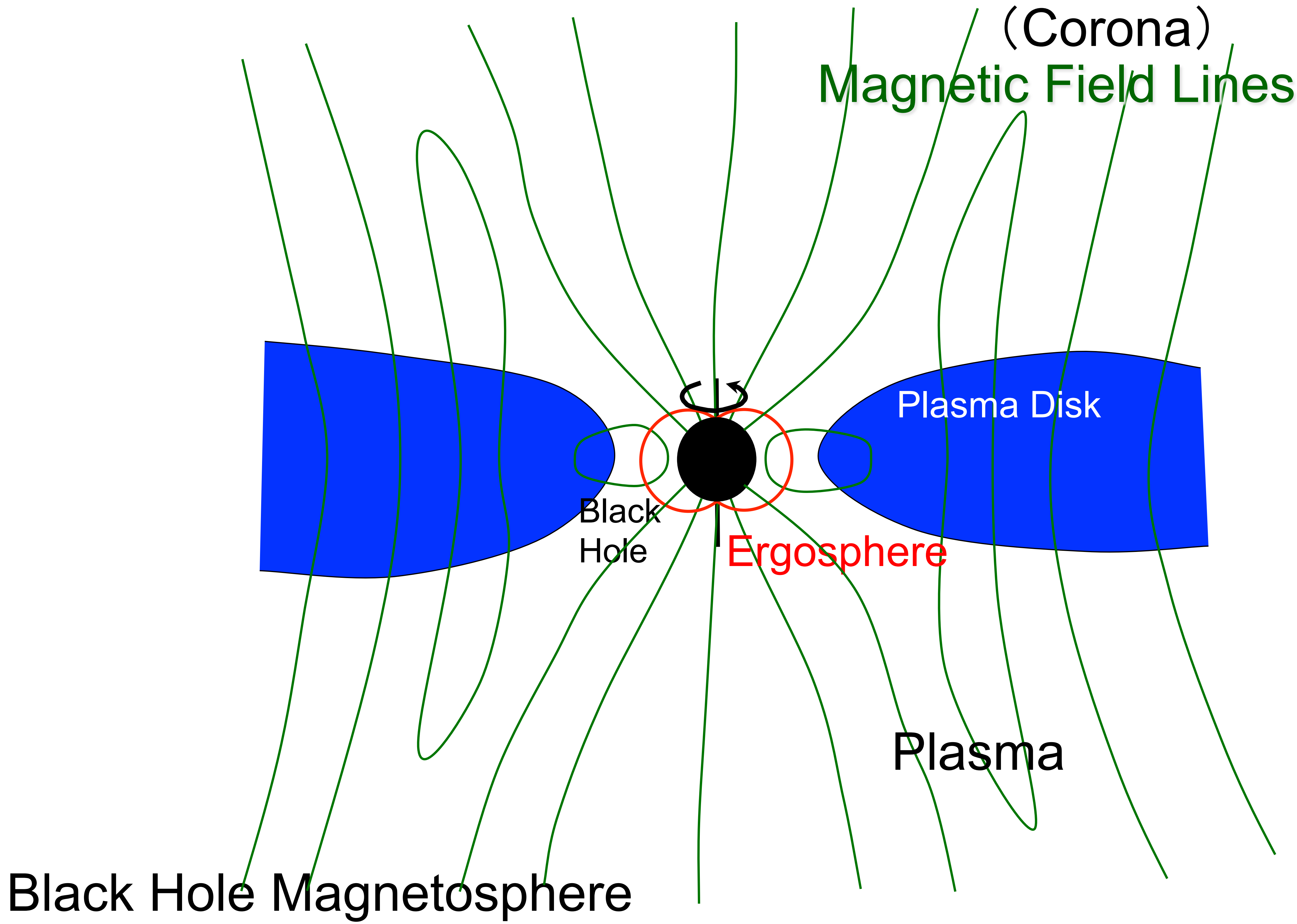


Black Hole Radio-Magnetosphere



Black Hole Magnetosphere

Black Hole Magnetosphere



Interaction between plasma and magnetic field

- Simplest approximation
  ⇒ magnetohydrodynamics (MHD)

- Plasma: ―non-relativistic one-component conducting fluid
- Field: ― Maxwell equations
– displacement current is negligible
– quasi-charge neutrality
- Ohm's law:
– Zero electric resistivity (ideal MHD)
– Non-zero electric resistivity (resistive MHD)

Important dynamics of magnetized plasma is determined by MHD!

Ideal MHD equations

(non-relativistic conservative form)

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v})$$ (conservation of particle number)

$$\frac{\partial \boldsymbol{P}}{\partial t} = -\nabla \cdot \boldsymbol{T} - \rho \nabla \Phi_{\mathrm{grav}}$$ (equation of motion)

$$\boldsymbol{P} = \rho \boldsymbol{v} \qquad \boldsymbol{T} = \rho \boldsymbol{v}\boldsymbol{v} + \left(p + \frac{B^2}{2} \right) \boldsymbol{I} - \boldsymbol{B}\boldsymbol{B}$$

$$\frac{\partial \varepsilon}{\partial t} = -\nabla \cdot \left(h_{nr} \boldsymbol{v} + \boldsymbol{E} \times \boldsymbol{B} \right) - \boldsymbol{P} \cdot \nabla \Phi_{\mathrm{grav}}$$ (equation of energy)

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \qquad \mathbf{J} = \nabla \times \mathbf{B}$$

(Maxwell equations)

$$\nabla \cdot \mathbf{B} = \mathbf{0}$$

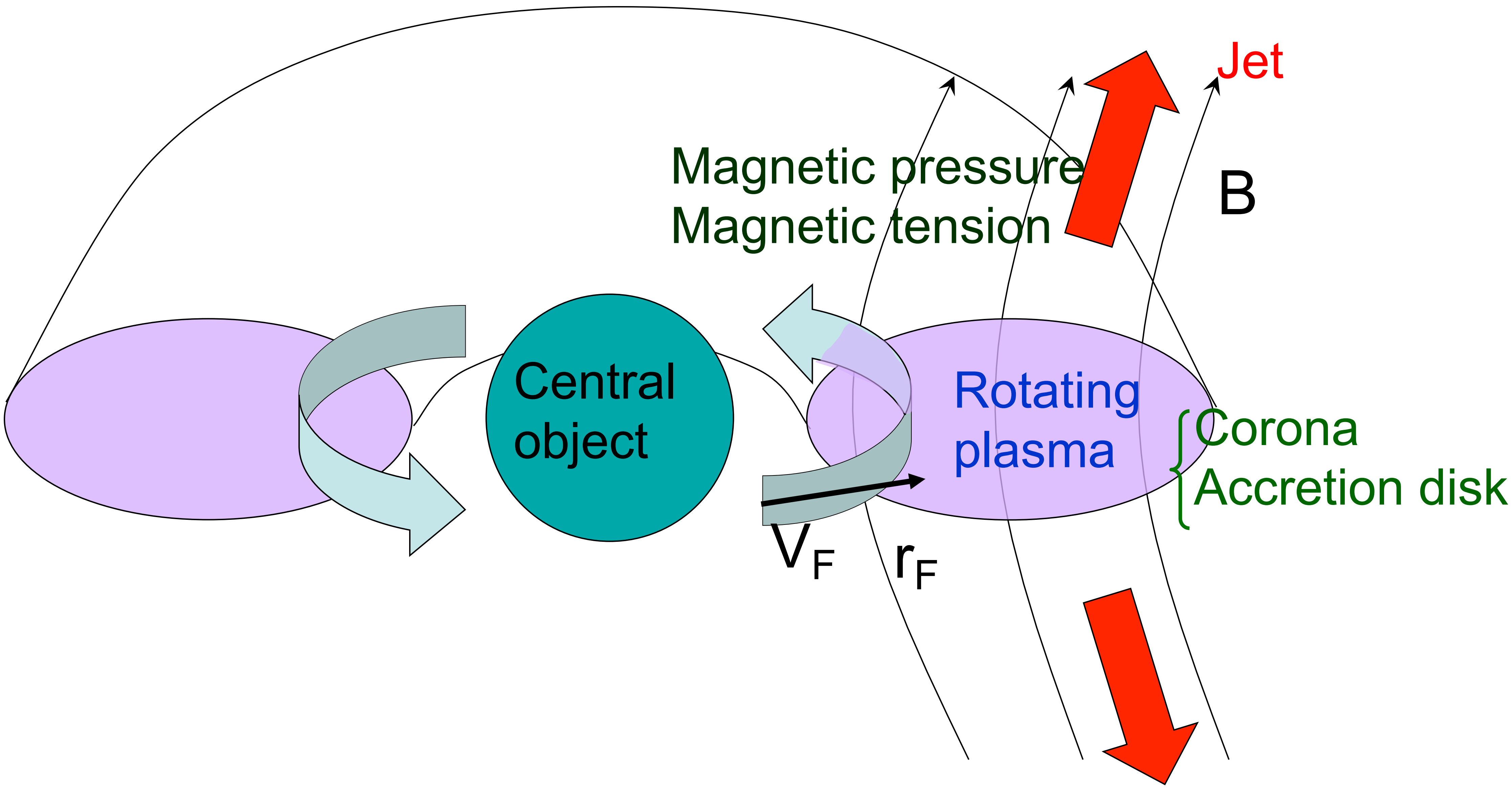
(ideal MHD condition)

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \mathbf{0}$$

$$h_{\mathrm{nr}} = \frac{\rho}{2} v^2 + \frac{\Gamma p}{\Gamma - 1}$$ (equation of state; non-relativistic enthalpy density)

Jet Formation Mechanism by Magnetic Field

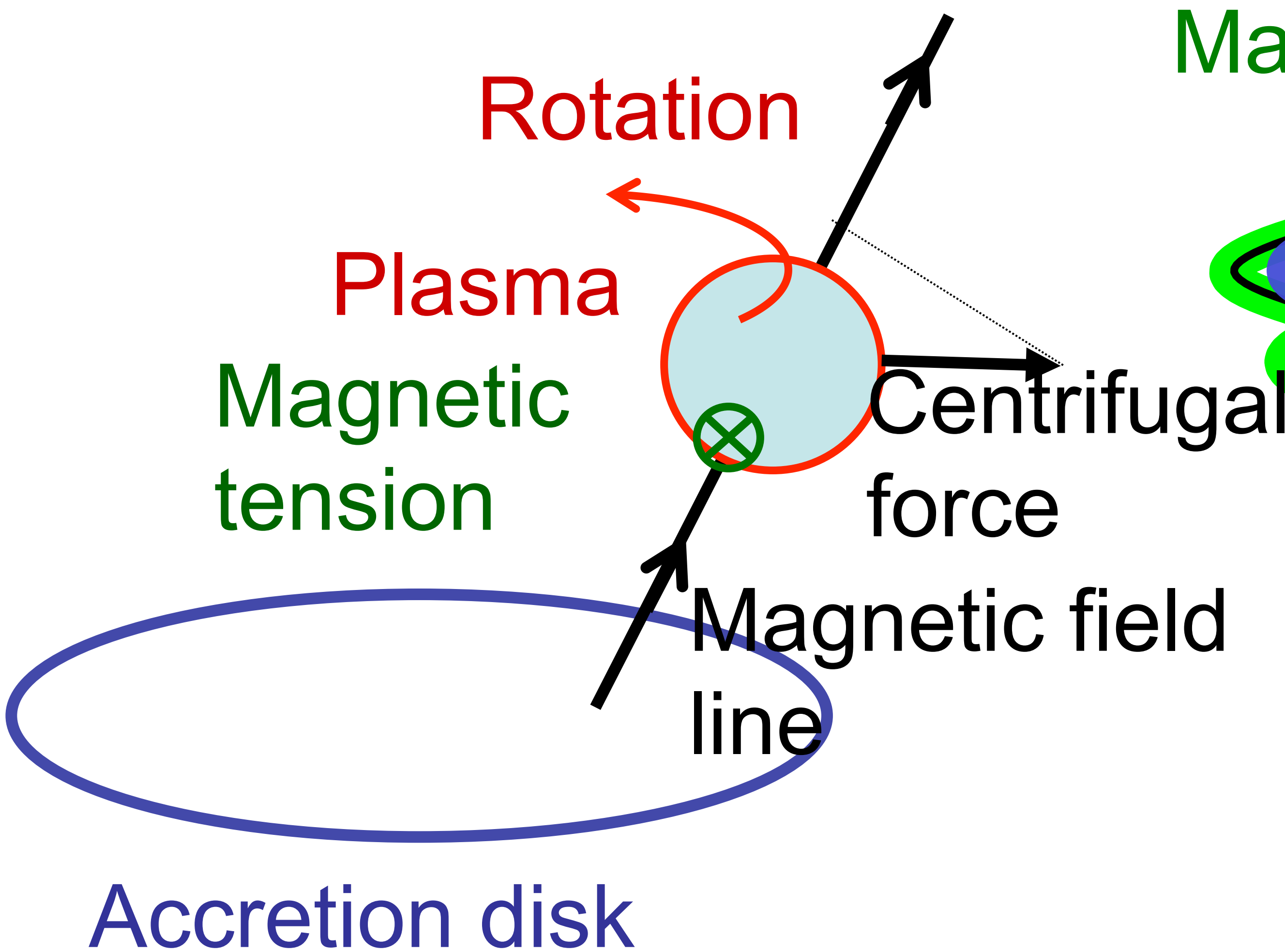
- Continuous acceleration mechanism
- Intermittent acceleration mechanism



Continuous acceleration mechanism

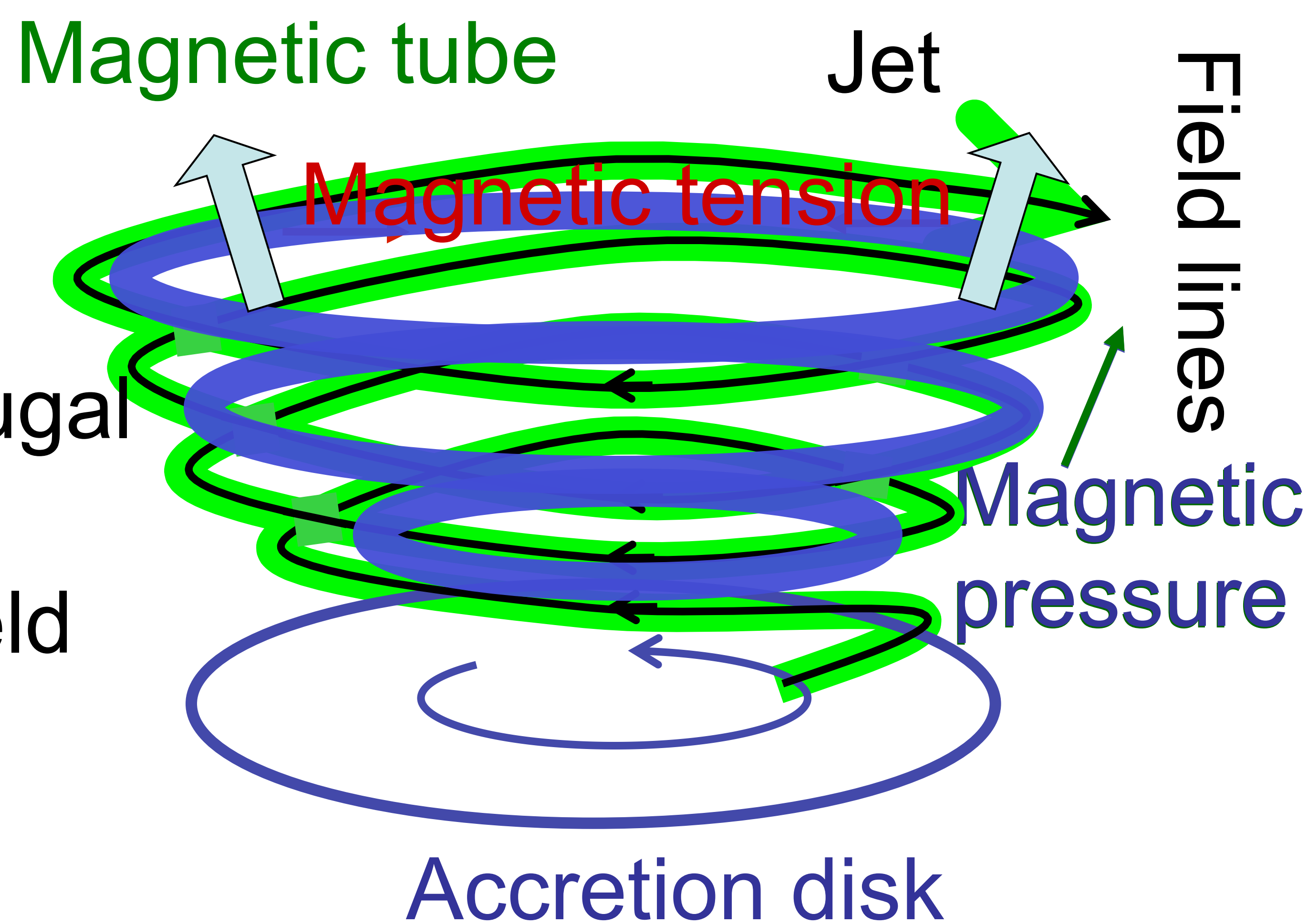
Blandford-Payne (1977) model

Magnetic tension → rotation →
centrifugal force →plasma acceleration

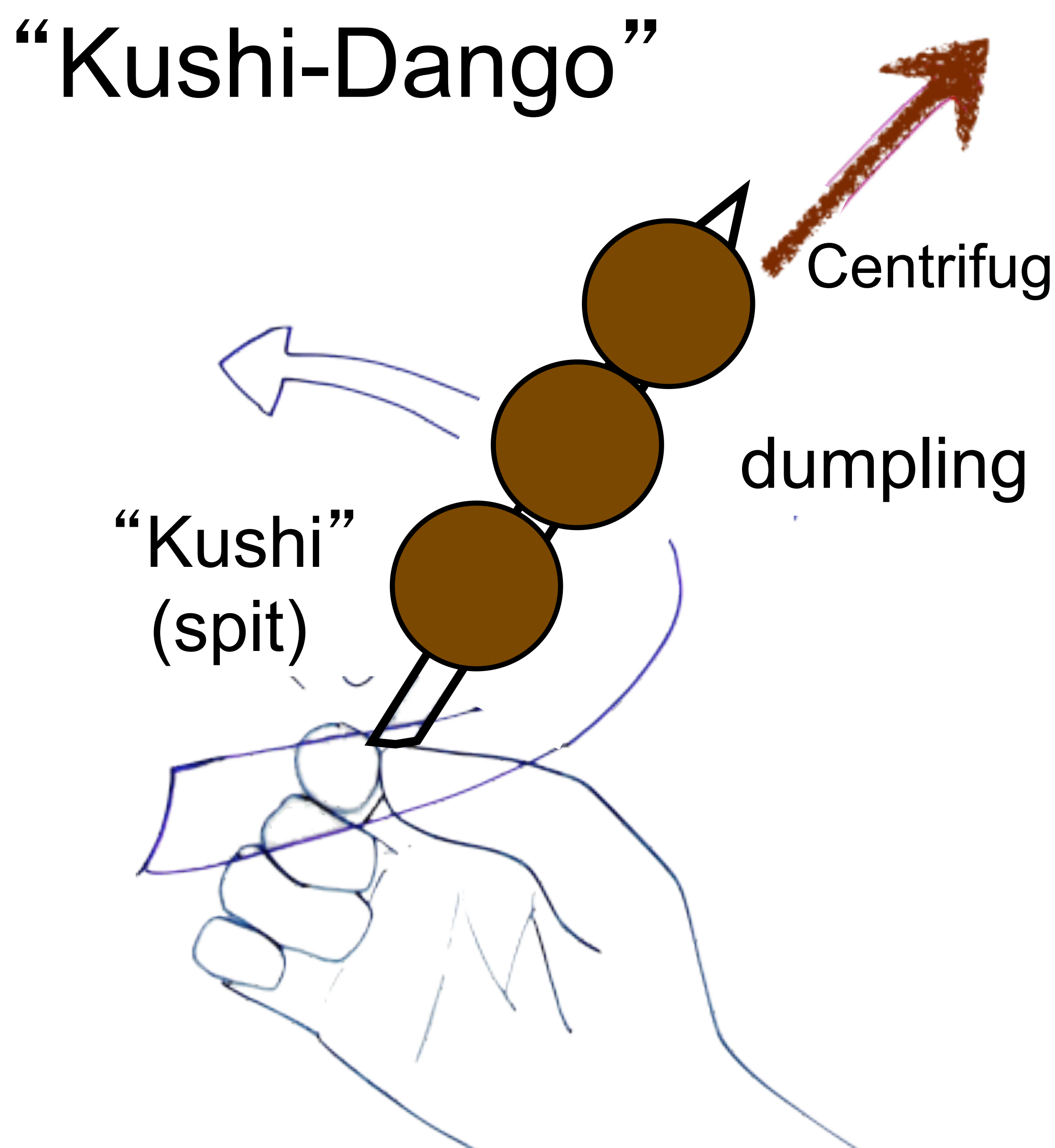


Uchida-Shibata (1985) model

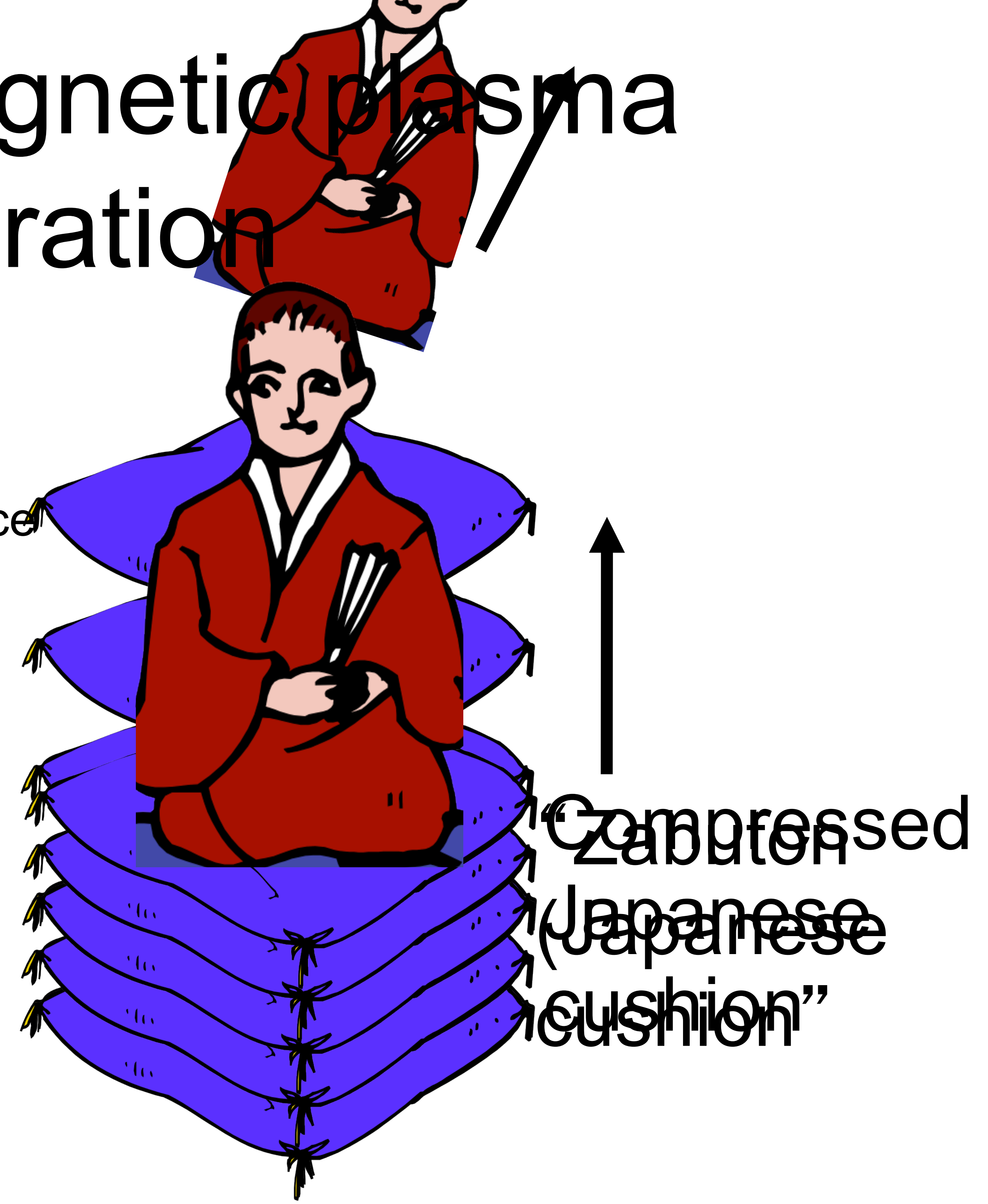
Magnetic pressure → plasma acceleration
Magnetic tension → outflow collimation
(pinch effect)



Analogy to magnetic plasma acceleration



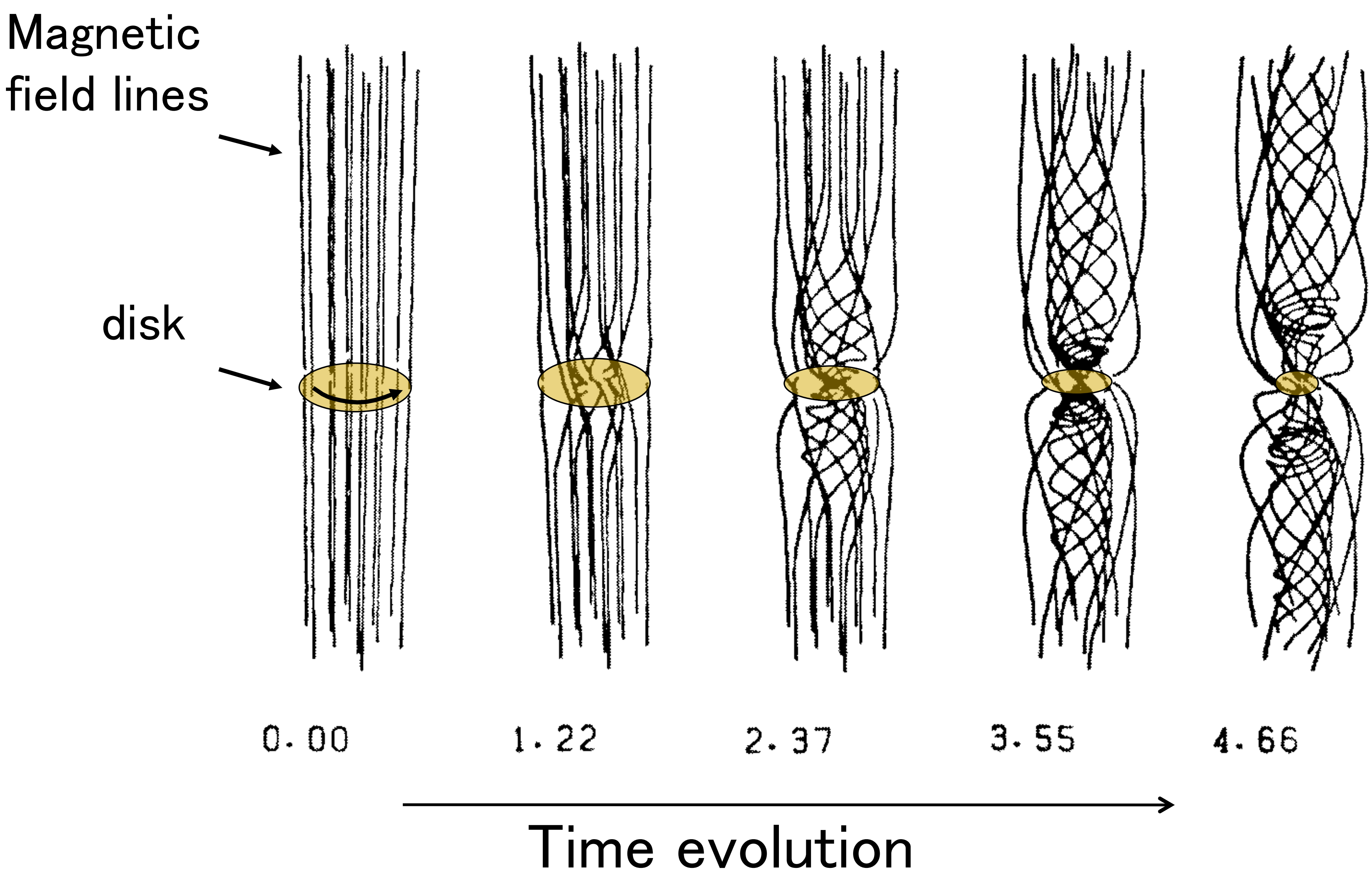
Acceleration by magnetic tension& centrifugal force （Blandford-Payne 1982）



Acceleration by Magnetic pressure (Uchida-Shibata 1985)

Numerical MHD simulation of jet formation (non-relativistic)

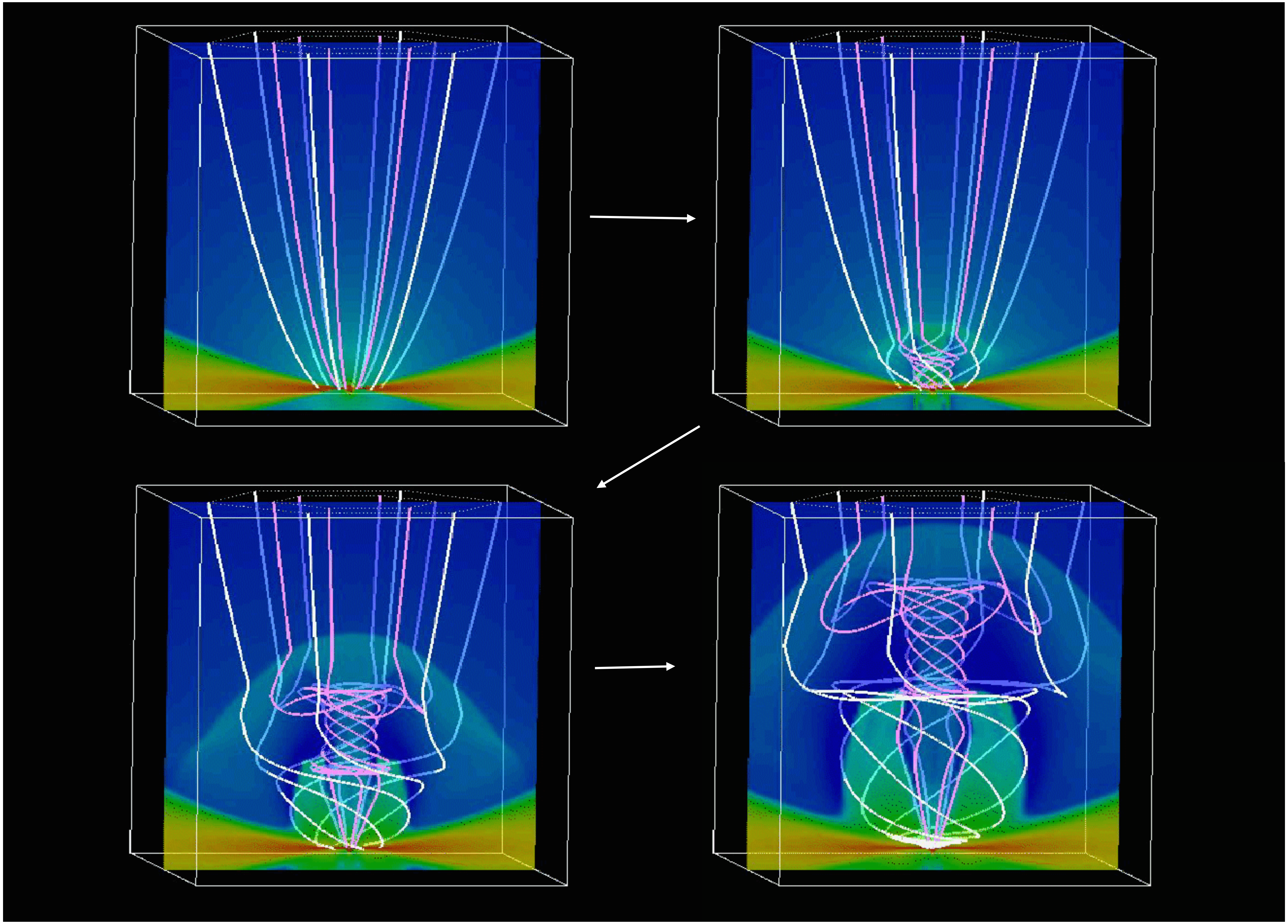
Shibata & Uchida 1986, PASJ



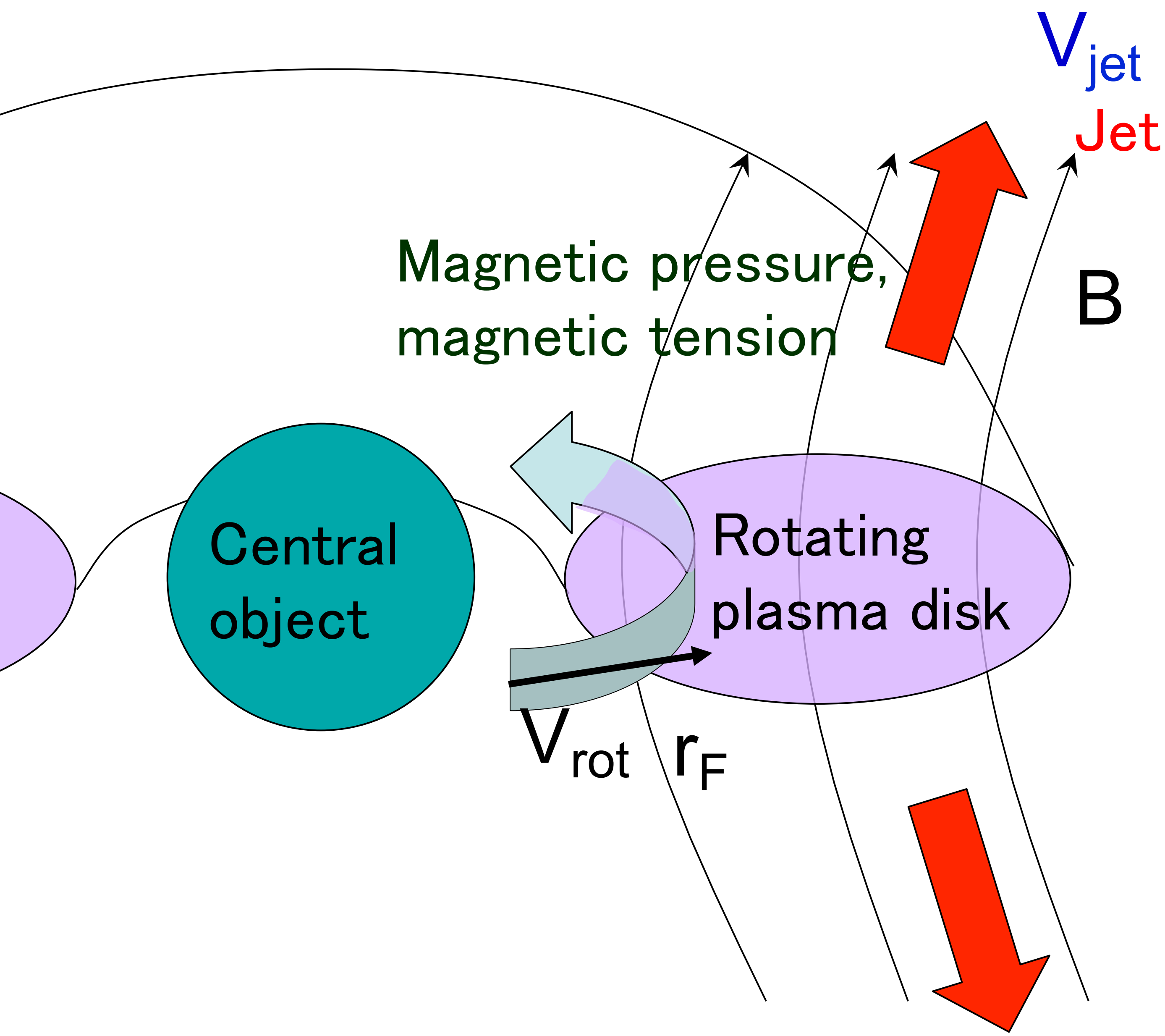
Numerical scheme：Lax-Wendrof method

Longer term numerical simulation of Newtonian MHD of jet propagation over broader region

Kudoh, Matsumoto & Shibata(1998)



Terminal velocity of jet caused by magnetic field



Non-relativistic stationary theory & numerical simulations show that in both cases (strong magnetic field case: acceleration by magnetic tension, weak magnetic field case: acceleration by magnetic pressure), the terminal velocity of the jet is given by

$$v_{jet} \approx v_{rot}$$

(Kudoh, Matsumoto, Shibata 1997)

Condition and region of relativistic jet formation by magnetic field

Relativistic jet:

$V_{jet} \sim c$

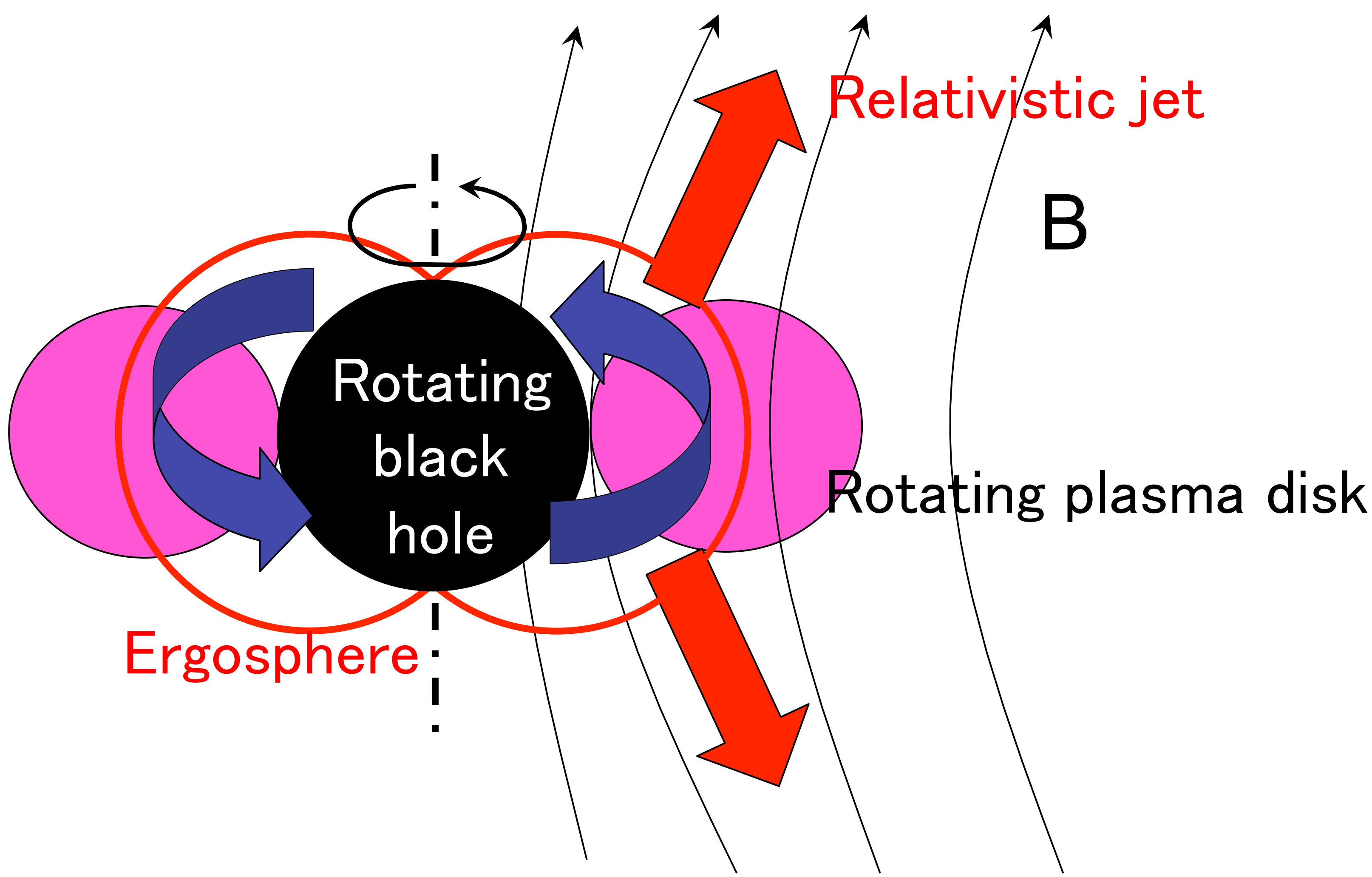
$V_{disk} \sim c$

Central object:

- Very rapidly rotating black hole (a=J/J_{max} ~1)
- $r_{rot} \sim r_H$ (Black hole horizon)

⇒ General relativistic MHD (GRMHD)

Simplest approximation

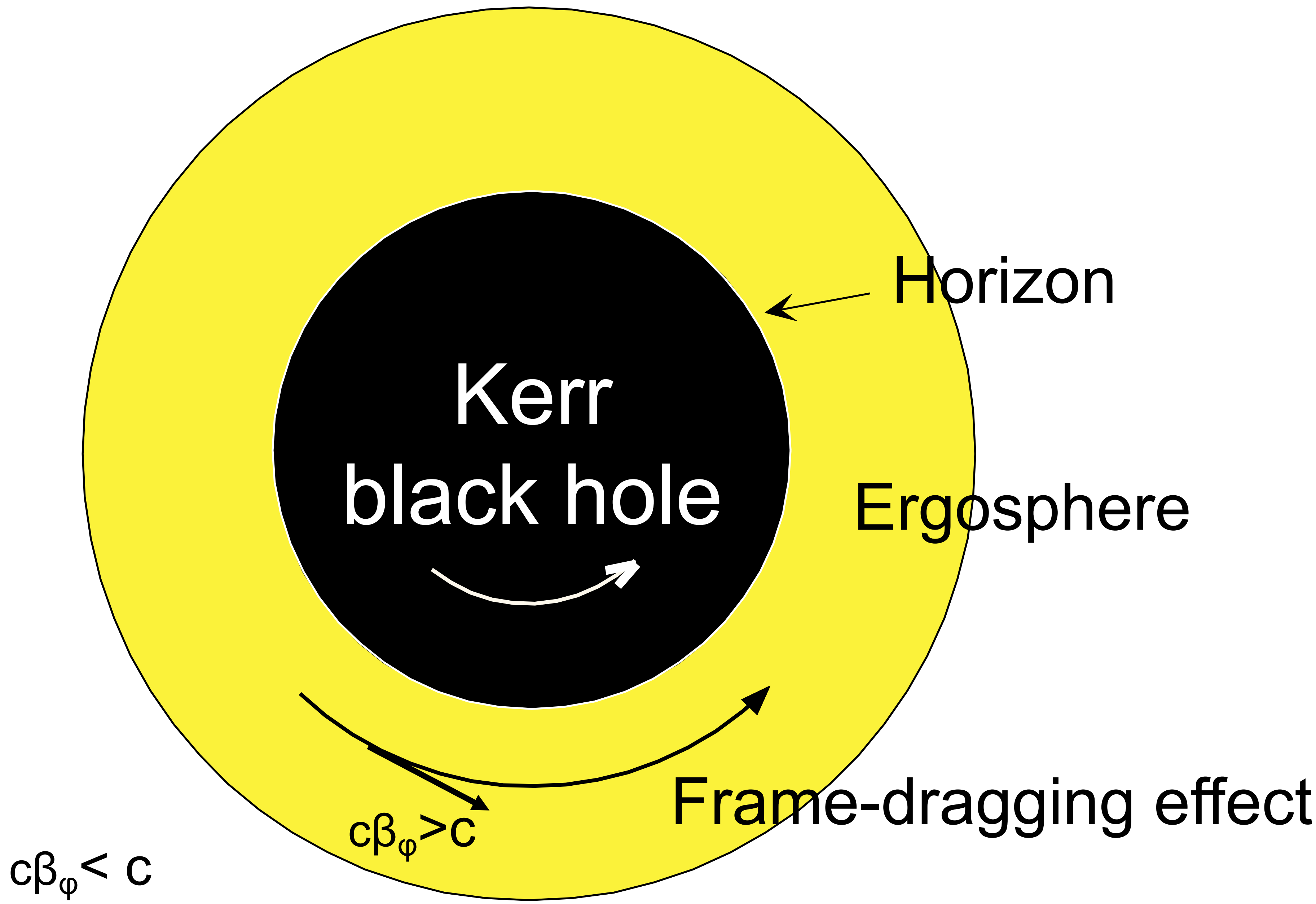


MHD

- Light speed doesn't appear explicitly .
  We can not recognize that jet from the vicinity of the black hole (BH) is relativistic or not.
- Neglect of equivalency of energy and mass.
- Neglect of Lorentz condensation and momentum of energy and fields.
- Neglect of general relativistic effects, e.g. extremely strong gravity, frame-dragging effect, and gravitational red-shift.

⇨ Requirement of general relativistic MHD (GRMHD)

Frame-dragging effect and ergosphere



In the ergosphere, any matter, energy, and information can't move or propagate in the opposite direction of the black hole rotation. Plasma in the ergosphere behaves like very heavy plasma disk!

General Relativistic MHD (GRMHD)

- GRMHD equations are similar to Newtonian relativistic MHD equations. Differences are
  - Displacement current (Ampere's law with displacement current)
  - Gravitational red shift （lapse function）
  - Very deep, steep gravitational potential（～Paczyński−Wiita potential Pseud−Newtonian potential）
  - Frame dragging effect（shift vector）

can not be described by artificial potential

- Ohm's law:
  - Zero electric resistivity (ideal GRMHD): recently explosively developed
  - Non-zero electric resistivity (resistive GRMHD): no calculation except for several special relativistic calculations
- Space-time metric: Solution of Einstein equation
  - Schwarzschild metric (non-rotating black hole)
  - Kerr metric/Kerr-Schild metric (rotating black hole)
  - (Time-varying metric（self-gravity of plasma and fields））

Analytic solution

← numerical solution

Covariant form of ideal GRMHD equations

Unit system $c = 1 \quad \mu_0 = 1$

- General relativistic equations of conservation laws:

proper particle number density

$$\nabla_\nu \left(n U^\nu \right) = 0 \quad \text{(particle number)}$$

4-velocity

$$\nabla_\nu T^{\mu\nu} = 0 \quad \text{(energy and momentum)}$$

Energy-momentum tensor

Maxwell equations:

Dual tensor of $F^{\mu\nu}$

$$\nabla_\nu \, {}^{*}F^{\mu\nu} = 0$$

Field strength tensor

4-current density

$$\nabla_\nu F^{\mu\nu} = -J^\mu$$

Ohm's law with zero resistivity (ideal MHD condition): $F_{\mu\nu} U^\nu = 0$

Kerr Metric: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$

$$g_{00} = -h_0^2; \quad g_{ii} = h_i^2; \; g_{0i} = -h_i^2 \omega_i \; (i = 1,2,3); \quad g_{ij} = 0 \; (i \neq j)$$

Lapse function: $\alpha = \sqrt{h_0^2 + \sum_i \left(h_i \omega_i \right)^2}$

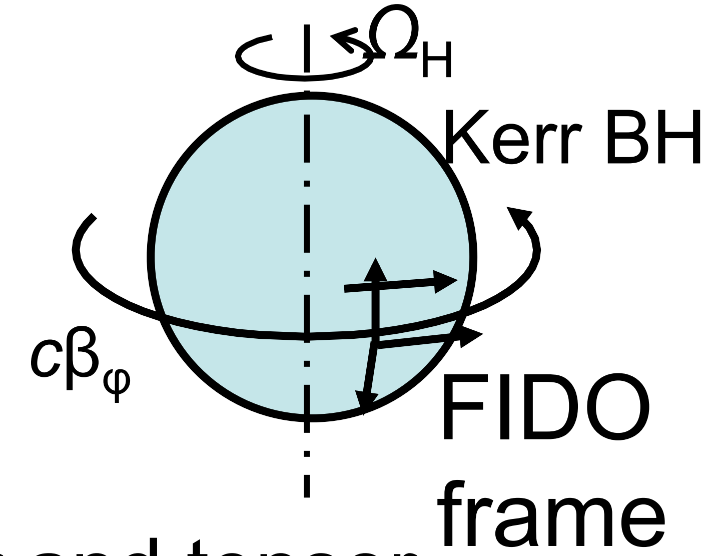
(gravitational time delay)

Shift vector: $\beta_i = h_i \omega_i / \alpha$

$\boldsymbol{\beta} = \left(\beta_1, \beta_2, \beta_3 \right)$

(velocity of dragged frame)

Several coordinates around rotating black hole



・Schwarzschild-Hilbert system (Boyer-Lindquist coordinates) ⇒ coordinates of global frame

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

・ Normal frame (spatially oblique in general)
⇒ vector and tensor

$$ds^2 = -d\tilde{t}^2 + \sum_i g_{ij} d\tilde{x}^i d\tilde{x}^j$$

← useful for numerical calculations (Kerr-Schild metric)

・ Fixed local Lorentz frame (Fiducial observer (FIDO) frame) (spatially orthogonal) ⇒ vector and tensor

$$ds^2 = -d\hat{t}^2 + \sum_i \left(d\hat{x}^i\right)^2$$

(Similar to that of Minkowski metric)

←heuristic

・ Co-moving frame ⇒scalar variables

Quantities on fiducial observer frame

(Fixed local Lorentz frame)

$$\gamma = \hat{U}^0 = \alpha U^0$$ (Lorentz factor)

$$v^i = \tfrac{1}{\gamma}\hat{U}^i = \tfrac{h_i}{\gamma}U^i - \beta^i \quad (i,j,k=1,2,3)$$ (3-velocity) $\longrightarrow \boldsymbol{v}$

$$\varepsilon = \hat{T}^{00} - \rho\gamma = \alpha^2 T^{00} - \rho\alpha U^0$$ (total energy density)

$$\hat{P}^i = \hat{T}^{i0} = \alpha h_i T^{i0} - \alpha^2 \beta^i T^{00}$$ (total momentum density) $\rightarrow \boldsymbol{P}$

$$\hat{T}^{ij} = h_i h_j T^{ij} - \alpha\beta^i h_j T^{0j} - \alpha\beta^j h_i T^{i0} + \alpha^2\beta^i\beta^j T^{ij}$$ $\rightarrow \mathbf{T}$

$$\hat{E}_i = \hat{F}^{i0} = \tfrac{1}{\alpha h_i}F_{i0} + \sum_j \tfrac{\beta_j}{h_i h_j}F_{ij}$$ (electric field) $\rightarrow \boldsymbol{E}$

$$\hat{B}_i = \sum_{j,k}\tfrac{1}{2}\varepsilon_{ijk}\hat{F}_{jk} = \sum_{jk}\tfrac{1}{2}\varepsilon_{ijk}\tfrac{1}{h_i h_j}F_{jk}$$ (magnetic field) $\rightarrow \boldsymbol{B}$

$$\rho_{\mathrm{e}} = \hat{J}^0 = \alpha J^0$$ (electric charge density)

$$\hat{J}^i = h_i J^i - \alpha\beta^i J^0$$ (electric current density) $\rightarrow \boldsymbol{J}$

$$c = 1, \quad \mu_0 = 1, \quad \varepsilon_0 = 1$$ (MKSA like)

3+1 Formalism of Ideal GRMHD Equation ~ similar to nonrelativistic ideal MHD

(conservative form)

Special relativistic mass density, γρ

$$\frac{\partial D}{\partial t} = -\nabla\cdot[\alpha D(\mathbf{v}+c\boldsymbol{\beta})]$$

(conservation of particle number)

general relativistic effect

Special relativistic total momentum density

$$\frac{\partial \boldsymbol{P}}{\partial t} = -\nabla\cdot[\alpha(\boldsymbol{T}+c\boldsymbol{\beta}\boldsymbol{P})] - \left(D+\frac{\varepsilon}{c^2}\right)\nabla(c^2\alpha) + \alpha\boldsymbol{f}_{\mathrm{curv}} - \boldsymbol{P}:\boldsymbol{\sigma}$$

(equation of motion)

special relativistic effect

$$\boldsymbol{P} = h\gamma^2\boldsymbol{v} + \boldsymbol{E}\times\boldsymbol{B} \qquad \boldsymbol{T} = h\gamma^2\boldsymbol{v}\boldsymbol{v} + \left(p+\frac{B^2}{2}+\frac{E^2}{2}\right)\boldsymbol{I} - \boldsymbol{B}\boldsymbol{B} - \boldsymbol{E}\boldsymbol{E}$$

Special relativistic total energy density

$$\frac{\partial \varepsilon}{\partial t} = -\nabla\cdot[\alpha(c^2\boldsymbol{P} - Dc^2\boldsymbol{v} + \varepsilon c\boldsymbol{\beta})] - (\nabla\alpha)\cdot c^2\boldsymbol{P} - \boldsymbol{T}:\boldsymbol{\sigma}$$

(equation of energy)

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla\times[\alpha(\boldsymbol{E} - c\boldsymbol{\beta}\times\boldsymbol{B})] \qquad \alpha(\boldsymbol{J}+\rho_e c\boldsymbol{\beta}) + \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \nabla\times\left[\alpha\left(\boldsymbol{B}+\frac{\boldsymbol{\beta}}{c}\times\boldsymbol{E}\right)\right]$$

Similarity with MHD has boosted the recent development of ideal GRMHD simulations!

$$\nabla\cdot\boldsymbol{B} = 0 \qquad \rho_e = \frac{\alpha}{c^2}\nabla\cdot\boldsymbol{E}$$

No coupling with other Eqs.

$$\mathbf{E}+\mathbf{v}\times\mathbf{B} = \mathbf{0}$$ (ideal MHD condition)

where

$$\alpha = \sqrt{h_0^2 + \sum_i (h_i\omega_i)^2}$$: (Lapse function),

$$\beta^i = \frac{h_i\omega_i}{\alpha}$$: (shift vector)

$\boldsymbol{\sigma}$: shear of $\boldsymbol{\beta}$

Ideal MHD equations

(non-relativistic conservative form)

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v})$$ (conservation of particle number)

$$\frac{\partial \boldsymbol{P}}{\partial t} = -\nabla \cdot \boldsymbol{T} - \rho \nabla \Phi_{\mathrm{grav}}$$ (equation of motion)

$$\boldsymbol{P} = \rho \boldsymbol{v} \qquad \boldsymbol{T} = \rho \boldsymbol{v}\boldsymbol{v} + \left(p + \frac{B^2}{2} \right) \boldsymbol{I} - \boldsymbol{B}\boldsymbol{B}$$

$$\frac{\partial \varepsilon}{\partial t} = -\nabla \cdot \left(h_{nr} \boldsymbol{v} + \boldsymbol{E} \times \boldsymbol{B} \right) - \boldsymbol{P} \cdot \nabla \Phi_{\mathrm{grav}}$$ (equation of energy)

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \qquad \mathbf{J} = \nabla \times \mathbf{B}$$

(Maxwell equations)

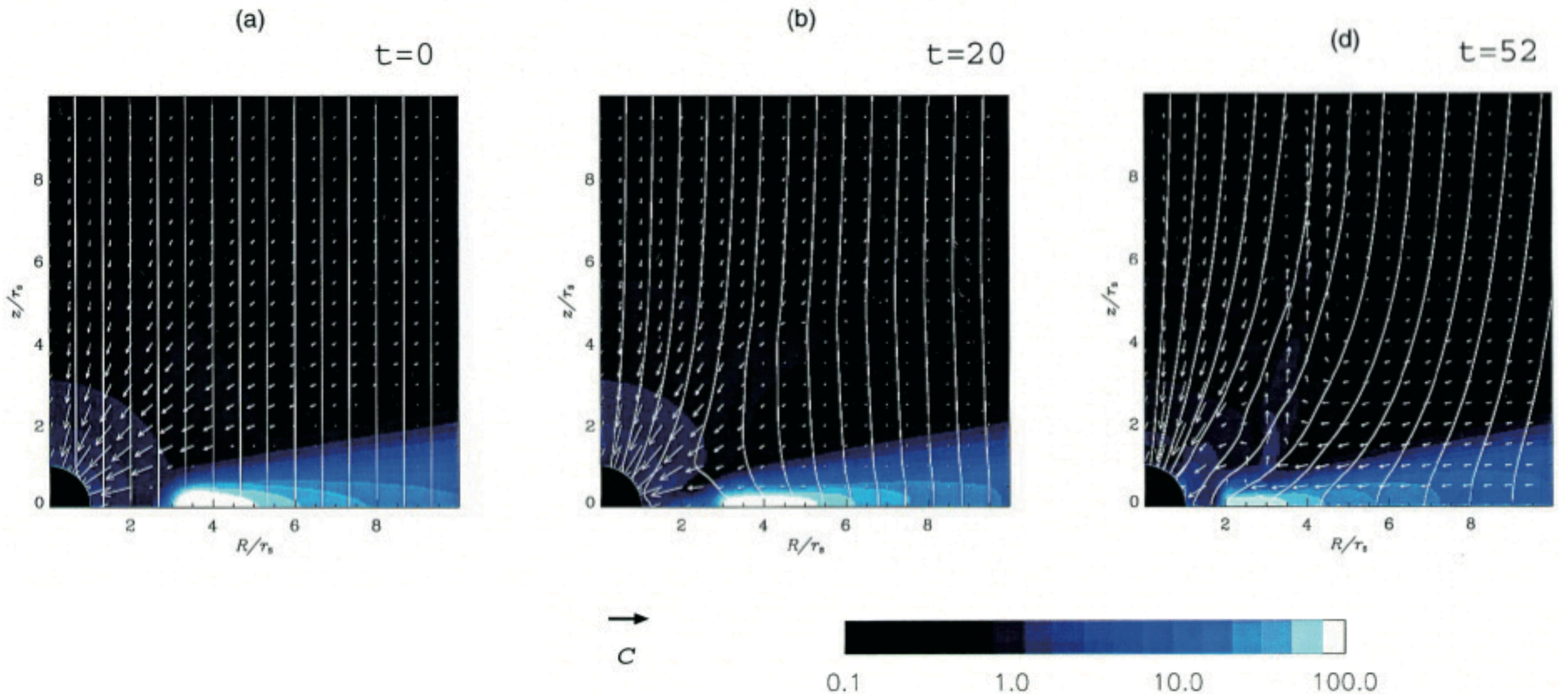
$$\nabla \cdot \mathbf{B} = \mathbf{0}$$

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \mathbf{0}$$ (ideal MHD condition)

$$h_{\mathrm{nr}} = \frac{\rho}{2} v^2 + \frac{\Gamma p}{\Gamma - 1}$$ (equation of state; non-relativistic enthalpy density)

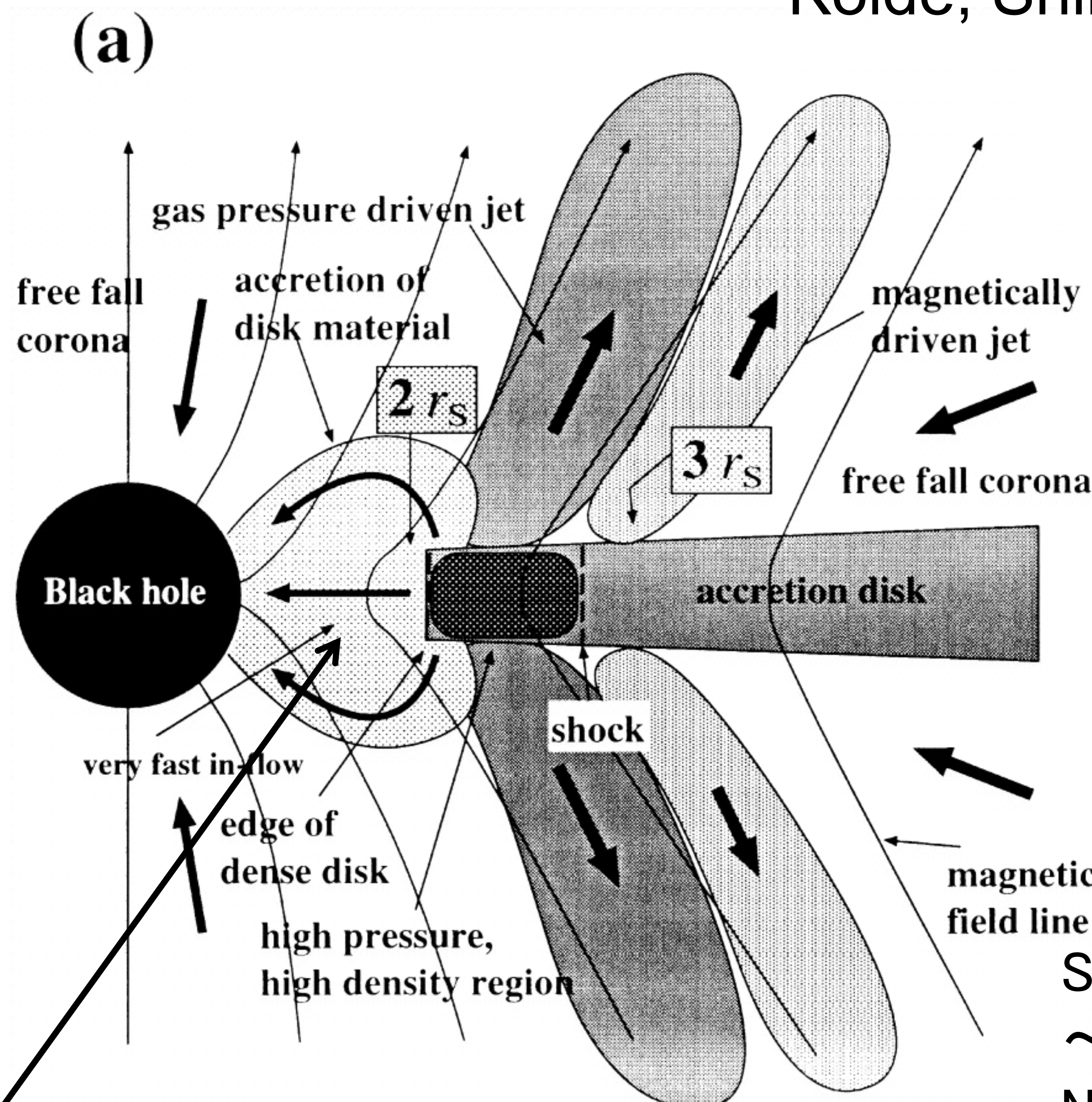
First GRMHD simulation of jet formation around non-rotating (Schwarzschild) black hole: GRMHD version of Shibata & Uchda (1986)

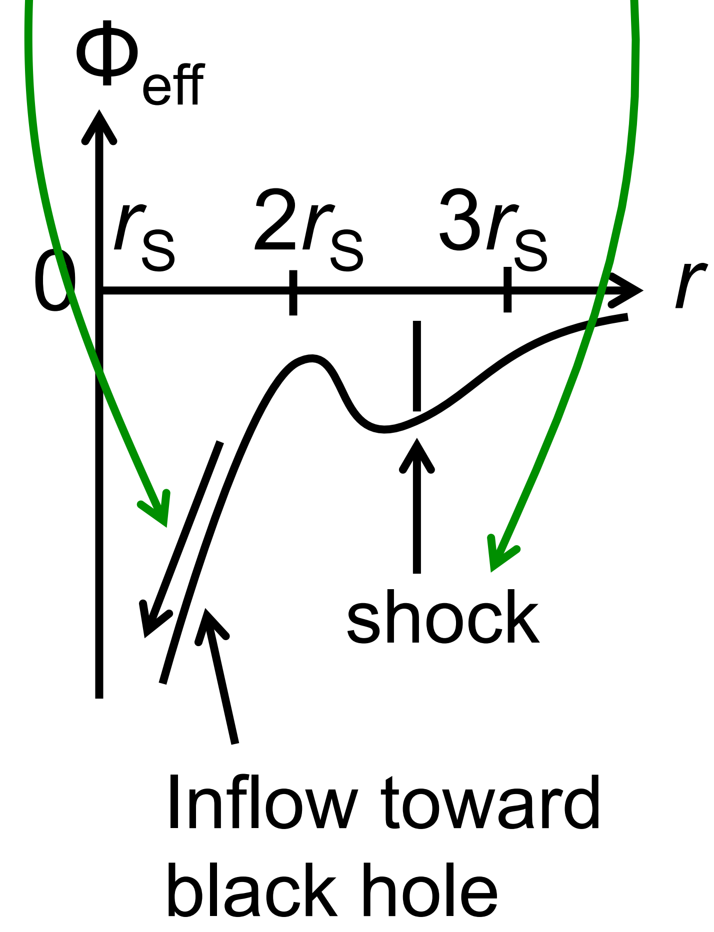
Koide, Shibata, & Kudoh (1999)



Deep, steep gravitational potential

Koide, Shibata, & Kudoh (1999)





Schwarzschild black hole case ～Paczyński–Wiita (pseudo-Newtonian) potential

No stable circular orbit around black hole inside of $3r_S$

First GRMHD simulation with rotating black hole

(Koide, Shibata, Kudoh, Meier 2002)

(1) Kerr black hole: maximally rotating rotation parameter, $a=J/J_{max}=0.9999$

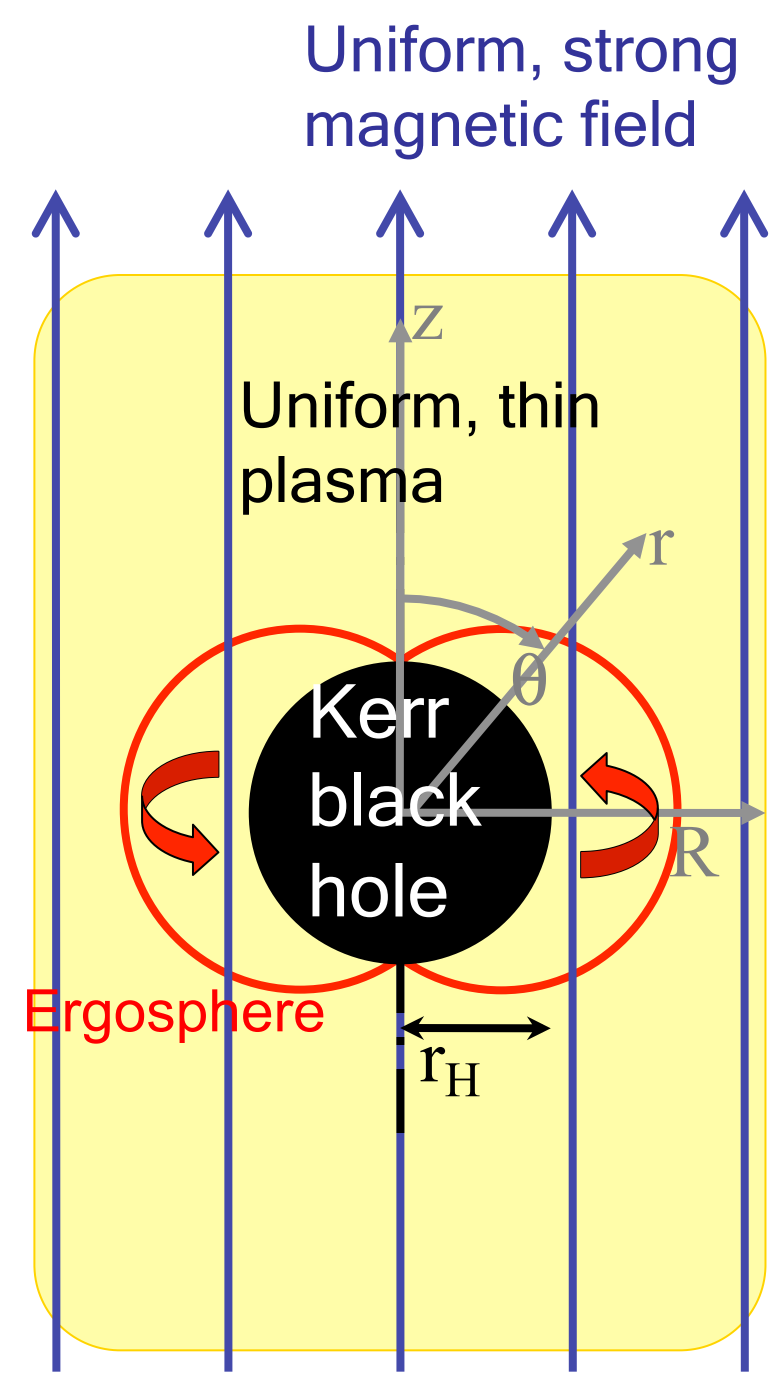
(2) Magnetic field: Uniform around Kerr black hole (Wald solution)

(3) Plasma: zero momentum, uniform, low density and pressure

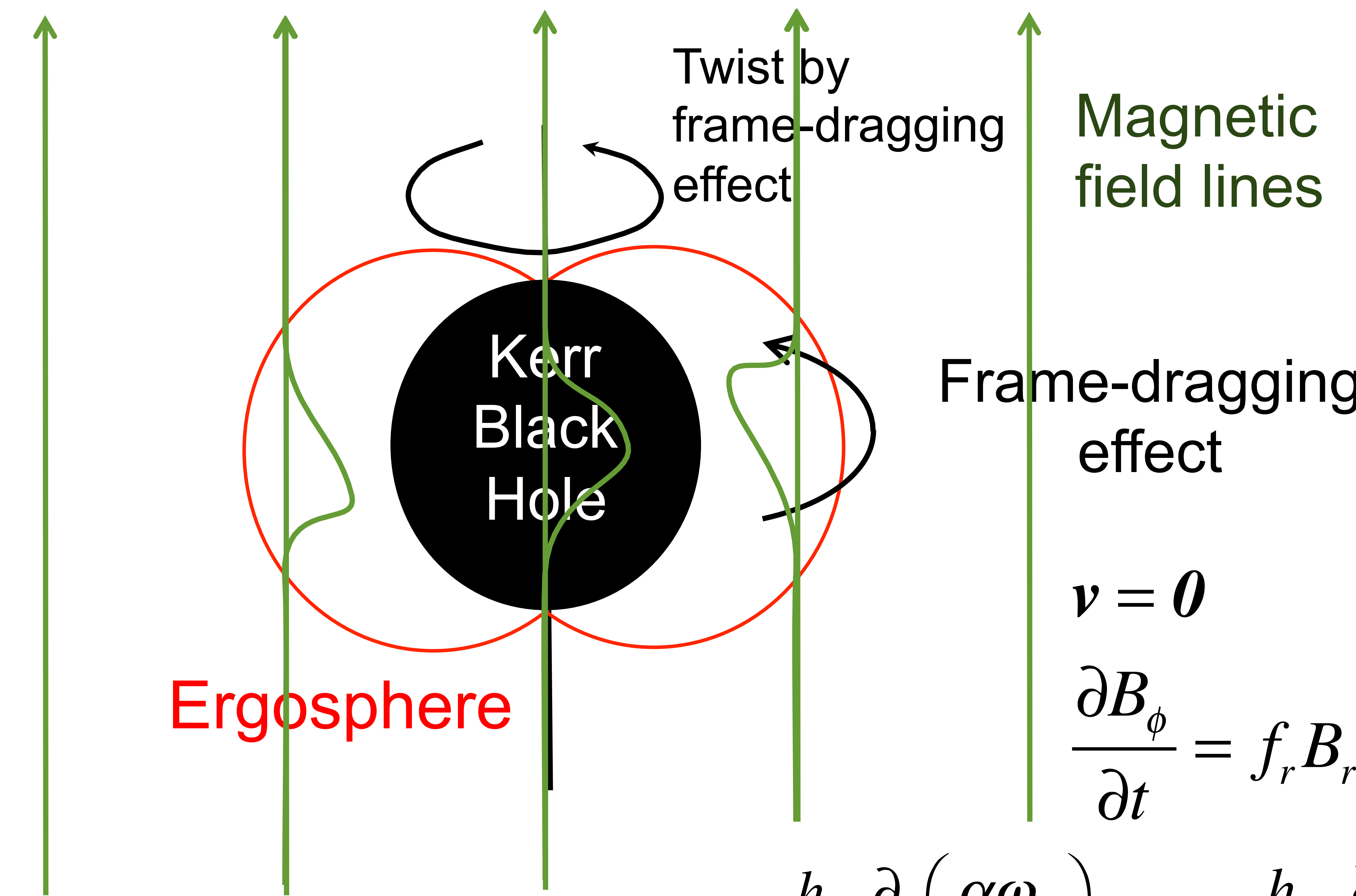
$$\rho_0=0.1B_0^2/c^2,\ p_0=0.06\rho_0 c^2$$

(4) Calculation region:
$1.05\ r_H \leqq r \leqq 40\ r_H$,
$0.01 \leqq \theta \leqq \pi/2$

– ・ Axisymmetry, symmetry with respect to equatorial plain



Frame-dragging dynamo: Twist of magnetic field line by ergosphere



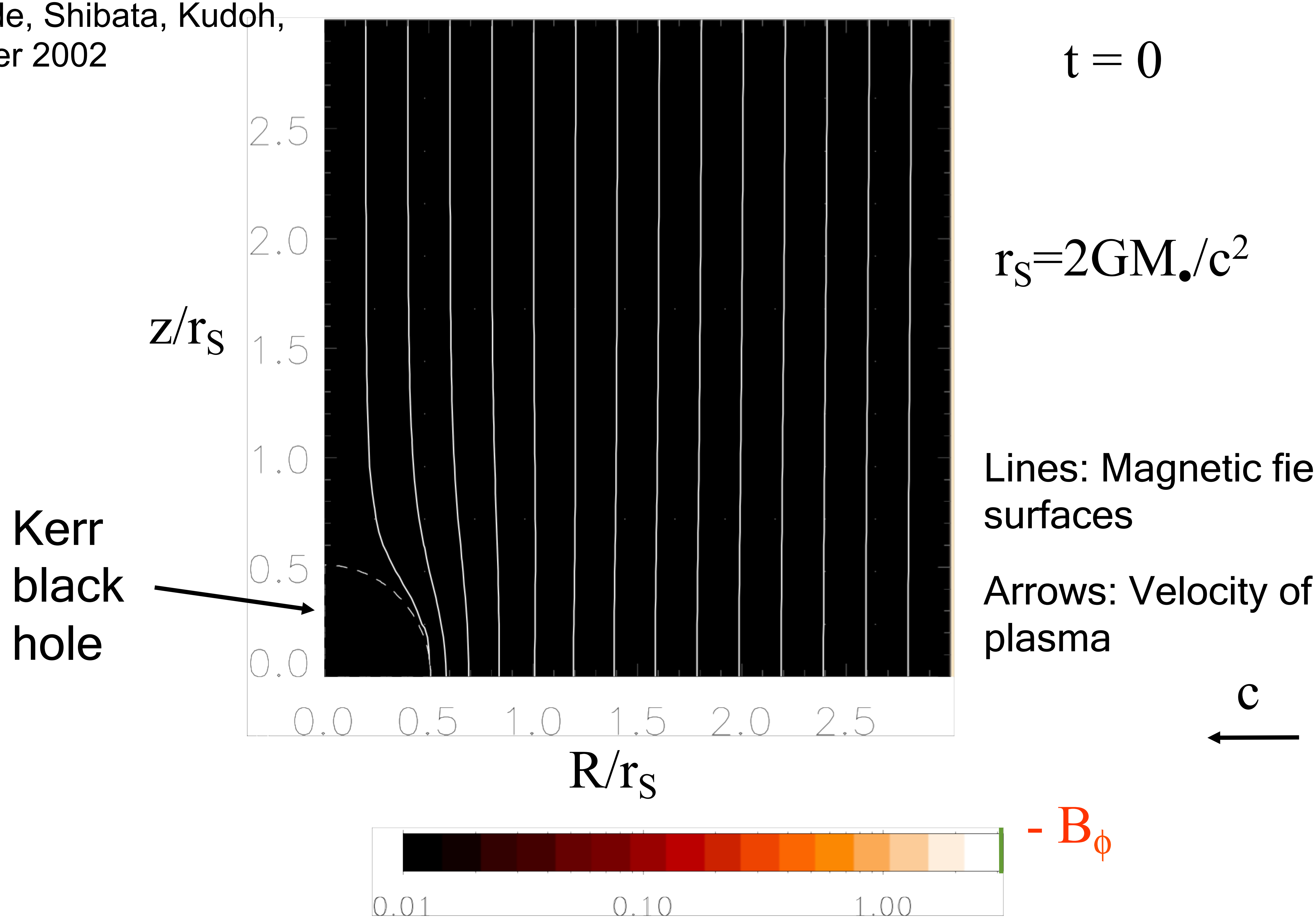
$$\boldsymbol{v} = \boldsymbol{0}$$

$$\frac{\partial B_\phi}{\partial t} = f_r B_r + f_\theta B_\theta$$

$$f_r = \frac{h_3}{h_1}\frac{\partial}{\partial r}\left(\frac{\alpha\omega_3}{h_3}\right), \quad f_\theta = \frac{h_3}{h_2}\frac{\partial}{\partial \theta}\left(\frac{\alpha\omega_3}{h_3}\right)$$

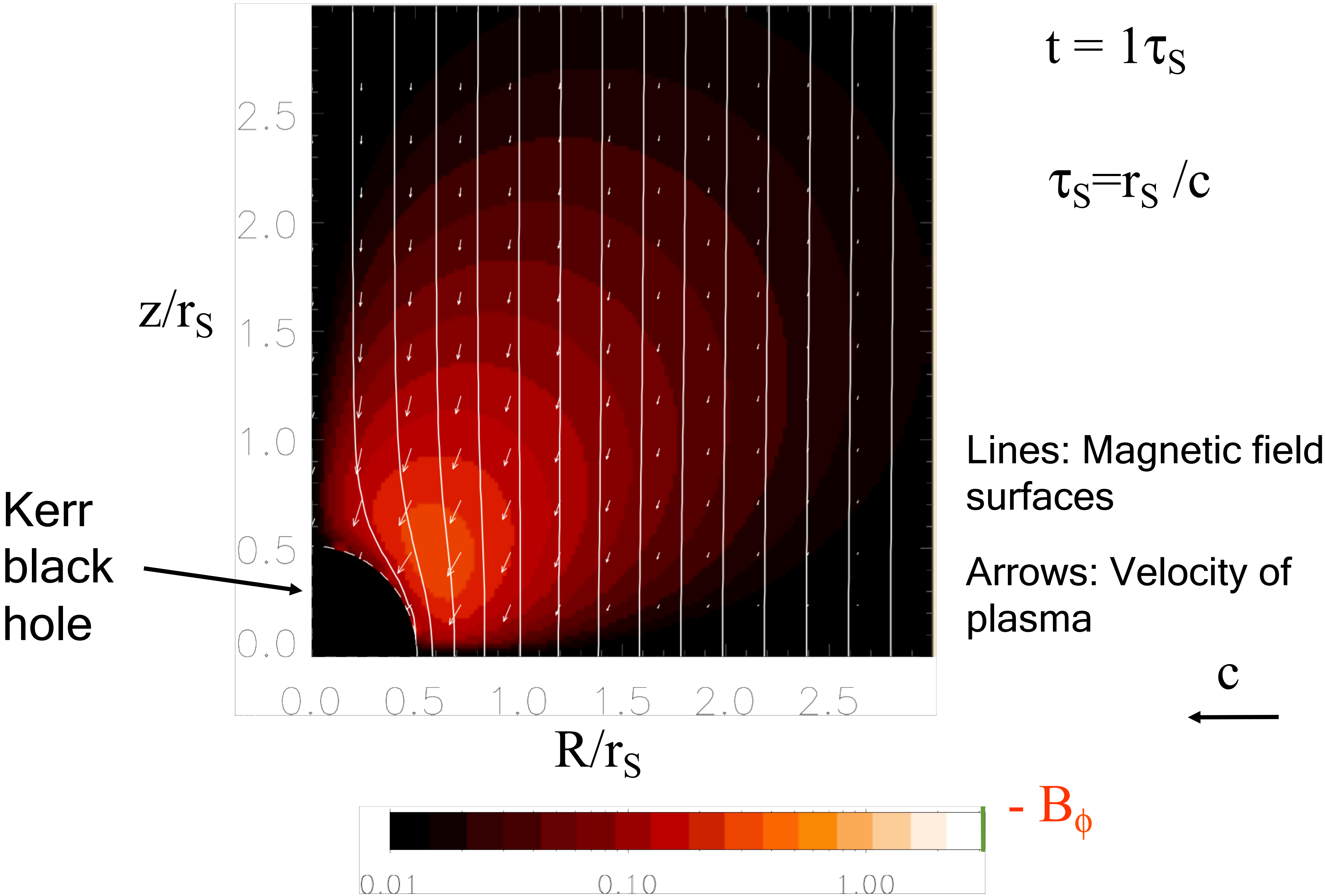
Kerr black hole, Uniform magnetic field, No Accretion disk

Koide, Shibata, Kudoh, Meier 2002

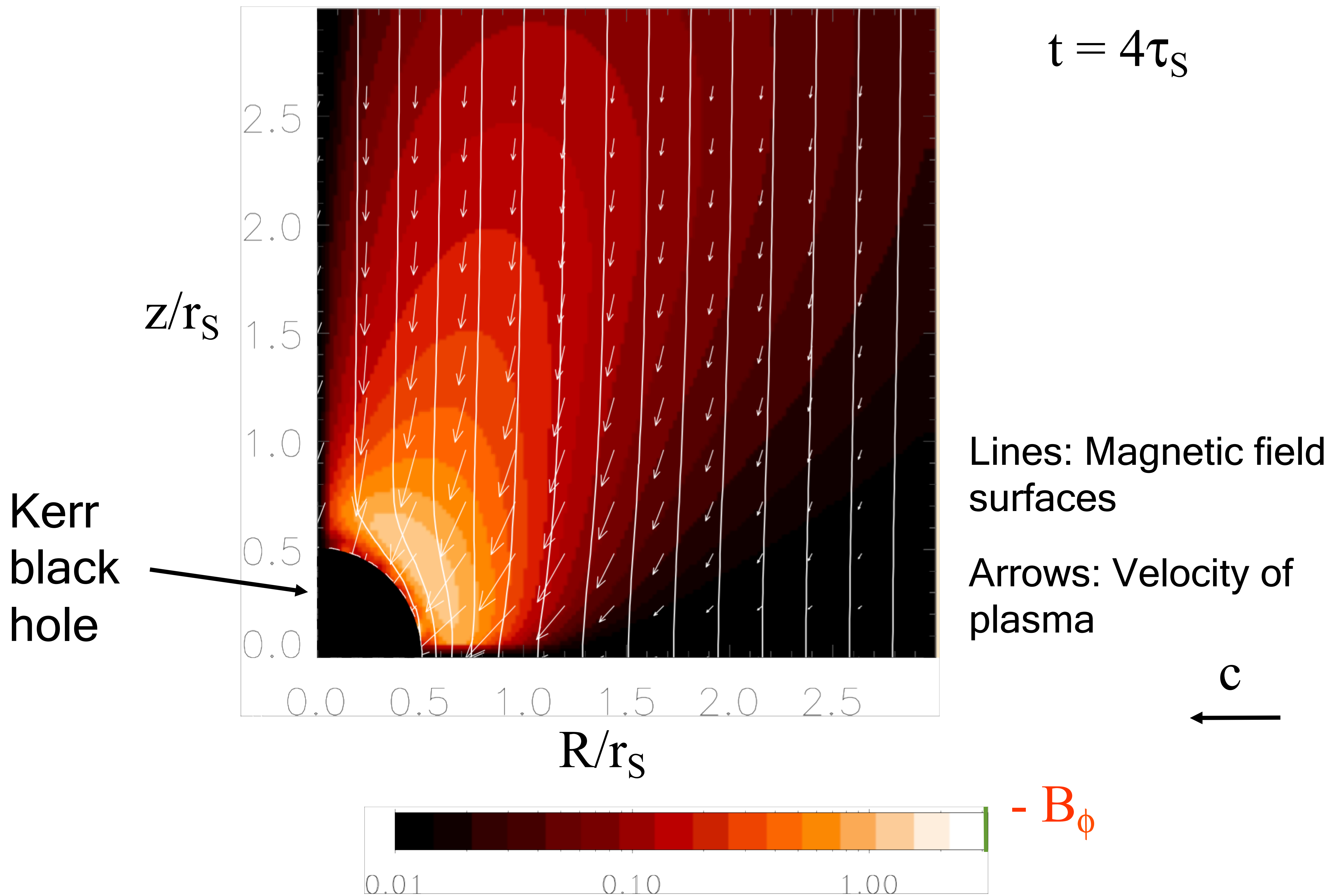


Lines: Magnetic field surfaces

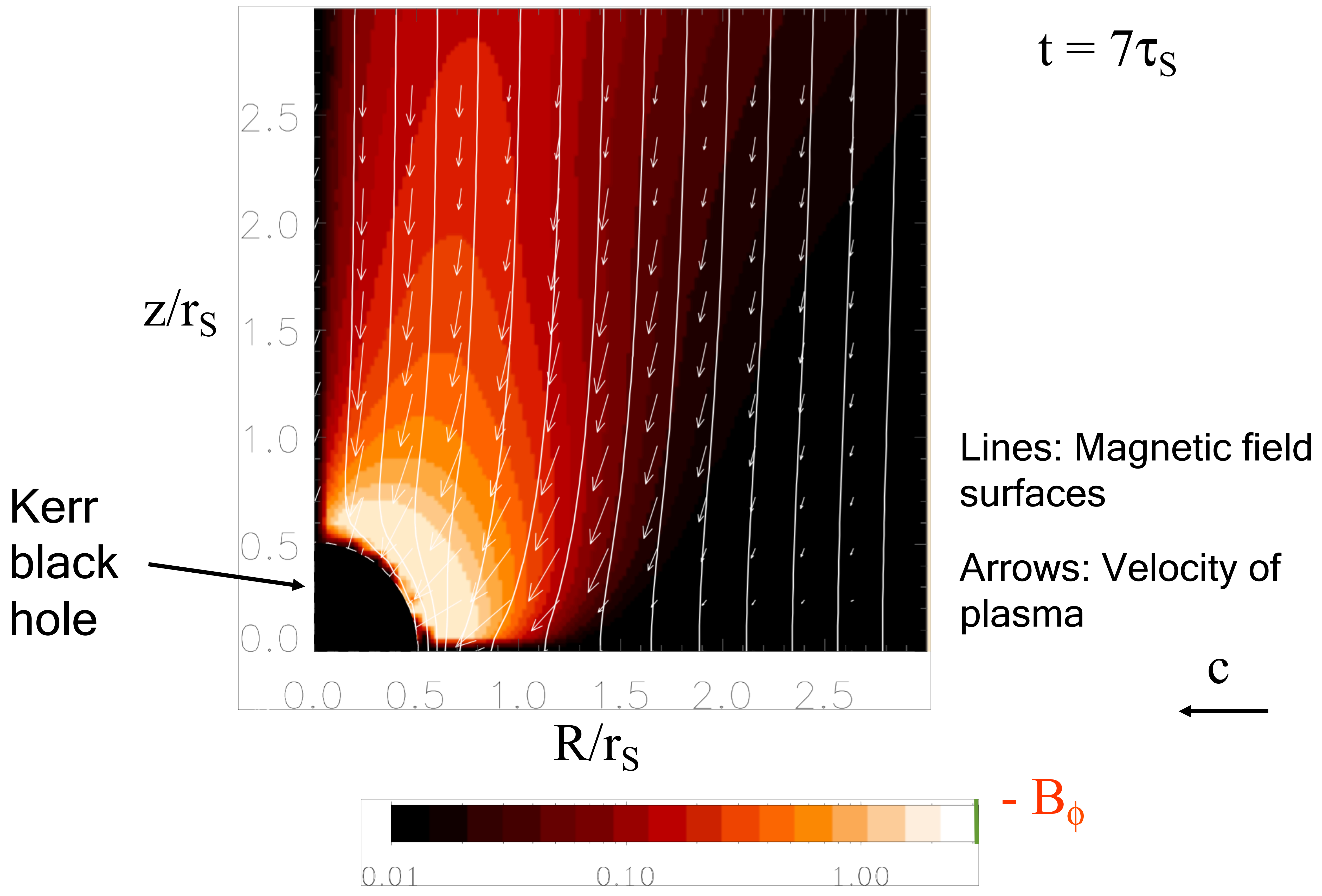
Arrows: Velocity of plasma



$t = 1\tau_S$
$\tau_S = r_S / c$
z/r_S
2.5
2.0
1.5
1.0
0.5
0.0
Kerr black hole
0.0
0.5
1.0
1.5
2.0
2.5
R/r_S
Lines: Magnetic field surfaces
Arrows: Velocity of plasma
c
0.01
0.10
1.00
$-B_\phi$

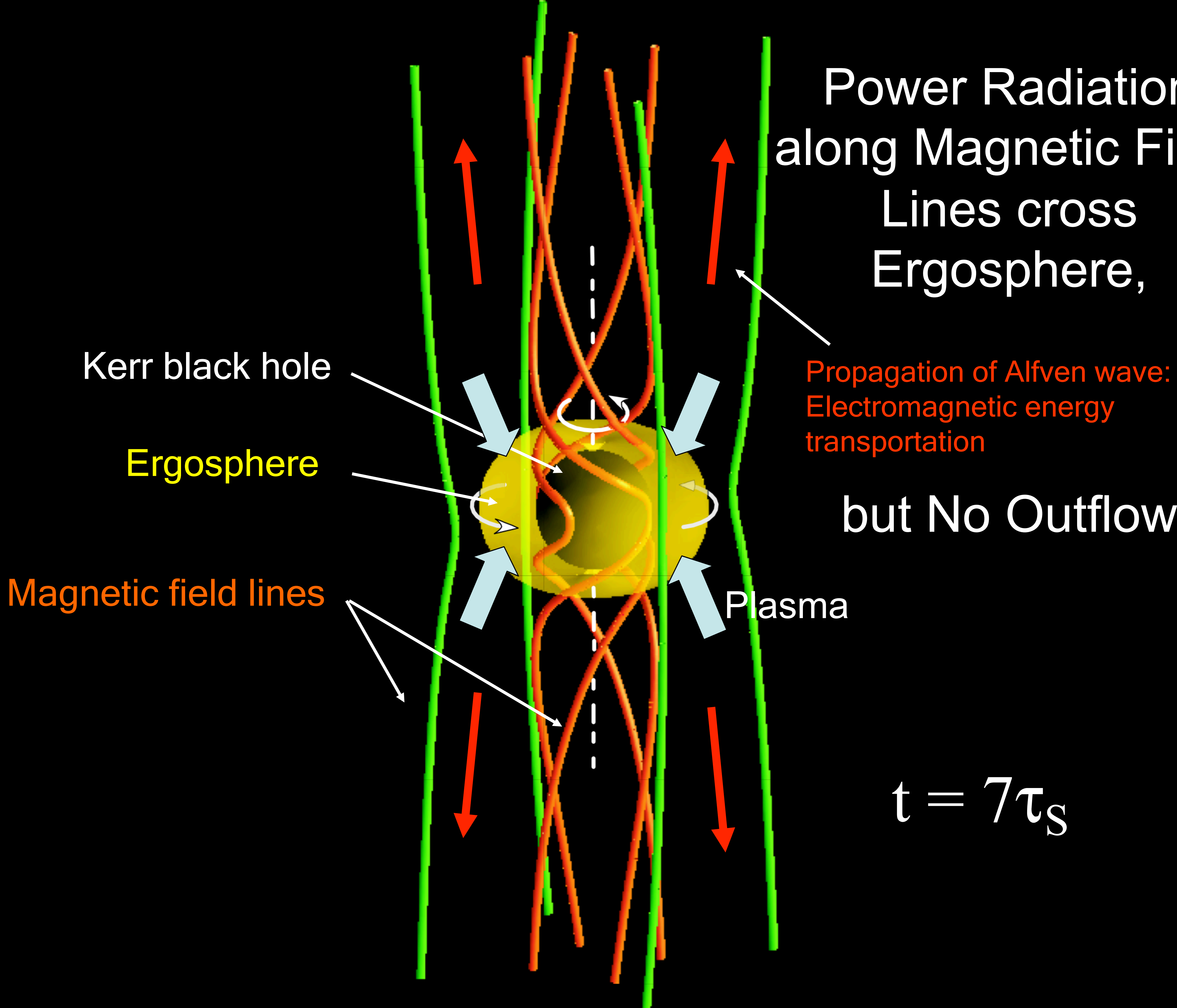


t = 4τS
z/rS
R/rS
Kerr black hole
Lines: Magnetic field surfaces
Arrows: Velocity of plasma
c
- Bϕ
0.0
0.5
1.0
1.5
2.0
2.5
0.01
0.10
1.00



t = 7τS
z/rS
R/rS
Kerr black hole
Lines: Magnetic field surfaces
Arrows: Velocity of plasma
c
- Bφ
0.0
0.5
1.0
1.5
2.0
2.5
0.01
0.10
1.00

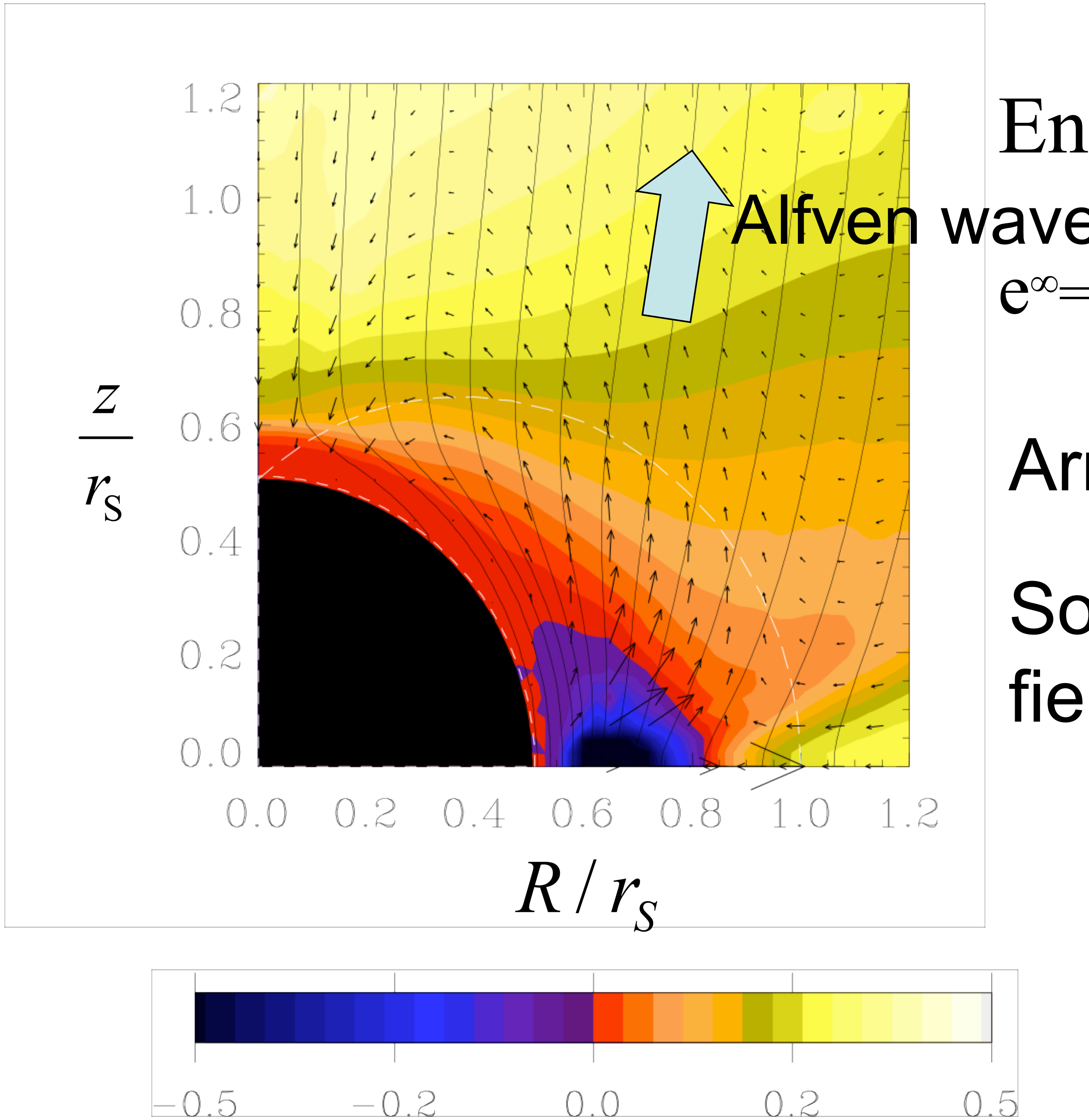
Power Radiation along Magnetic Field Lines cross Ergosphere,



but No Outflow

$$t = 7\tau_S$$

Formation of Negative Energy Region



Energy-at-infinity density, e^{∞}

Energy-at-infinity

$$e^{\infty} = \alpha e + \omega_3 l_z$$

Arrow: Power flux density

Solid line: magnetic field surface

MHD Penrose process (Hirotani, Takahashi, Nitta, Tomimatsu 1992)

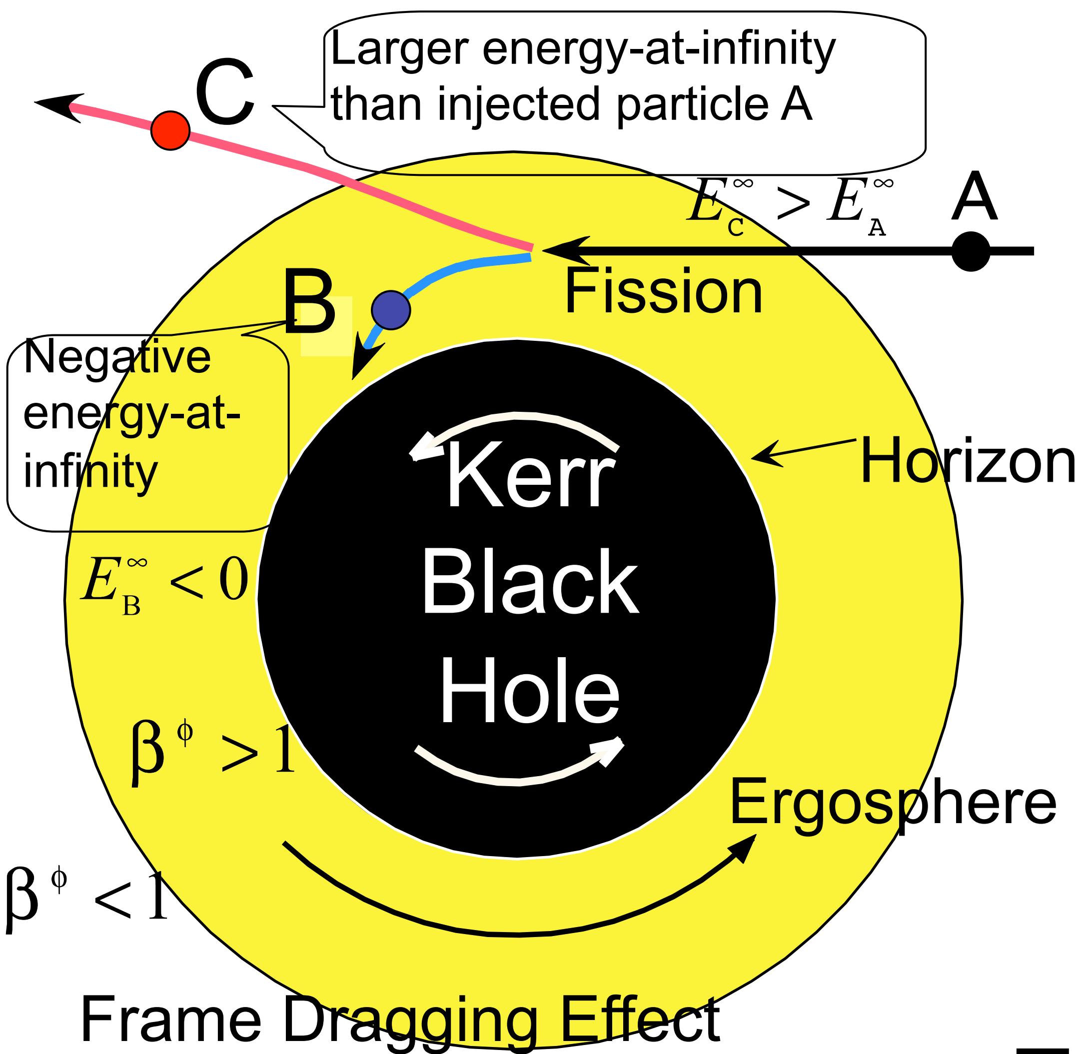
Penrose Process

Extraction of energy of rotating black hole through particle fission in ergosphere

Conservation of angular momentum $L_A^\infty = L_B^\infty + L_C^\infty$

Conservation of energy $E_A^\infty = E_B^\infty + E_C^\infty$

A→B+C (fission ⇒ redistribution of angular momentum)



$$E^\infty = \alpha \hat{E} + \beta^\phi L^\phi$$

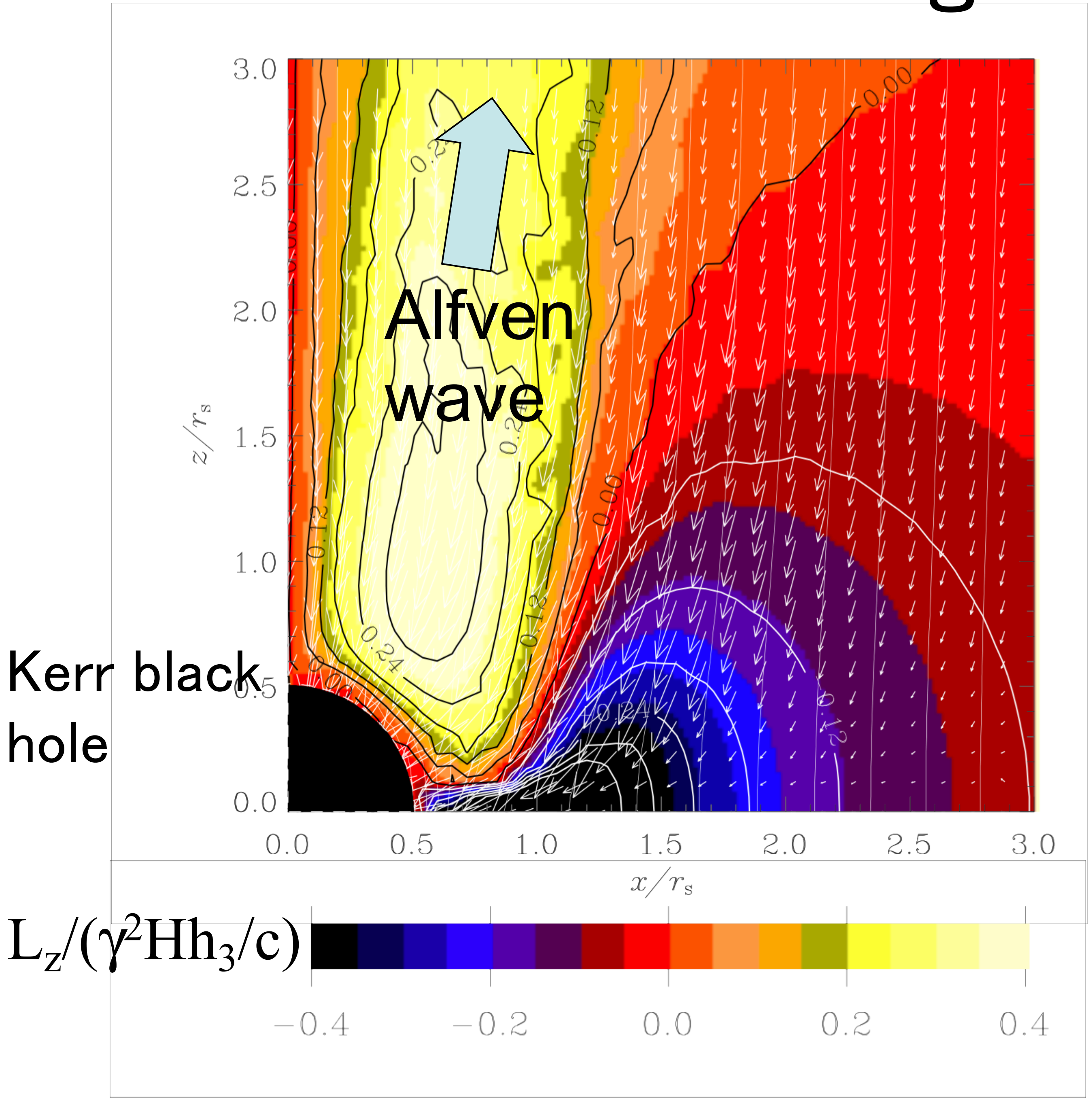
When $L_B^\phi < -\hat{E}_B / \beta^\phi$ in ergosphere, $E_B^\infty < 0$.

If particle B is swallowed by black hole, total mass of black hole decreases. The particle C obtains energy larger than that of injected particles: $E_C^\infty = E_A^\infty - E_B^\infty > E_A^\infty$

$$M'_{BH} = M_{BH} + E_B^\infty < M_{BH}$$

⇒ Energy extraction from black hole

Redistribution of angular momentum



$L_z/(\gamma^2 H h_3/c)$

H: relativistic enthalpy

h_3: component of metric

Penrose process

Energy-at-infinity

$$E^{\infty} = \underbrace{\alpha E}_{> 0} + \underbrace{\omega_3 L_z}_{\downarrow}$$

Negative energy

Inward negative energy: plasma,

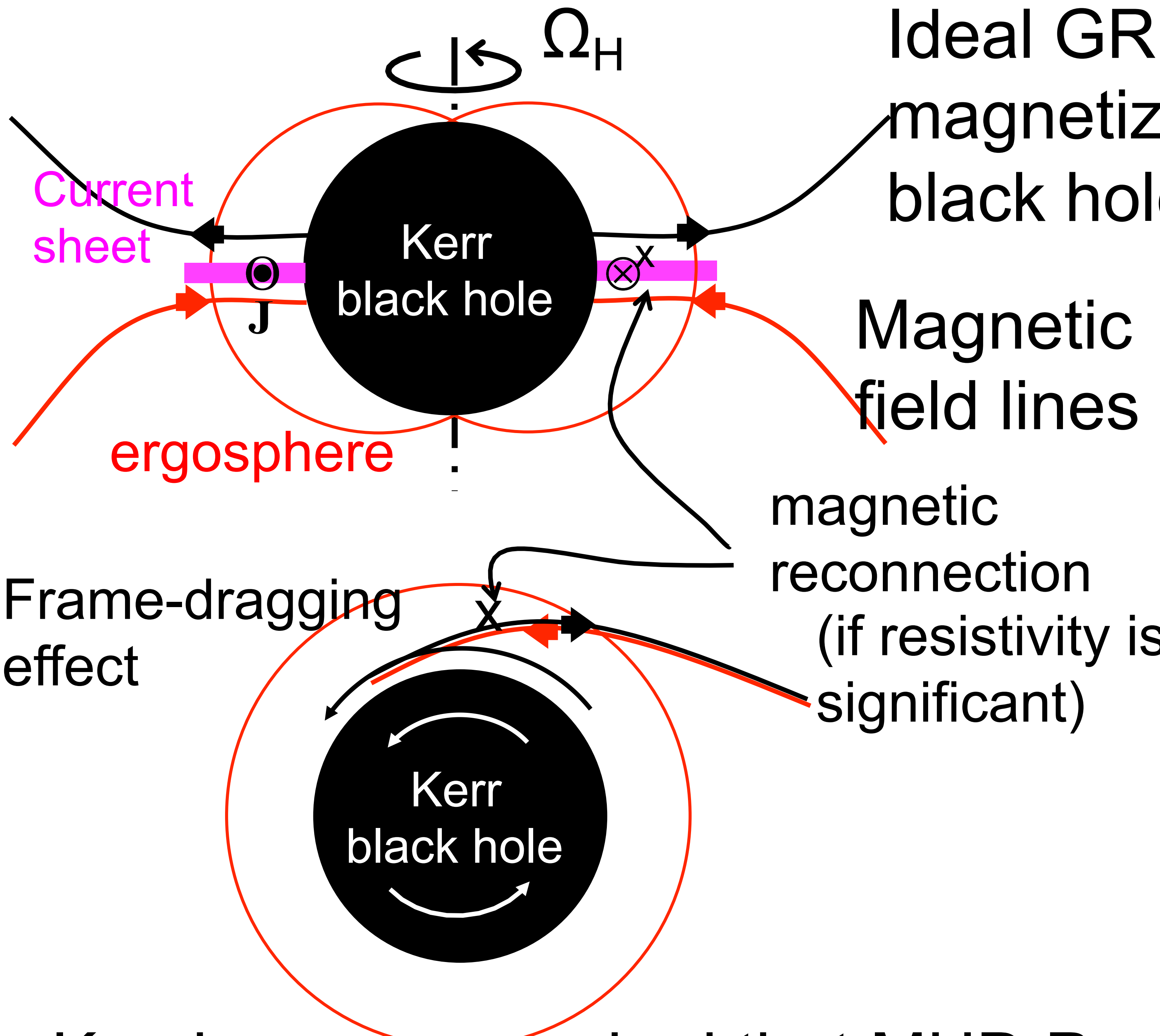
Outward energy: Alfven wave (plasma and magnetic field),

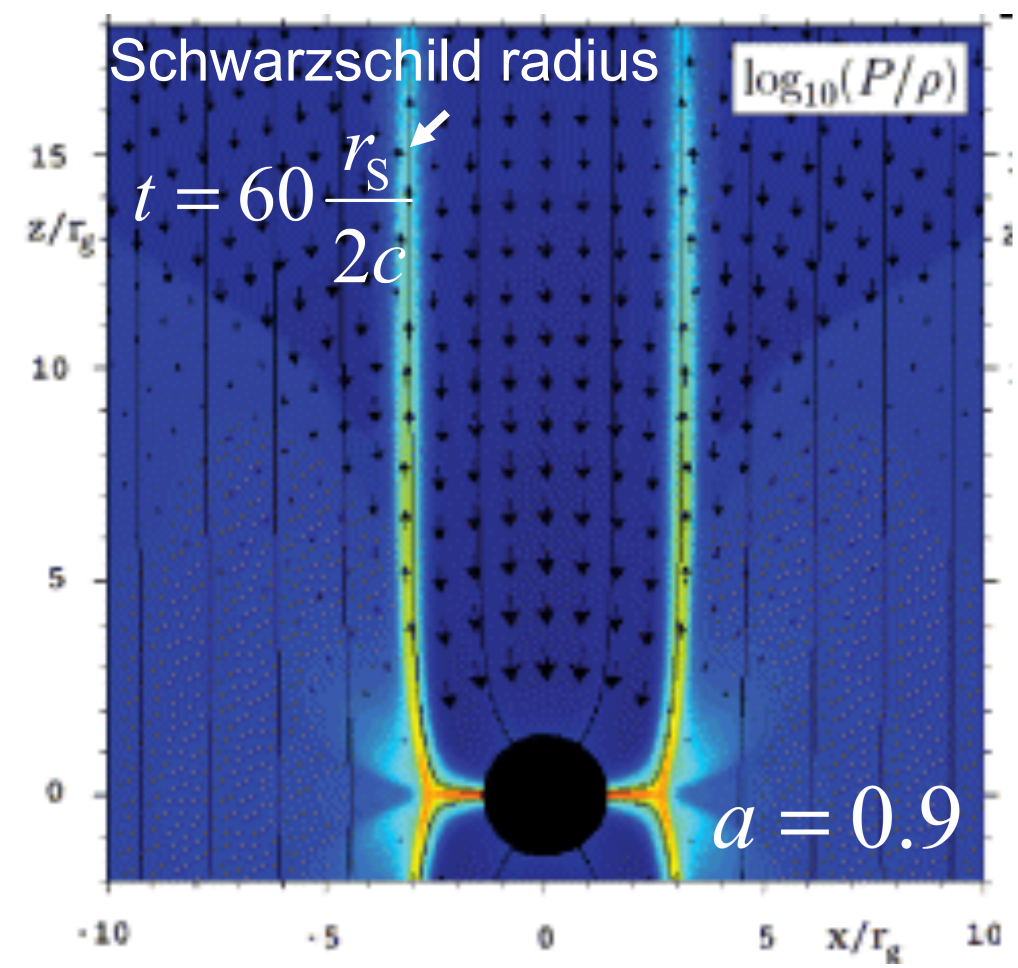
Redistribution of angular momentum: magnetic tension

～ MHD Penrose process (Takahashi et al. 1990, Hirotani et al. 1992)

Longer term simulation of uniformly magnetized plasmas around Kerr BH

Ideal GRMHD simulation of uniformly magnetized plasma around rotating black hole (Komissarov 2005)

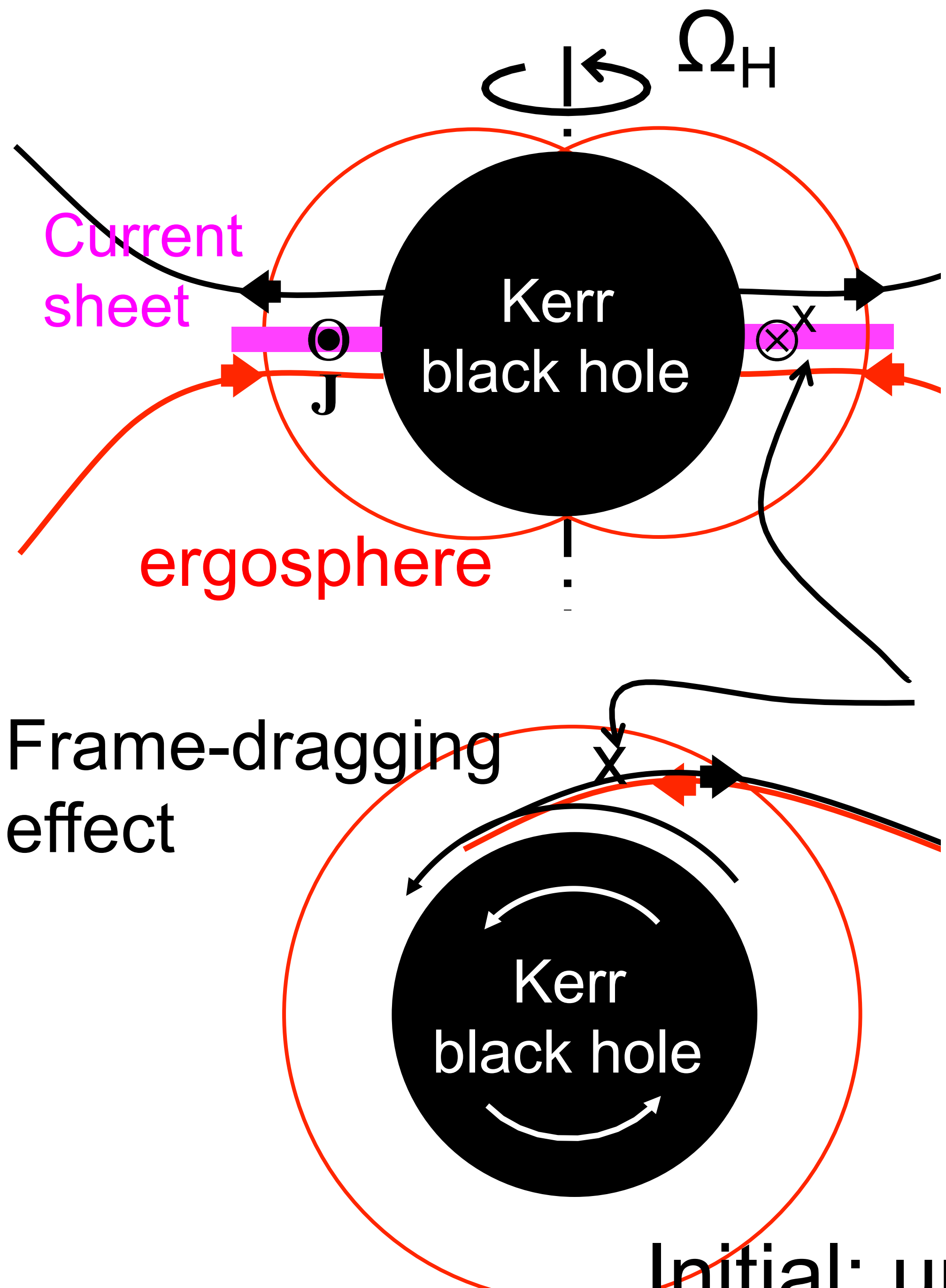


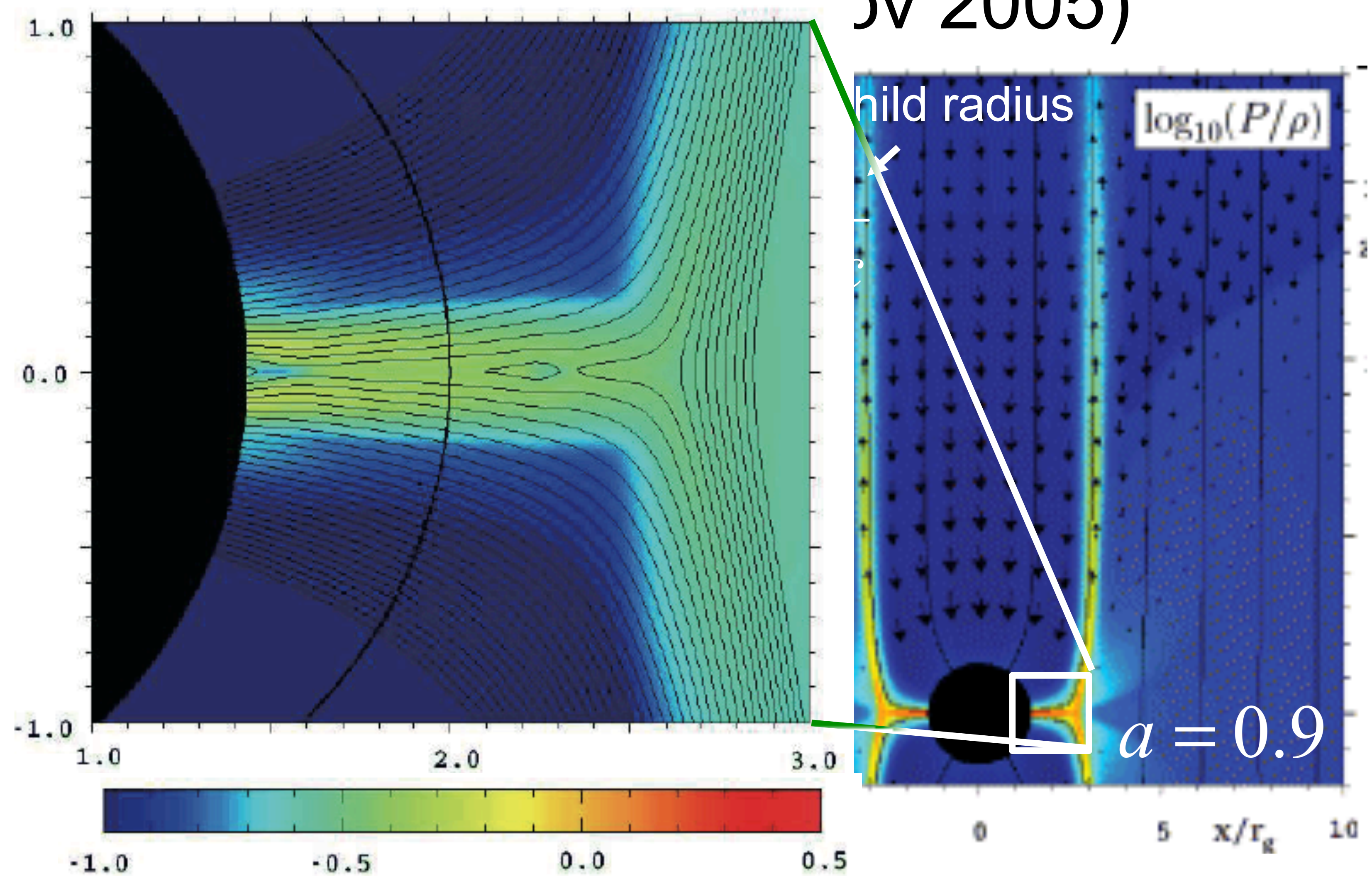


Komissarov remarked that MHD Penrose process is transient. In final, steady sate, Purely electromagnetic mechanism (Blandford-Znajek process like) extracts rotation energy of the black hole.

Longer term simulation of uniformly magnetized plasmas around Kerr BH

Ideal GRMHD simulation of uniformly magnetized plasma around rotating black hole (Komissarov 2005)



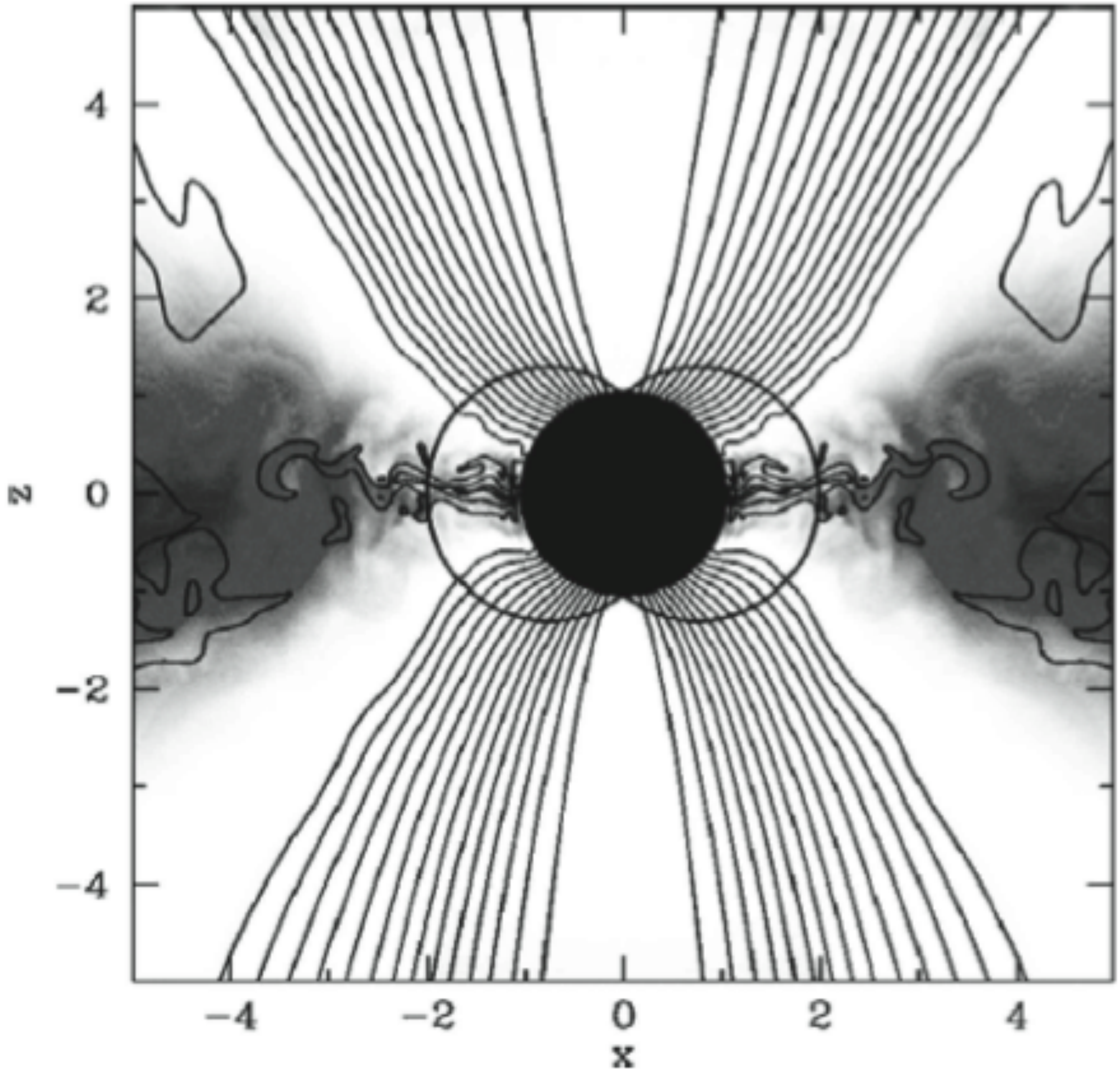


Initial: uniform magnetic field (Wald solution)

⇒ sprit monopole-like magnetic field in ergosphere

⇒ magnetic reconnection in ergosphere

Long term simulation of GRMHD simulation of initially uniform magnetic field around maximally rotating black hole



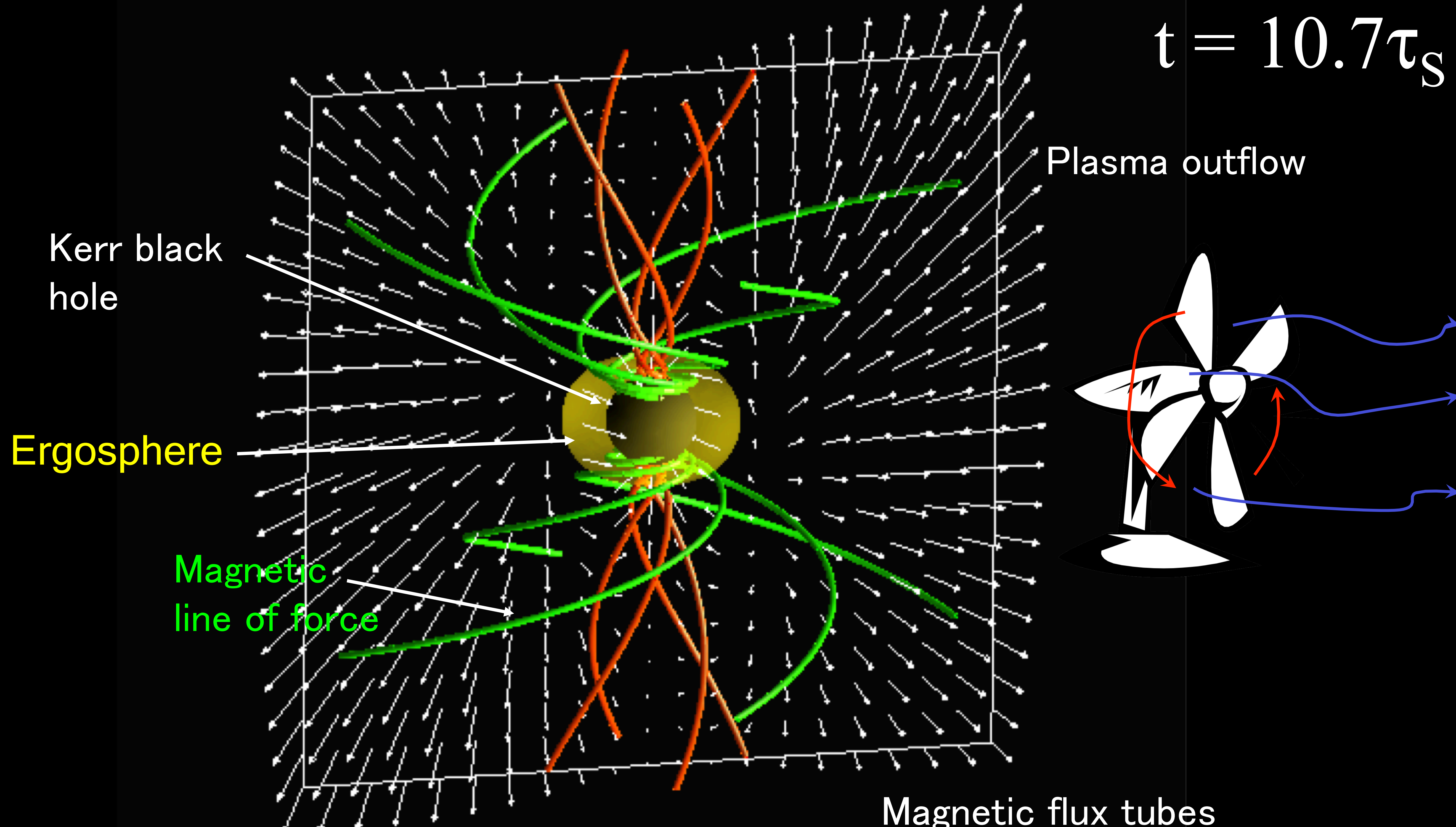
$a = 0.99995$

$t = 1000 r_S$

Komissarov & McKinney 2009

No outflow(?)

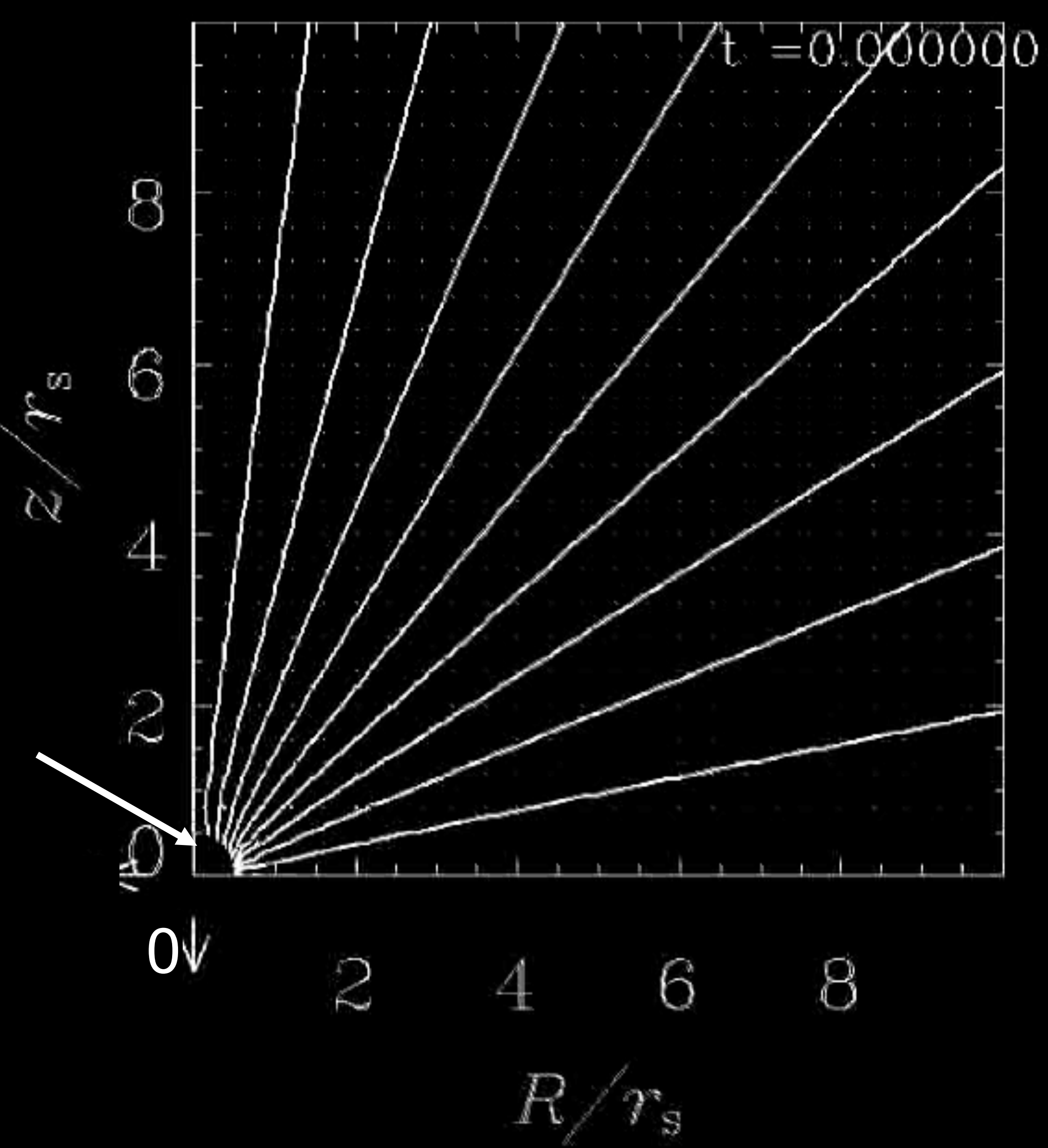
Plasma outflow by magnetic field from black hole ergosphere



(Koide 2004)

Magnetic flux tubes accelerate plasma just like a propeller screw!

Plasma outflow by magnetic field from black hole ergosphere



Kerr black hole

0

V_{max}=0.86c
(Lorentz factor 2.0)

$r_S=2GM_{BH}/c^2$

$\tau_S=r_S/c$ (Unit of time)

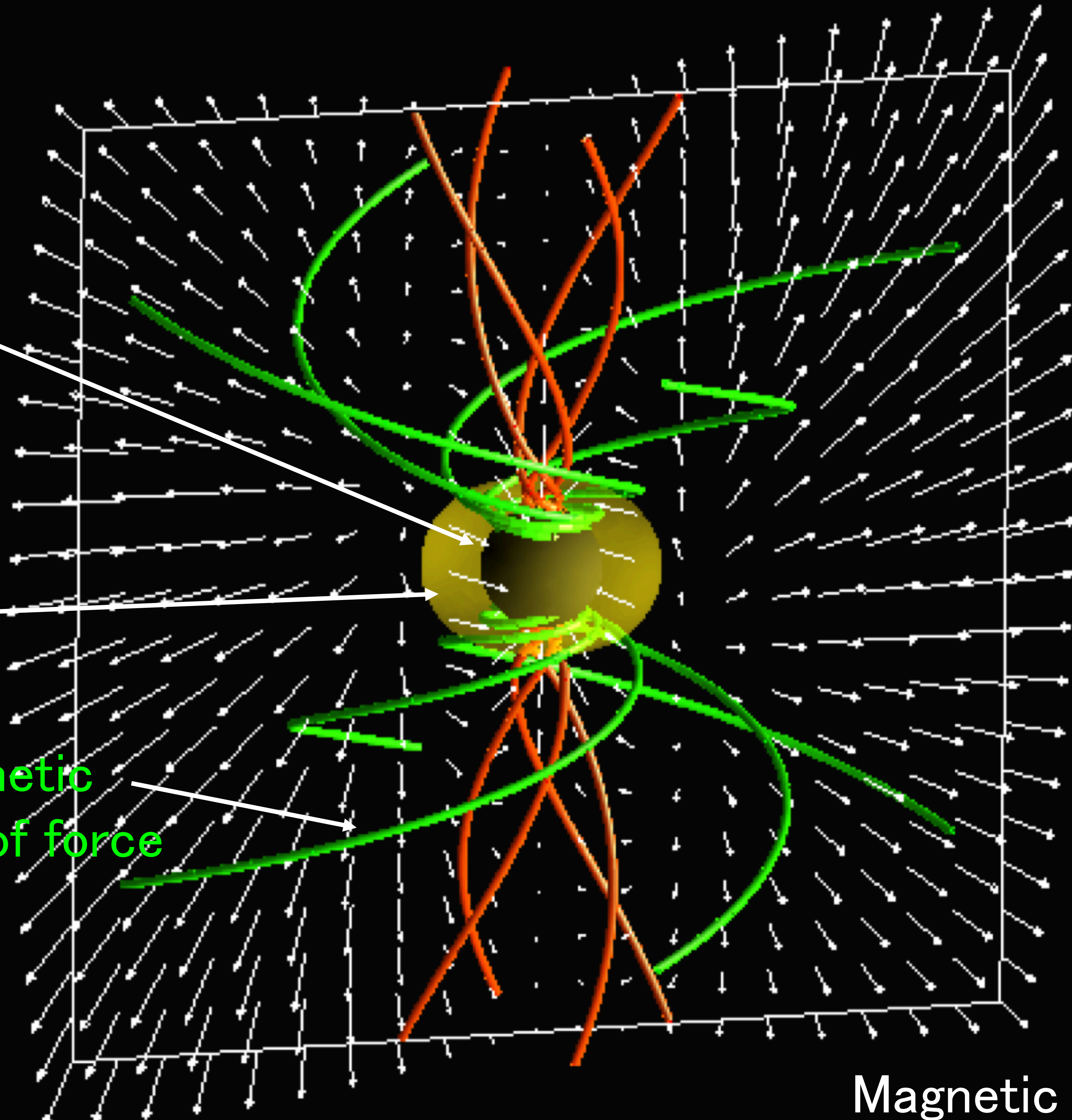
Lines: magnetic line of force

arrow: plasma velovity

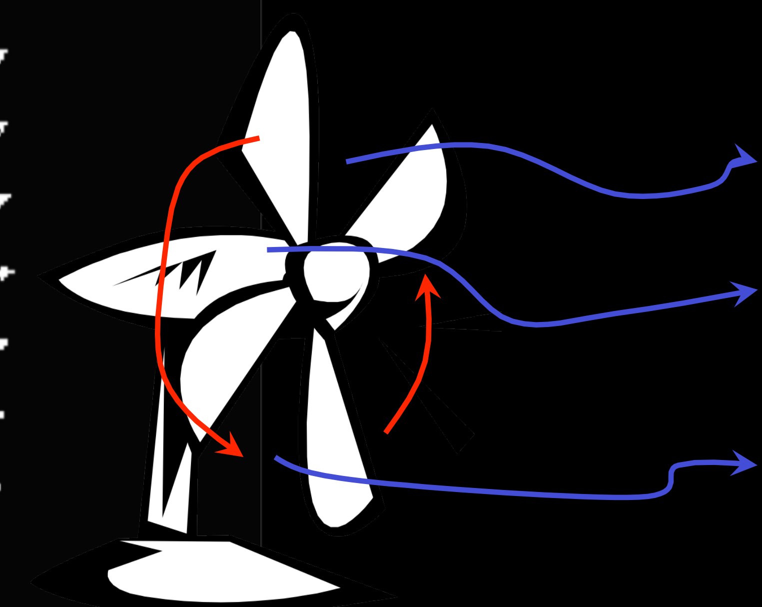
color: B_φ^2

Plasma outflow by magnetic field from black hole ergosphere

$t = 10.7\tau_S$



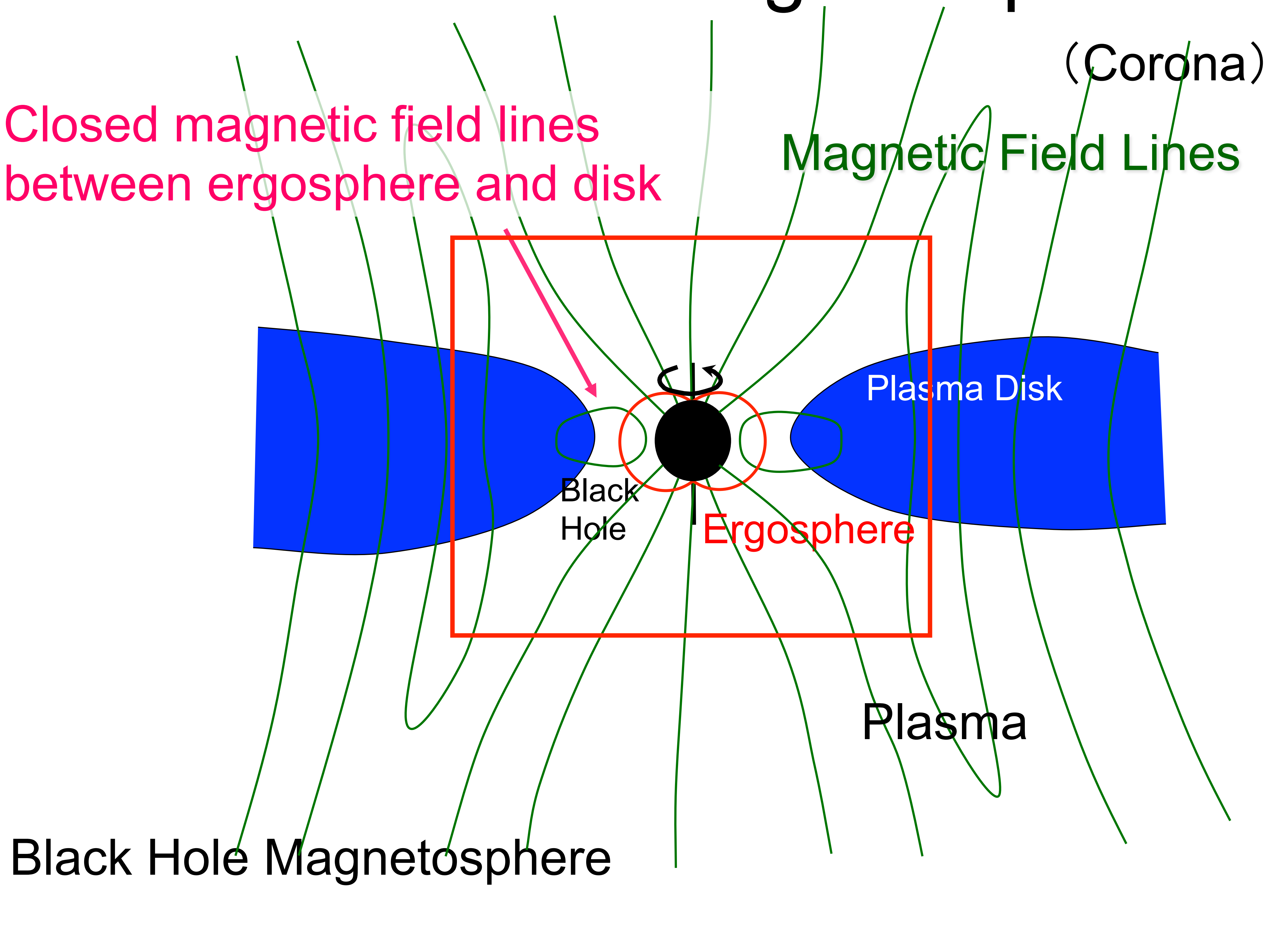
Plasma outflow



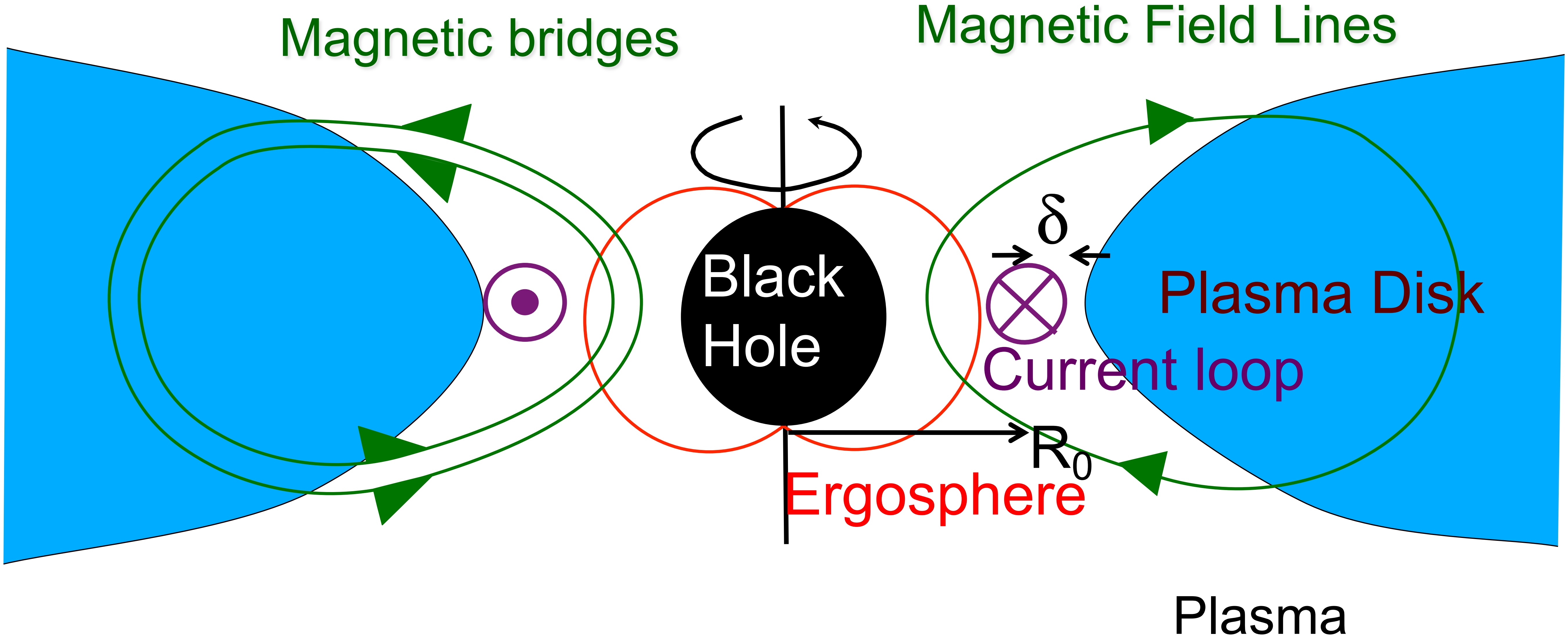
(Koide 2004)

Magnetic flux tubes accelerate plasma just like a propeller screw!

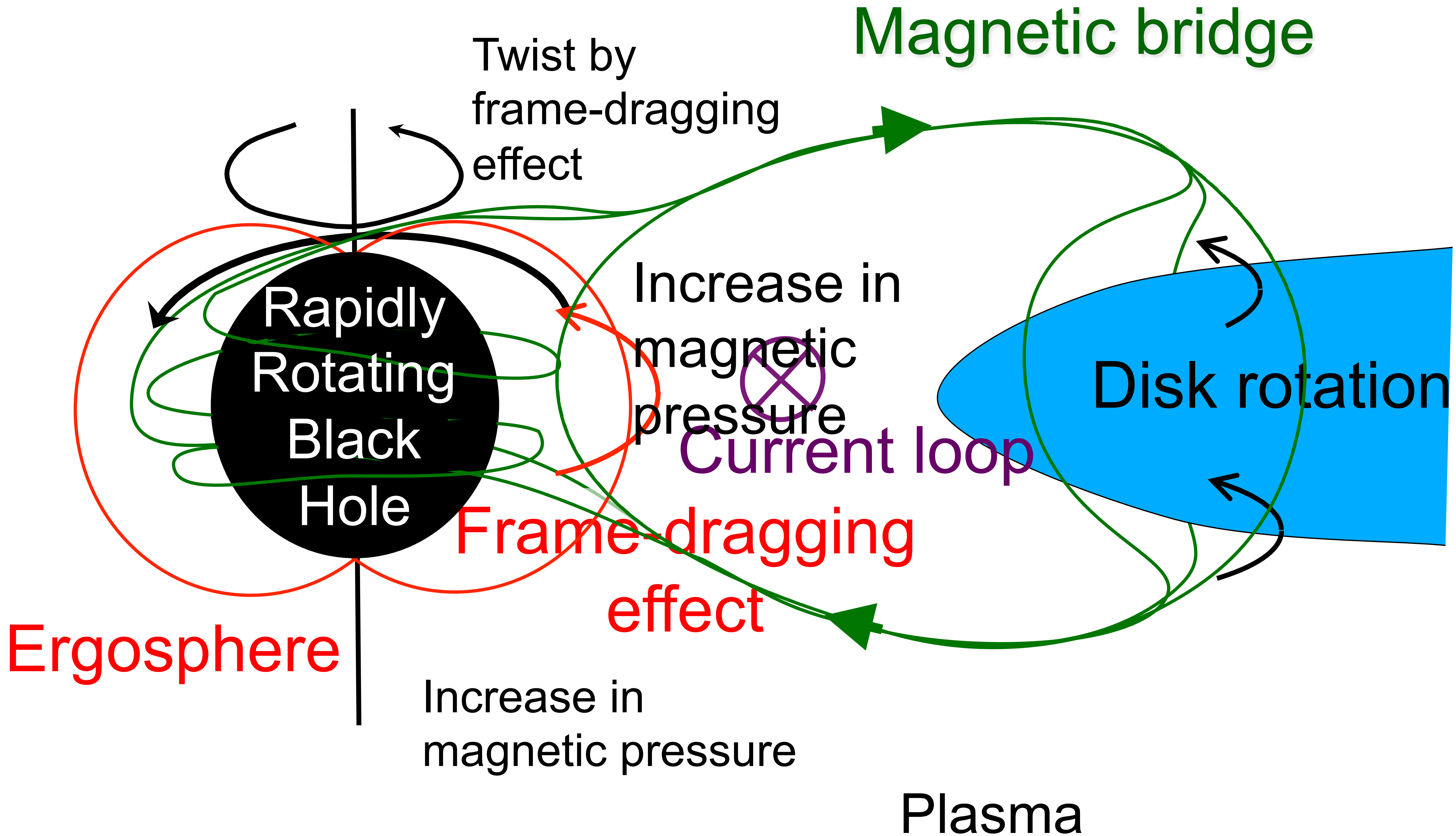
More realistic magnetic configuration of black Hole Magnetosphere



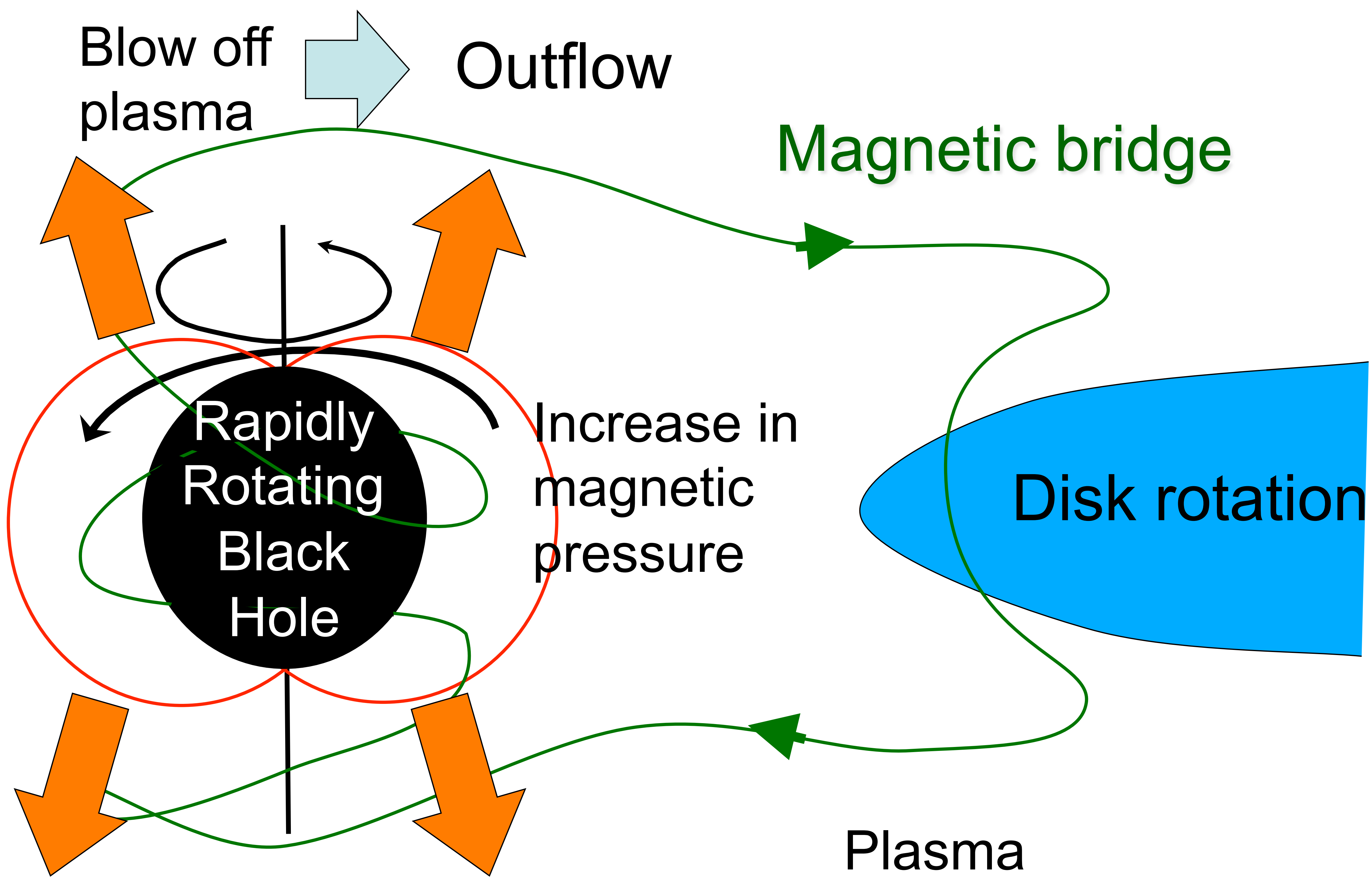
Magnetic Field induced by Current Loop around Black Hole



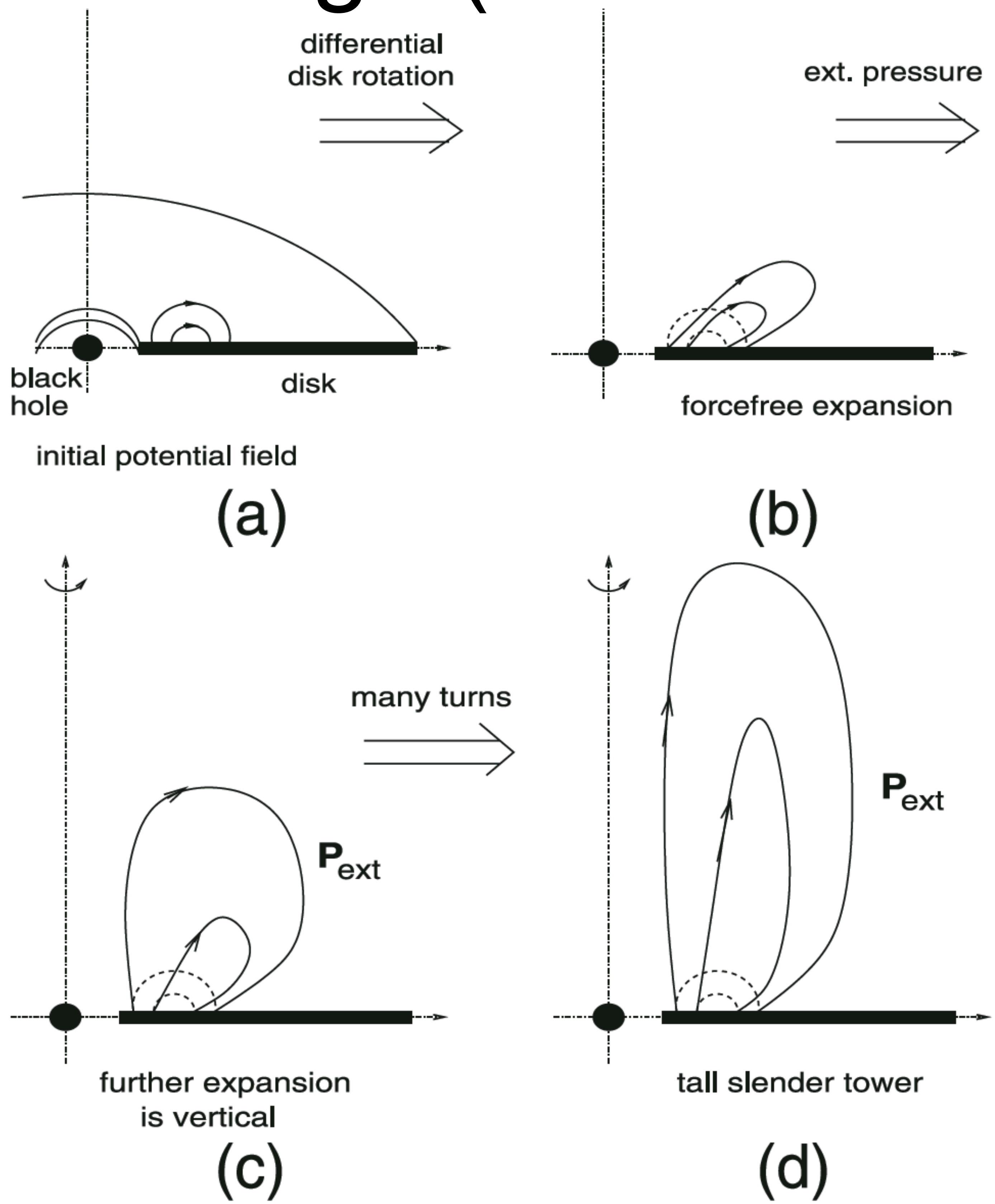
Twist of magnetic bridge by ergosphere



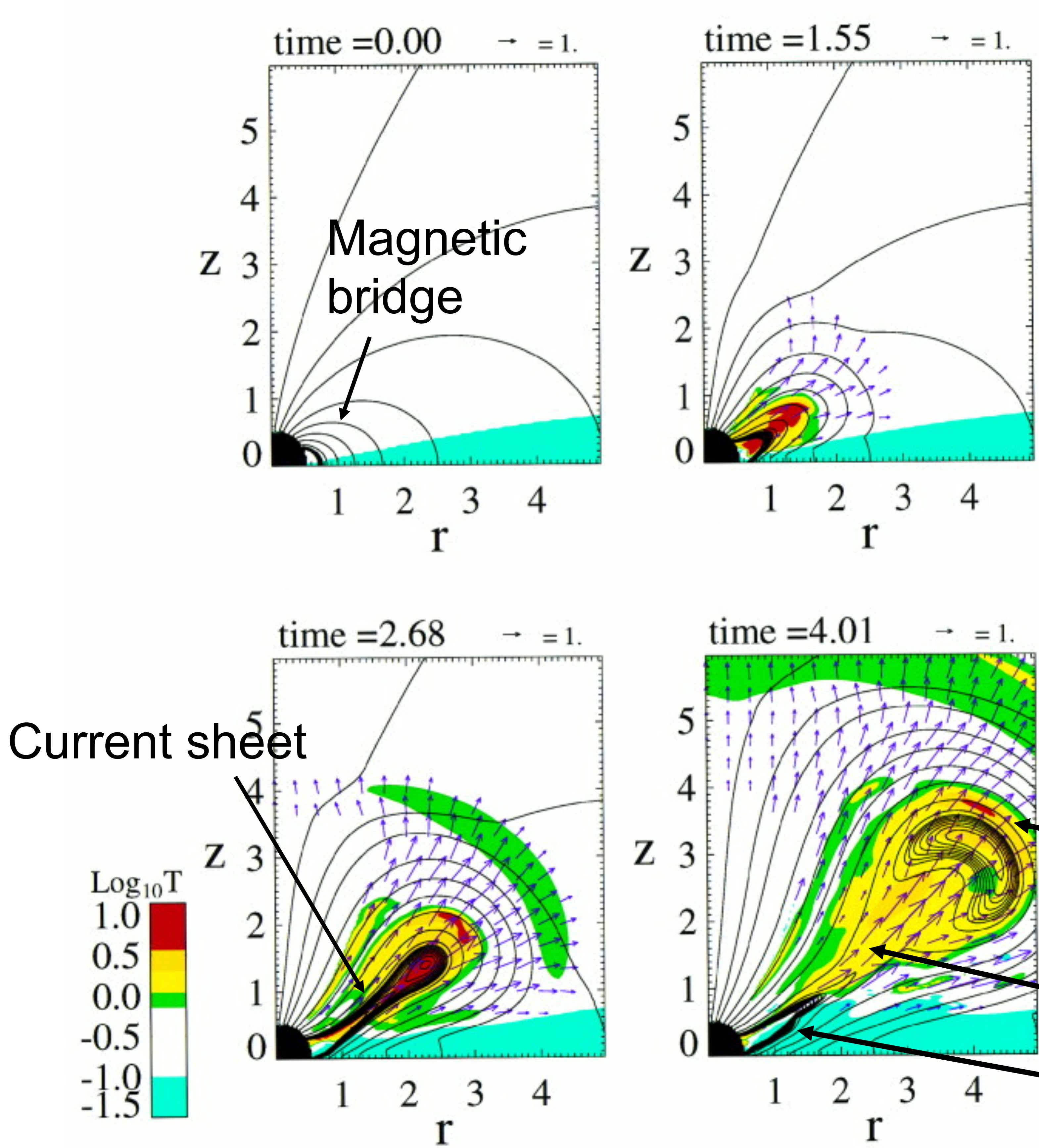
Twist of magnetic bridge by ergosphere



Magnetic tower produced from magnetic bridge (non-relativistic MHD)



Nonrelativistic MHD Simulation with Dipole-Magnetic Field and Disk



Hayashi, Shibata, and Matsumoto (1996)

Anomalous resistivity:

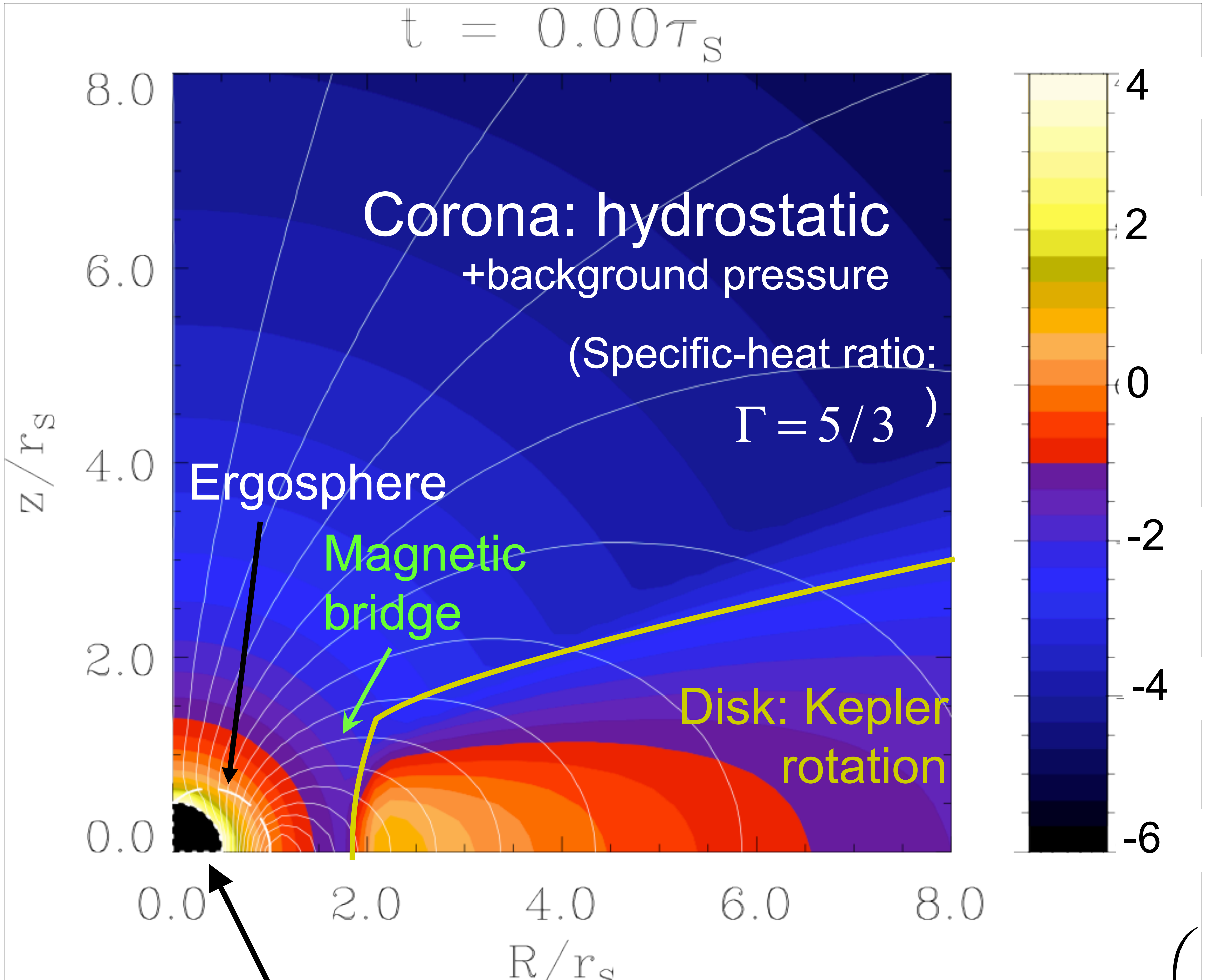
$$\sigma = 100 \quad (J/\tilde{n} > v_d)$$
$$\sigma = \infty \quad (J/\tilde{n} \le v_d)$$

Plasmoid: ejected blob

Magnetic reconnection

(loop will be heated up)

Initial condition of Ideal GRMHD simulation



Almost maximally rotating Black hole

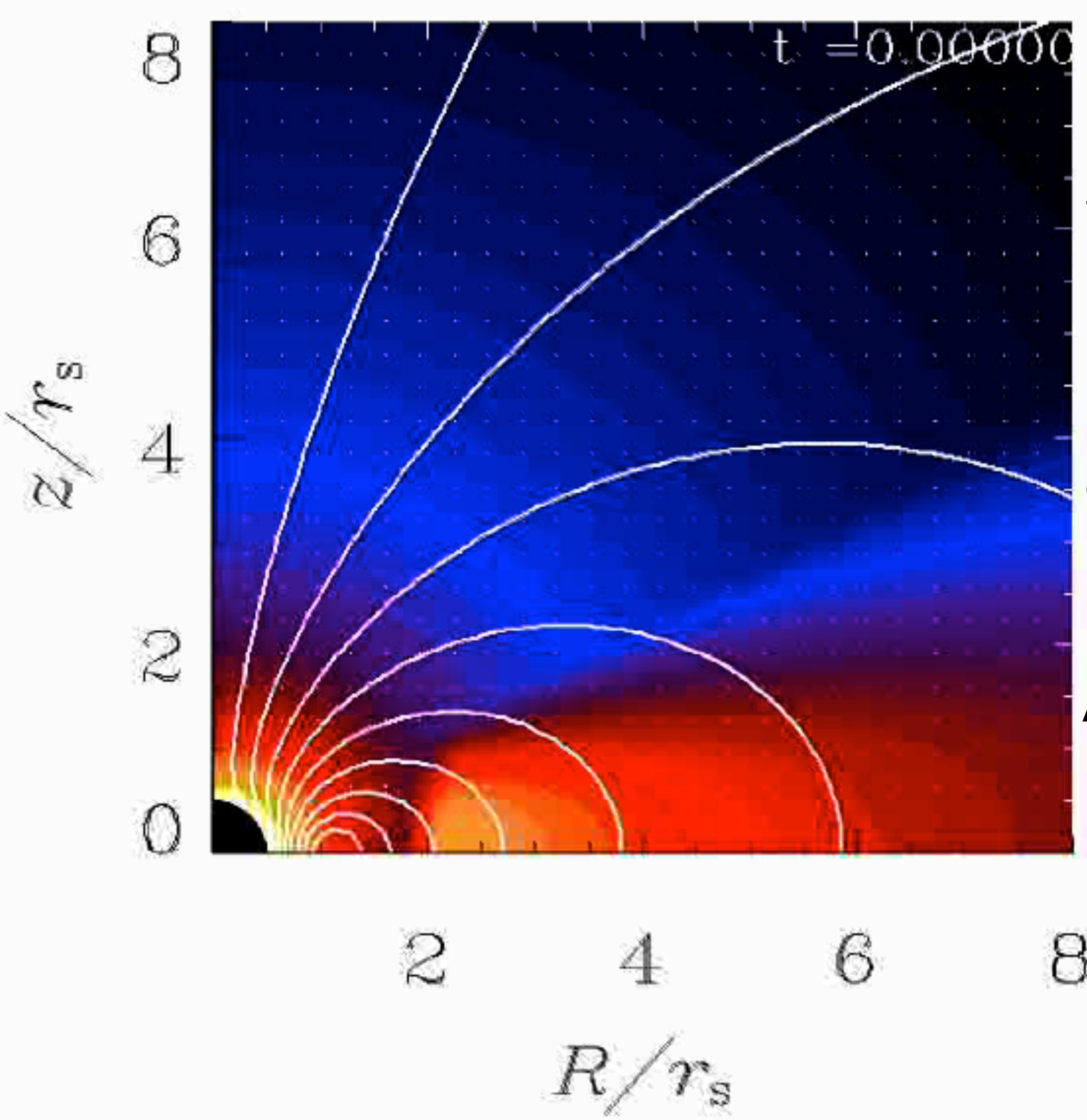
t =0

Solid white line:
Magnetic field line

Color: $\log \rho$

$$\left(a = \frac{J}{J_{\max}} = 0.99995 \right)$$

Time evolution: Mass density, magnetic configuration

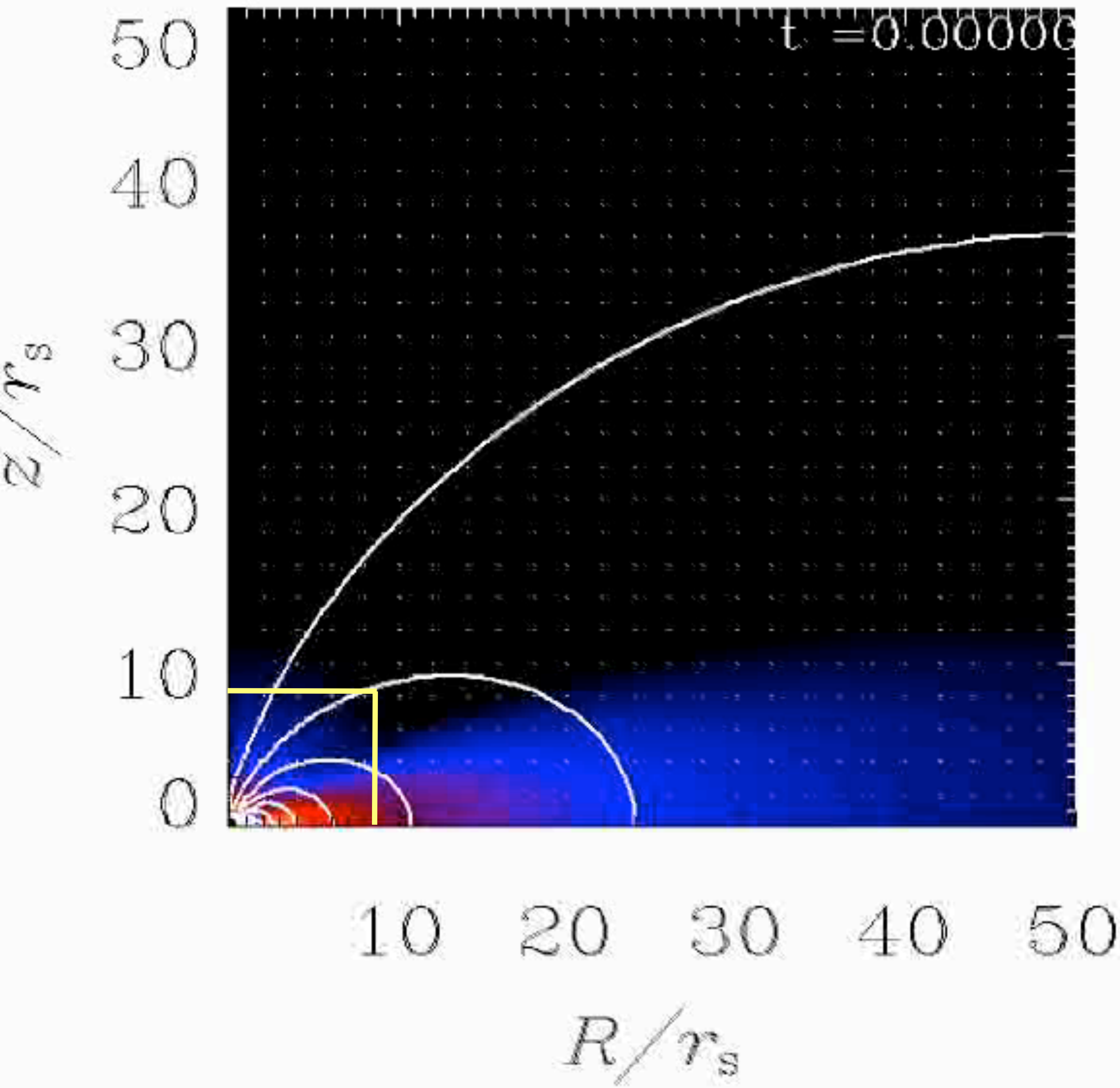


Solid white line:
Magnetic field surface

Color: $\log \rho$

Arrow: velocity

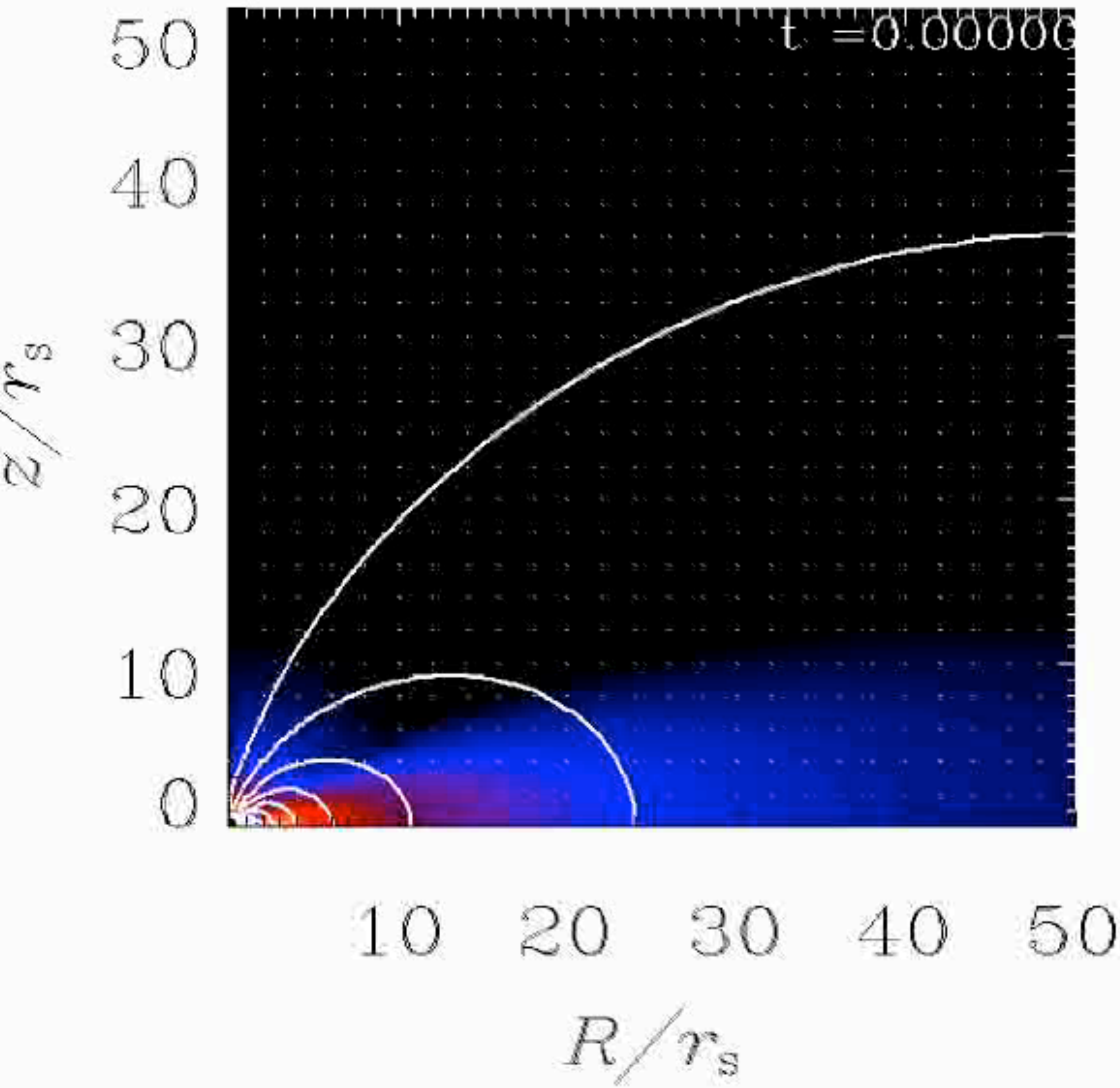
Physical Review D 74, 044005 (Aug., 2006)



Solid line:
Magnetic field surface

Color: $\log \rho$

Arrow: Velocity



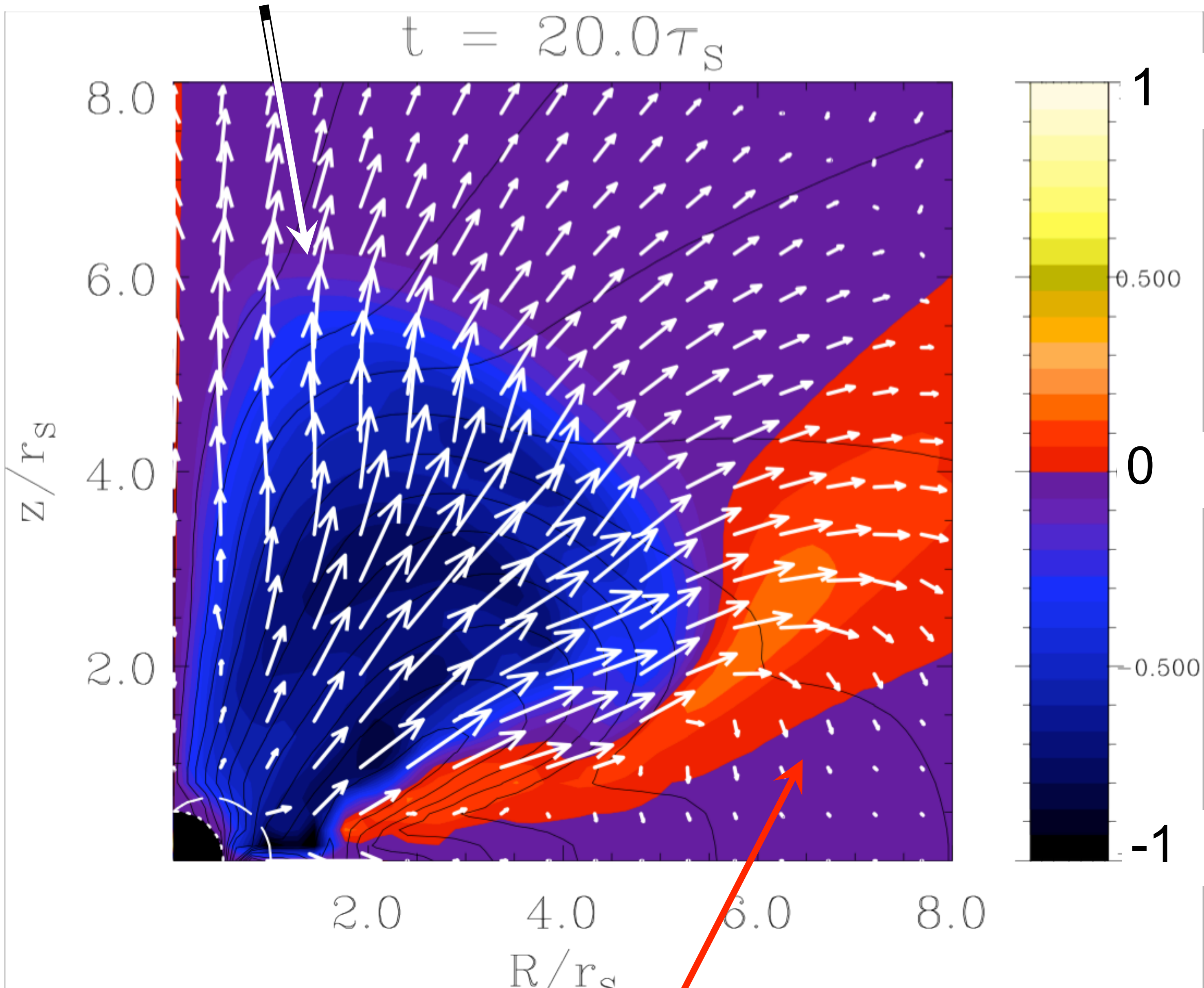
Solid line:
Magnetic field surface

Color: $\log \rho$

Arrow: Velocity

Azimuthal component of magnetic field, magnetic configuration clarifies outflow driven by frame-dragging effect!

Frame-dragging effect



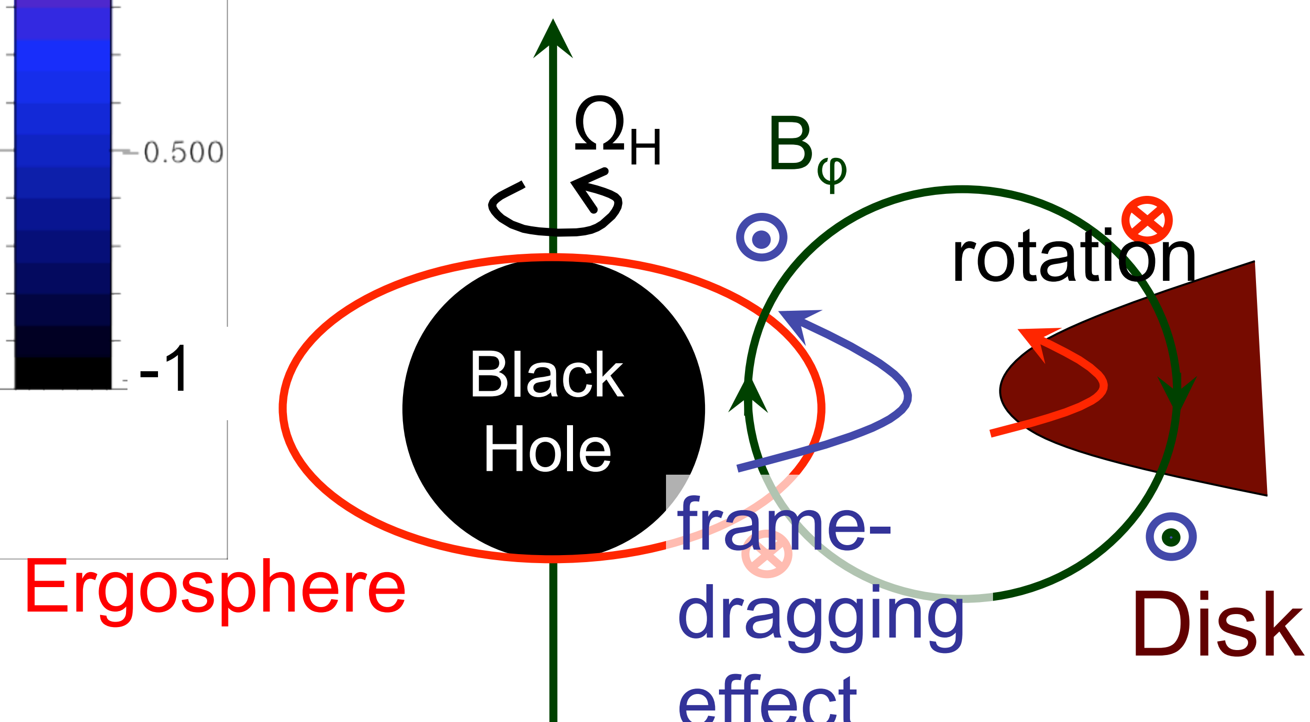
Disk rotation

$t = 20\tau_S$

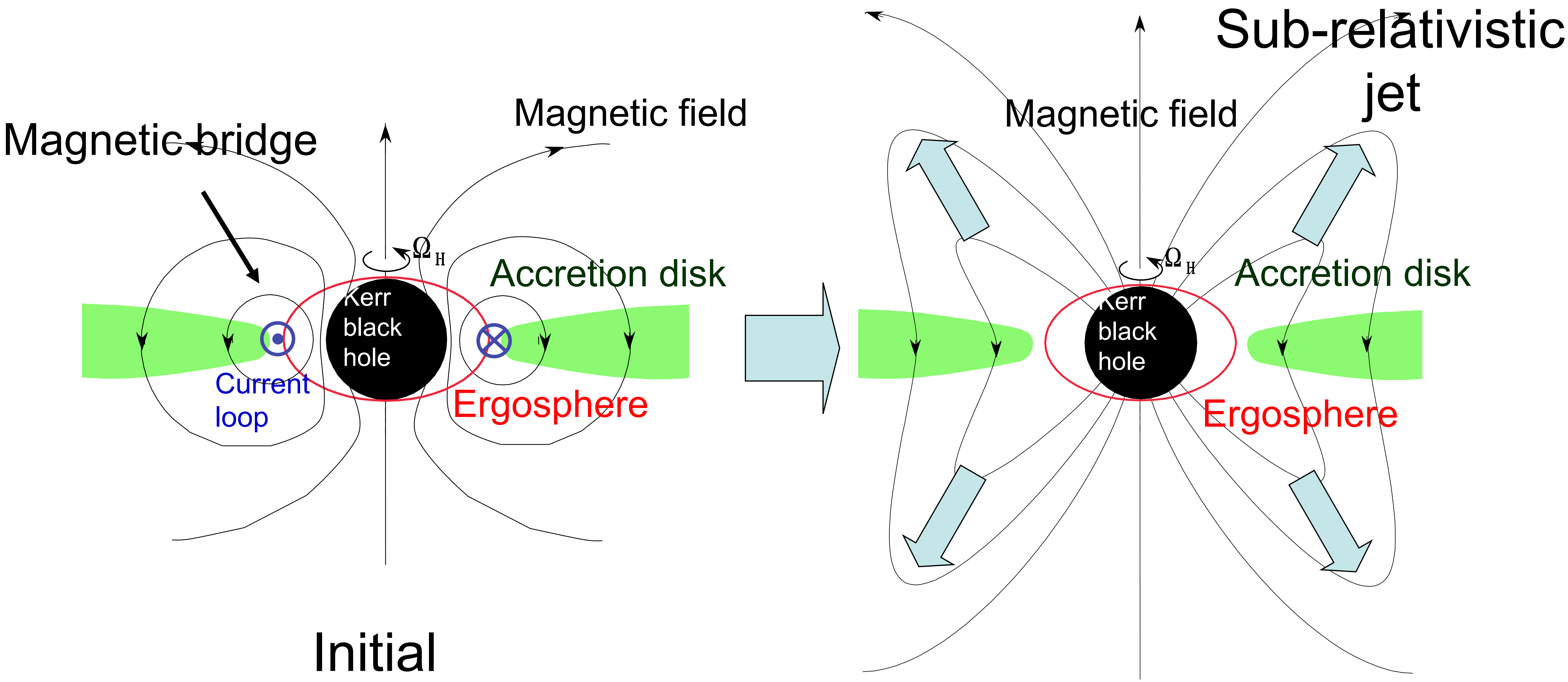
Solid line:
Magnetic field line

Color: $B_\phi / \rho^{1/2}$

Arrow: Velocity



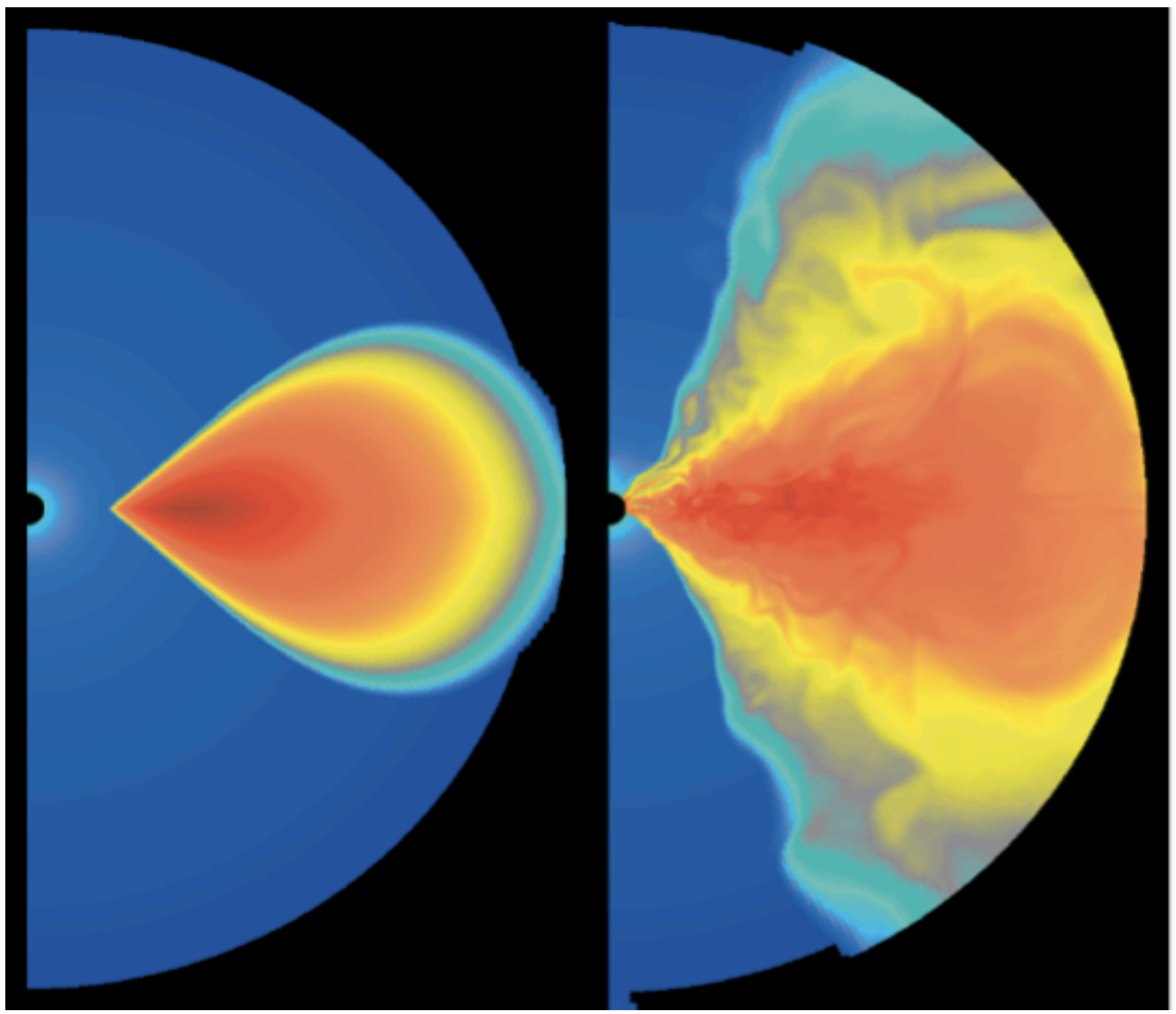
Schematic picture of phenomena caused by the magnetic bridge near the black hole



Longer term simulation of ideal GRMHD

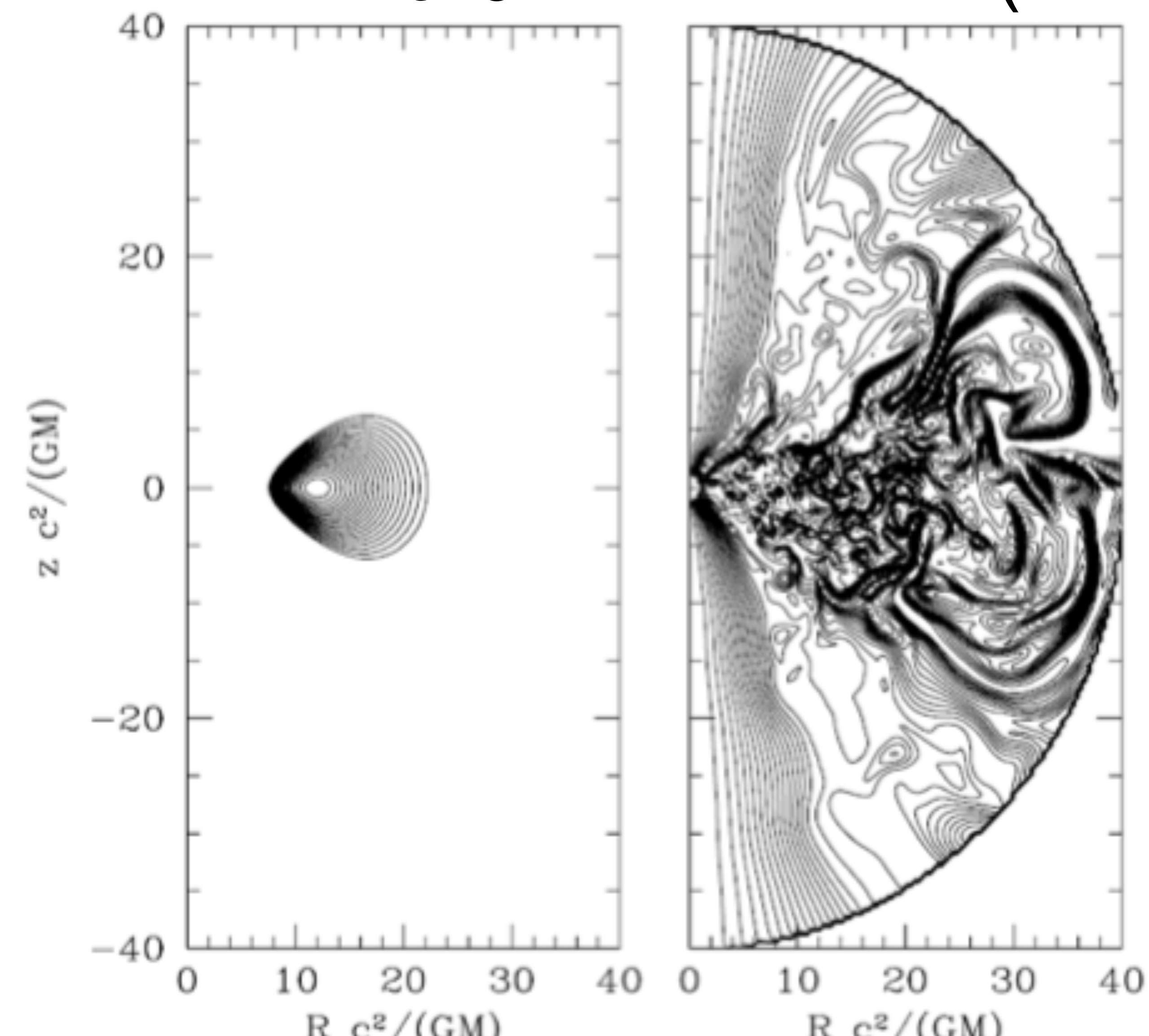
Log ρ

t=0 t=2000c/(GM)



Magnetic surfaces

t=0 t=2000c/(GM)



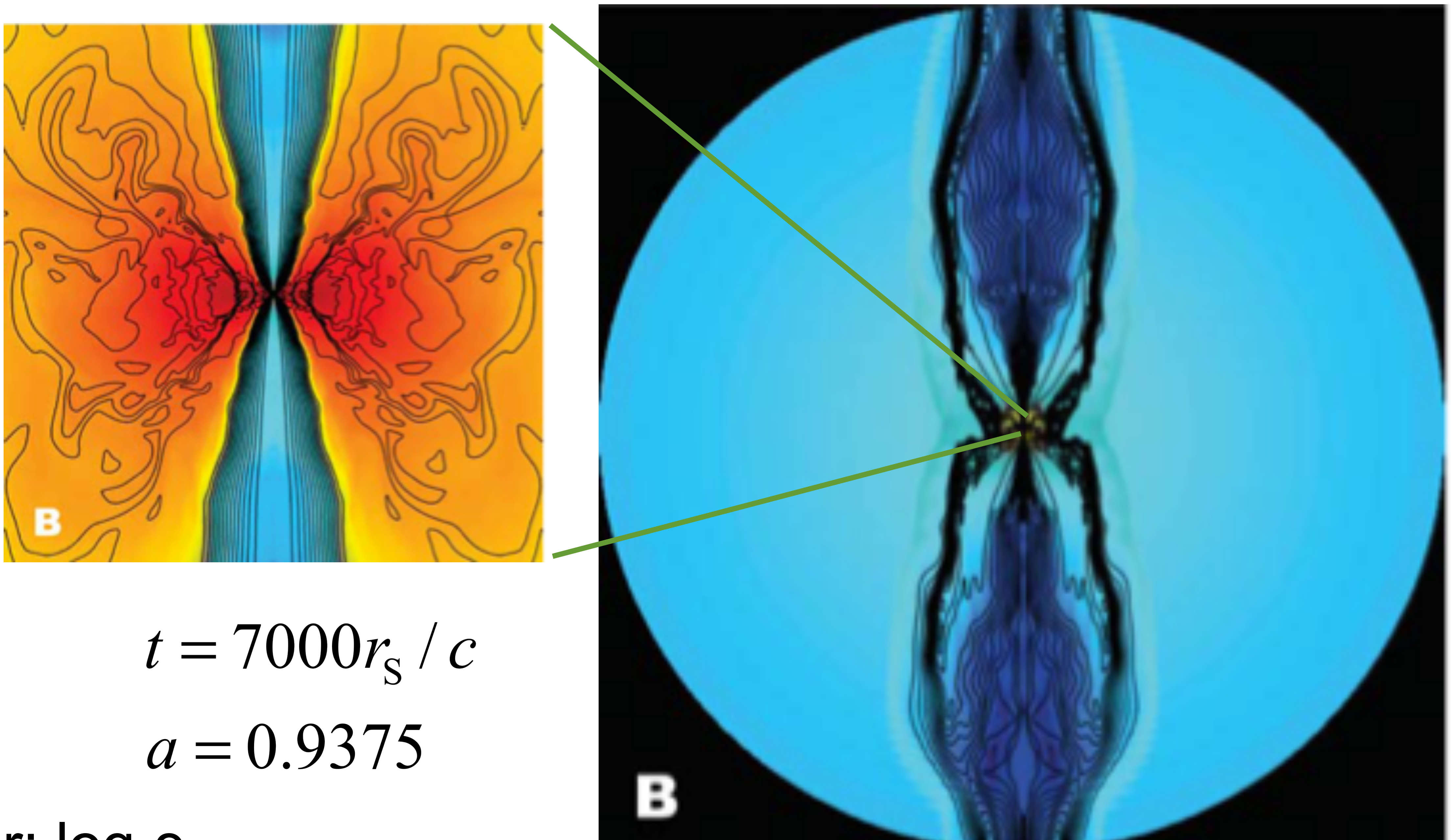
Artificial formation of magnetic islands

McKinney & Gammie 2004

Formation of Relativistic Jet
Ideal GRMHD longer-term simulation

Lorentz factor：$\Gamma \sim 5$ (v_{jet}=0.98c)

$100 r_S$



$10^4 r_S$

$t = 7000 r_S / c$

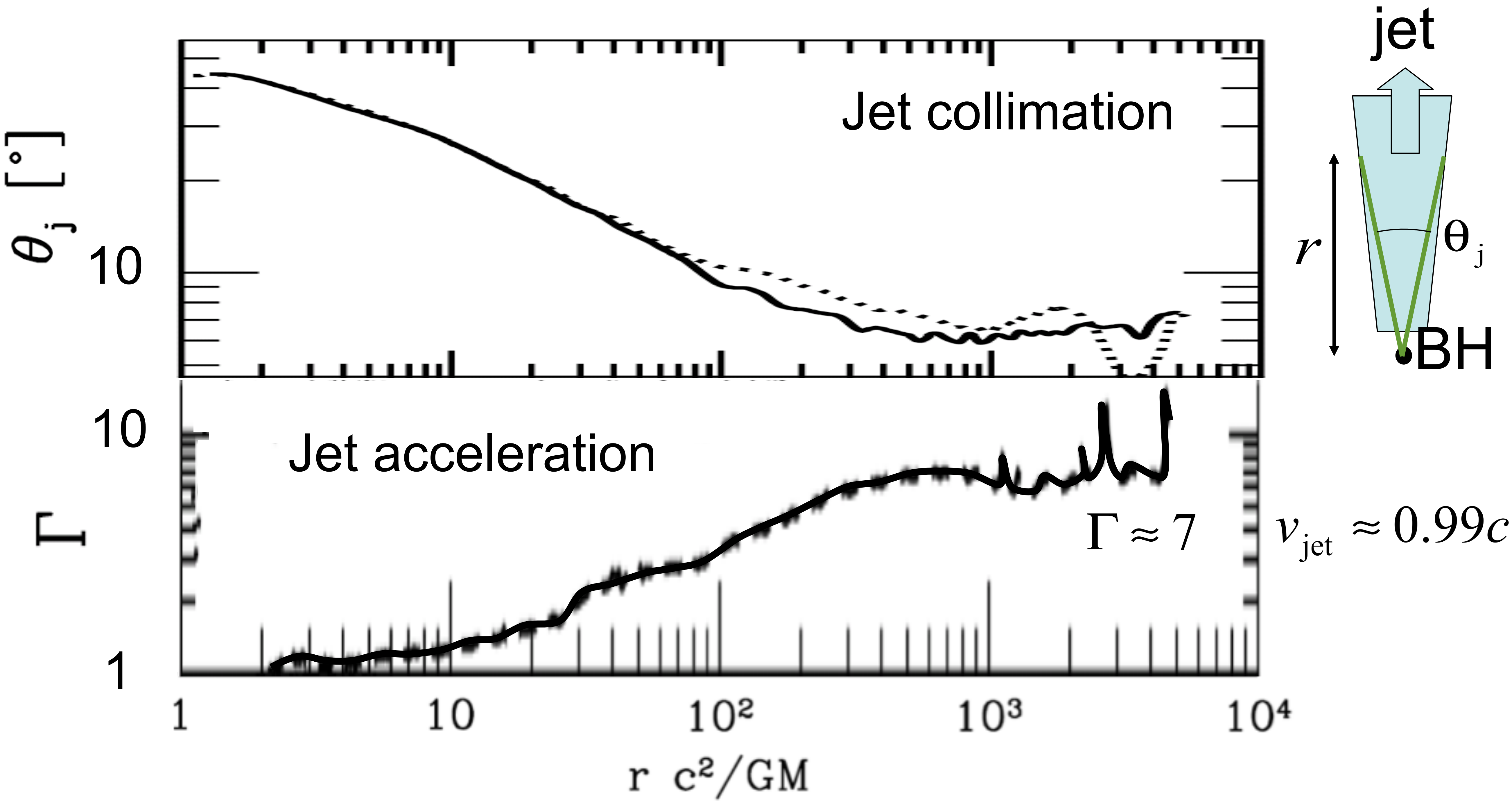
$a = 0.9375$

Color: log ρ

Lines: magnetic surfaces

- J. C. McKinney 2006, MNRAS

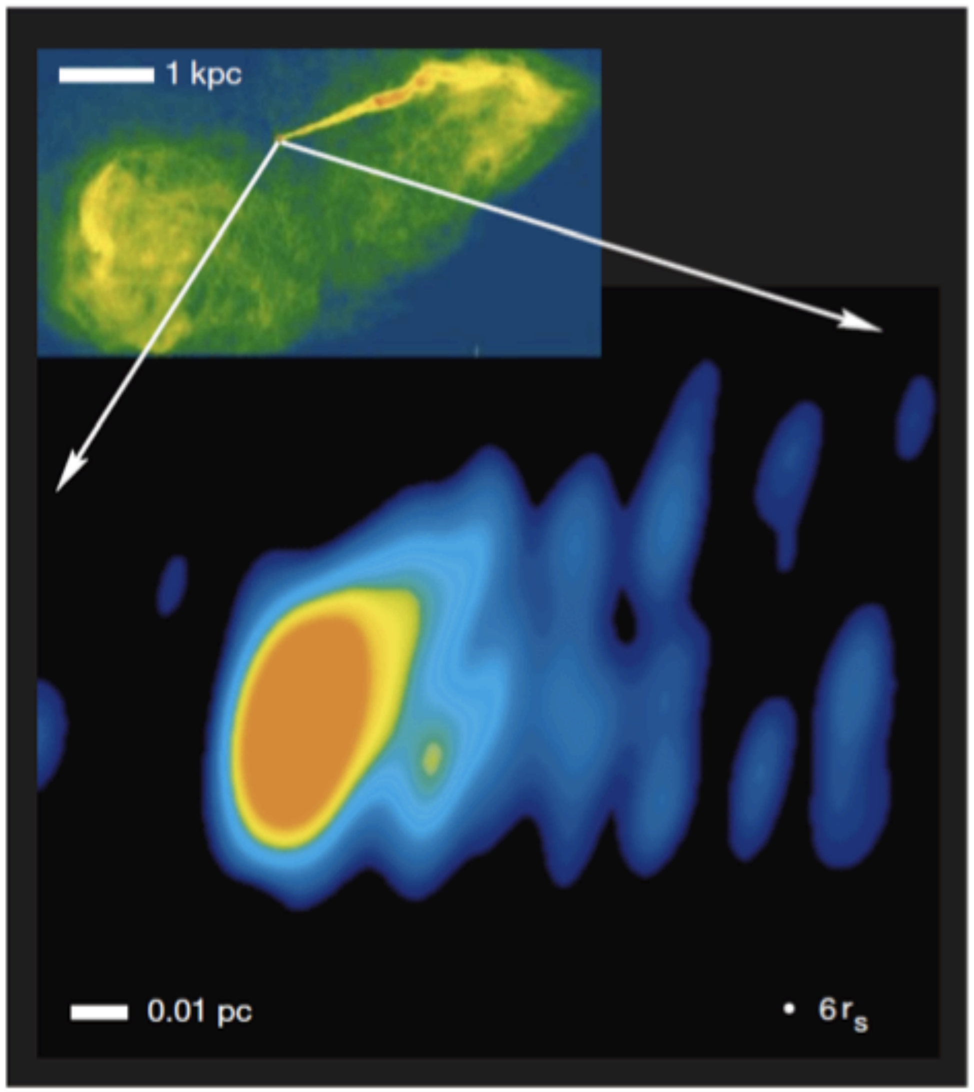
Longitudinal structure of relativistic jet



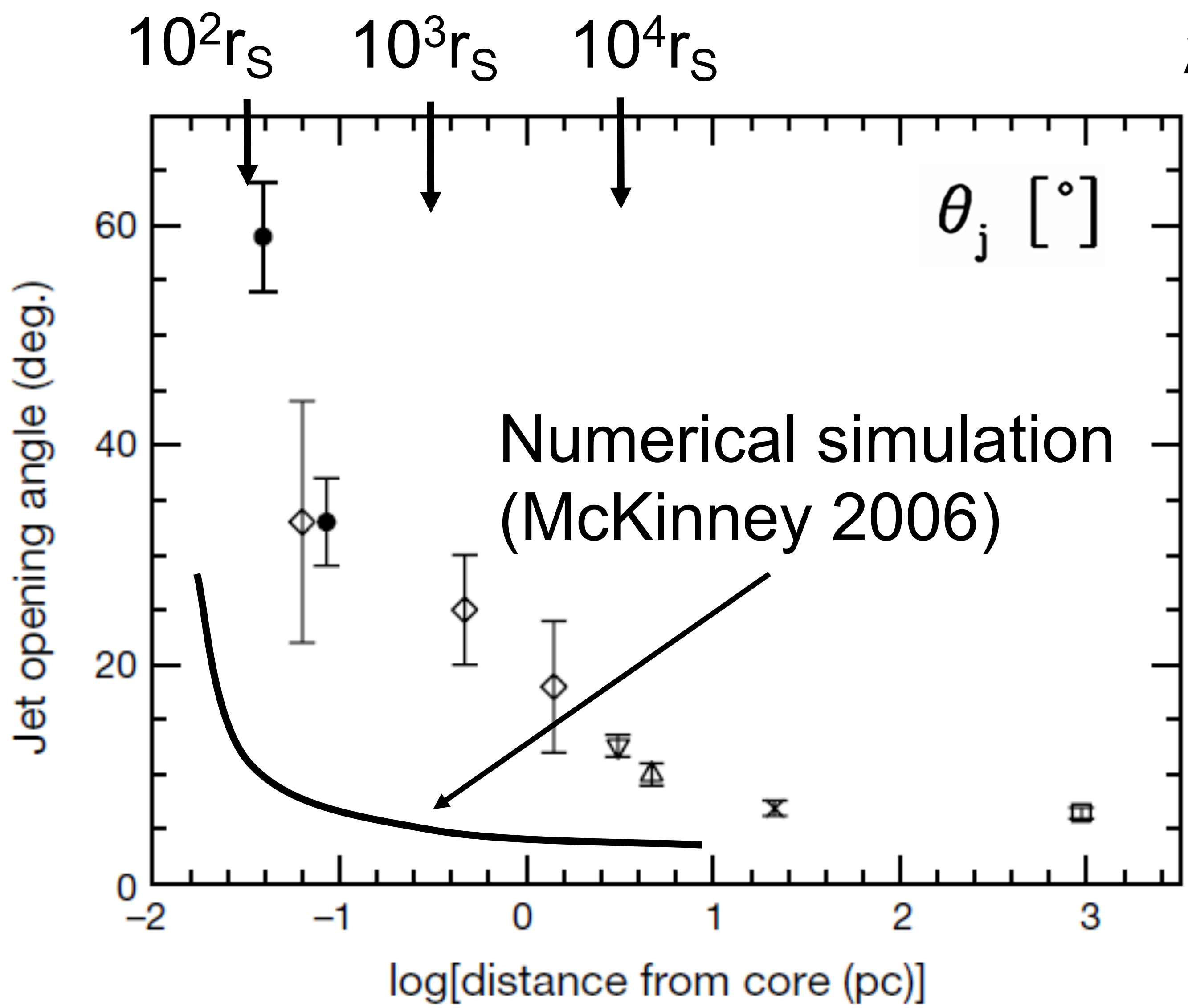
- J. C. McKinney 2006, MNRAS

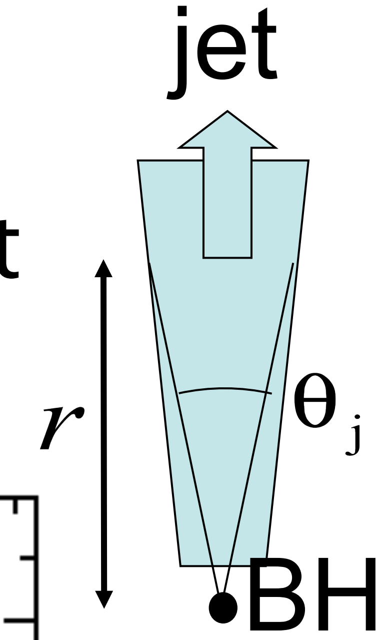
Comparison between numerical result and observation of jet from AGN of M87

Radio Image (VLBI)



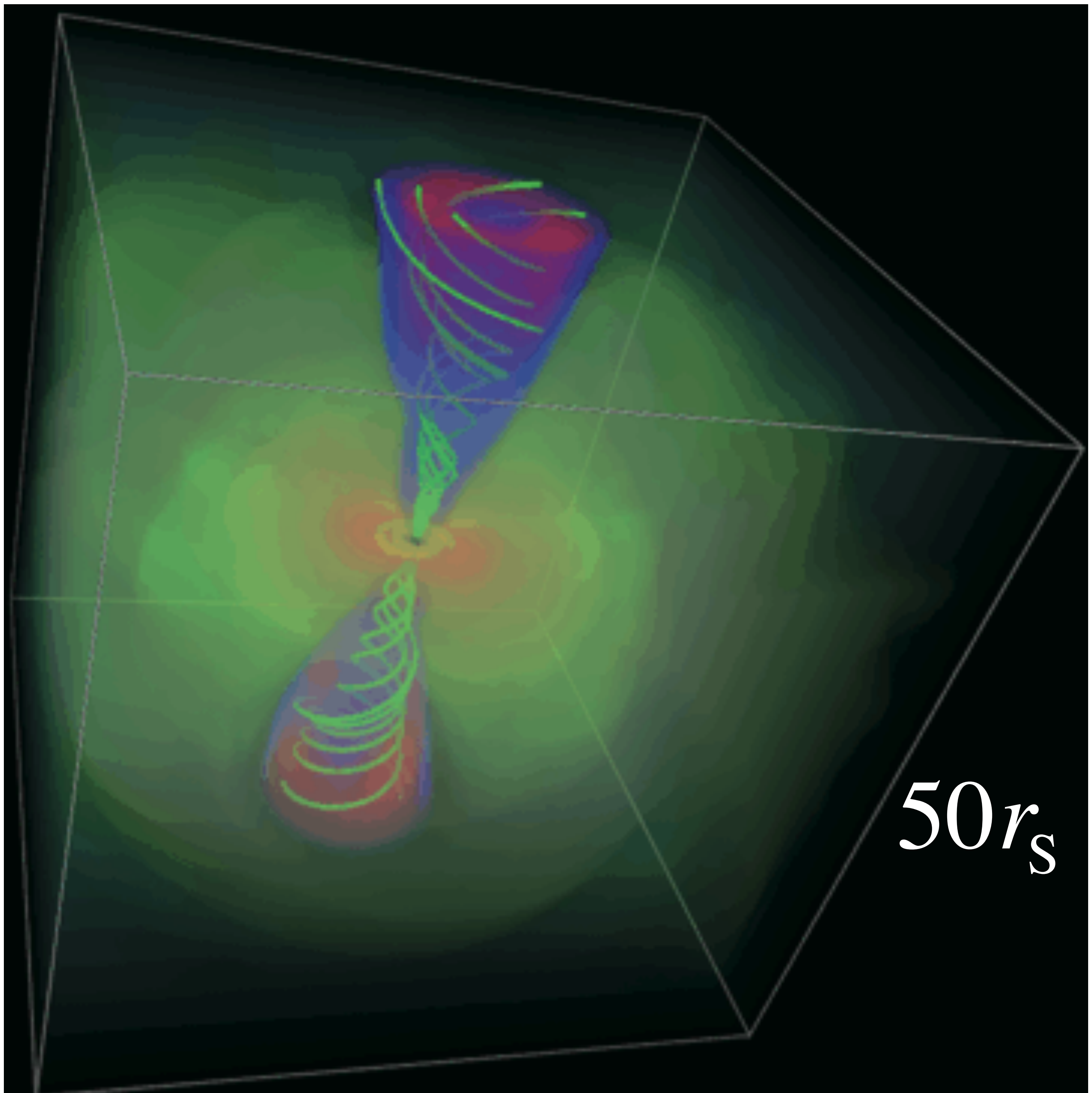
Collimation of relativistic jet





W. Junor, J. A. Biretta, and M. Livio 1999, Nature

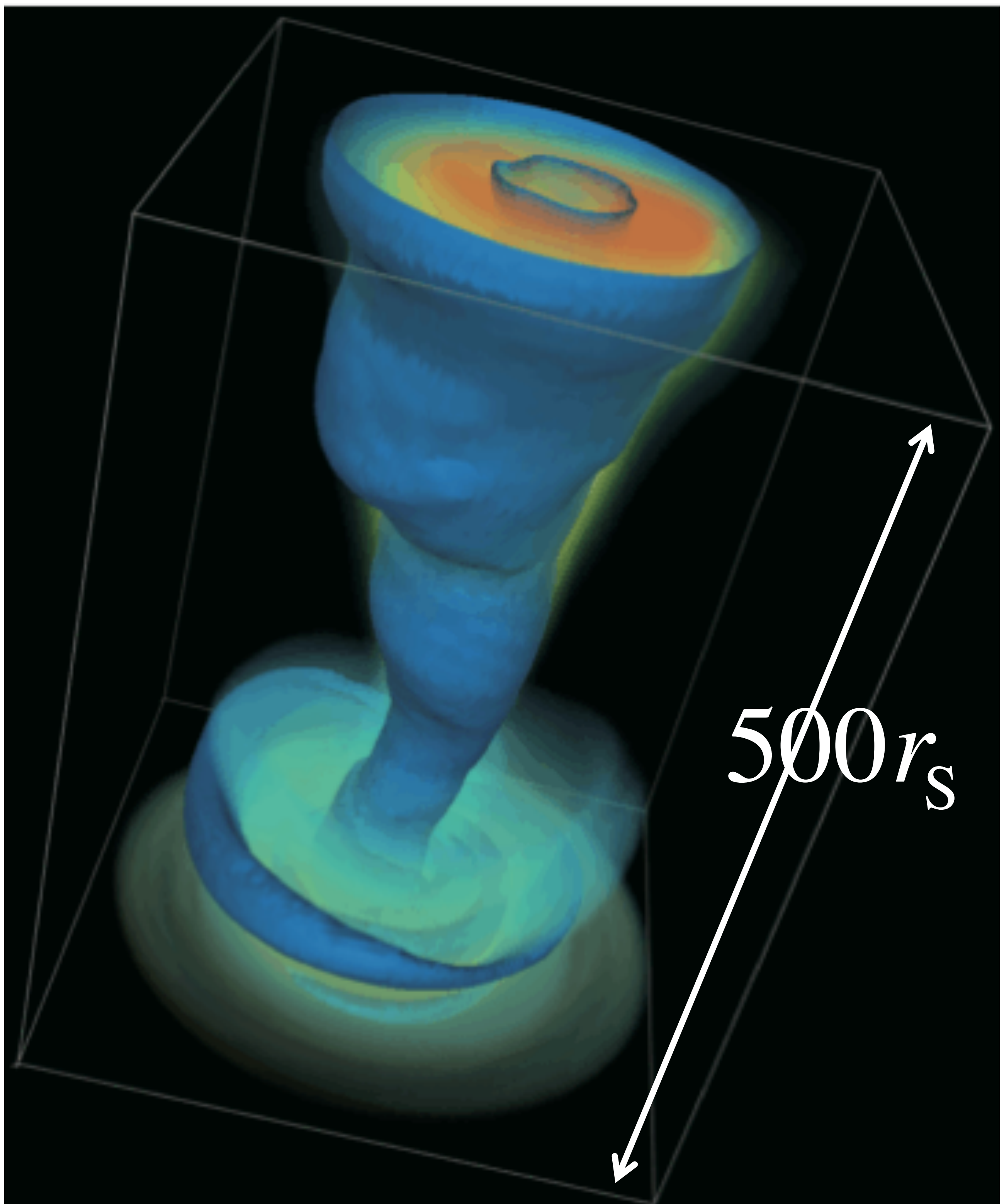
Recent result of GRMHD simulations McKinney & Blandford (2009)



$$t = 2000\tau_S$$

Jet is stable against kink (current-driven) instability, in spite of strong twist of magnetic filed lines.

Recent result of GRMHD simulations McKinney & Blandford (2009)



3D very long term GRMHD simulation

$$t = 2000\tau_S$$

Almost the same initial condition of McKinney (2006) except for 3D

Relativistic jet propagates stably against Kelvin-Helmholtz instability and kink (current-driven) instability.

ここで話はストップ．3月5日（火）