Detecting Black Holes by Waves

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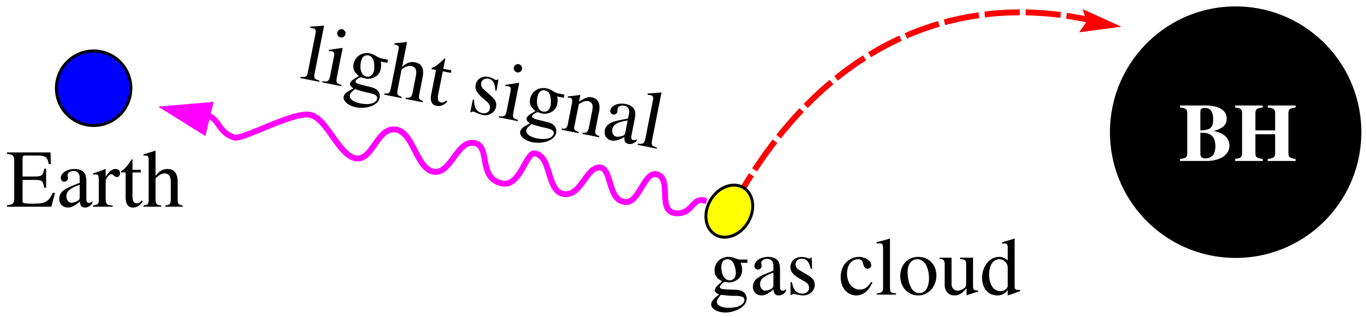


Conference on BH Magnetosphere, 2013.3.4–7 at 清華大學, 新竹市, 台湾

1. Intro. : I want to see the black hole.

- What is the meaning of "seeing BH"

  { To verify the existence of BH horizon by detecting Gen.Rel. effects of BH
  { To measure the mass and angular momentum (and charge)

- Suggestions in this talk:

  ◇ **Observable verifying the existence of BH horizon**

  ◇ **Observable giving a clear relation of the mass and angular momentum of BH horizon**

  ◇ **2nd best approach other than the above ones.**

  ⋯ these are not the BH shadow.

Note: Our observable is measurable in any wave propagating on black hole spacetime. (grav. wave, EM wave, etc...)

• **Comment :** From the point of view of Physics of Strong Gravity

Validity of General Relativity for <u>strong</u> gravitating cases
has not been checked experimentally.

→ Many candidates of theory for strong gravitation
other than Gen.Rel. (alternative theories) have been suggested.

→ Some detail property of BH horizon may reflect
a specific property of each alternative theory.

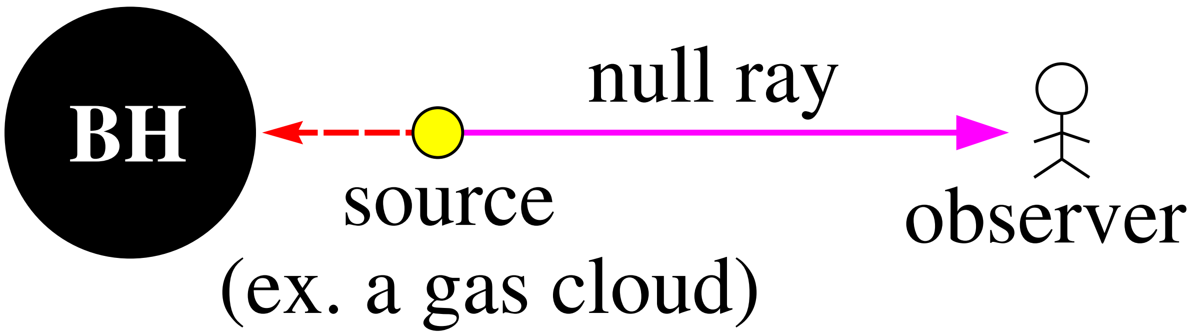
→ The direct observaion of effects of strong gravity (curved spacetime)
is important for physics of strong gravity.

This talk assumes only the Kerr BH,
and any alternative theory which predicts Kerr BH.

2. Set up a situation : a simple example

- Consider the radially alinged situation

$$\begin{cases} \text{Black hole} & : \text{Schwarzschild} \rightarrow \text{radius } R_{\rm BH} \\ \text{Source} & : \text{radial free fall (timelike geodesic)} \\ \text{Wave} & : \text{propagate on a radial orbit (null geodesic)} \\ \text{Observer} & : \text{rest at a distant point} \end{cases}$$

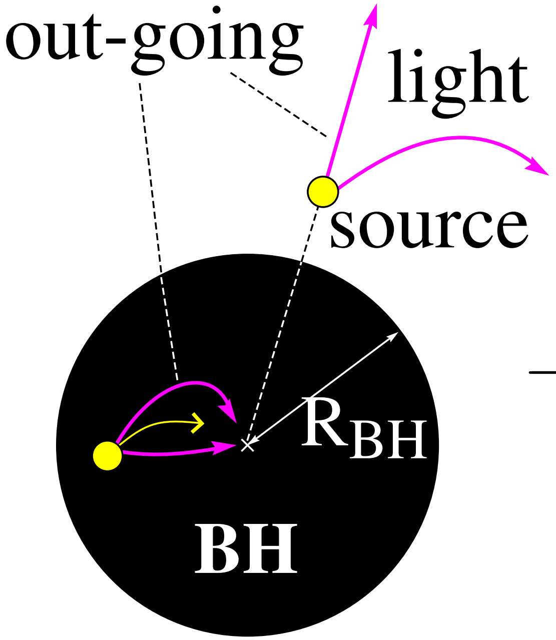


◇ I hope that a similar situation will arise by the gas cloud G2 in 2013.

3. The observable verifying the existence of BH horizon

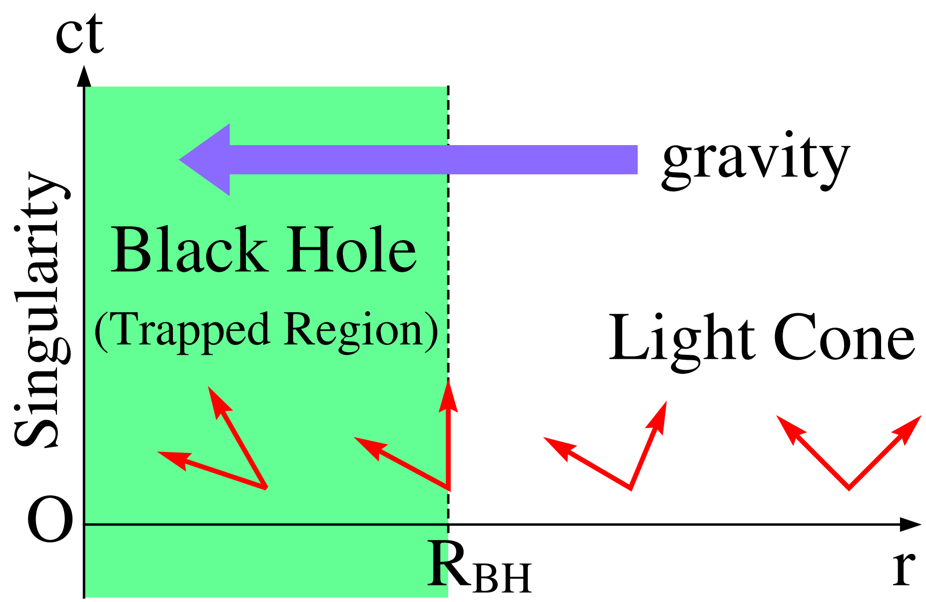
- **Physical feature of BH horizon**

[Theoretical Facts] :
- ◇ **Light cones describe the causal structure of spacetime.** (Any matter evolves inside the cone.)
- ◇ **Gravity controls the direction and opening angle of light cones.** ($\because$ Equivalence principle)



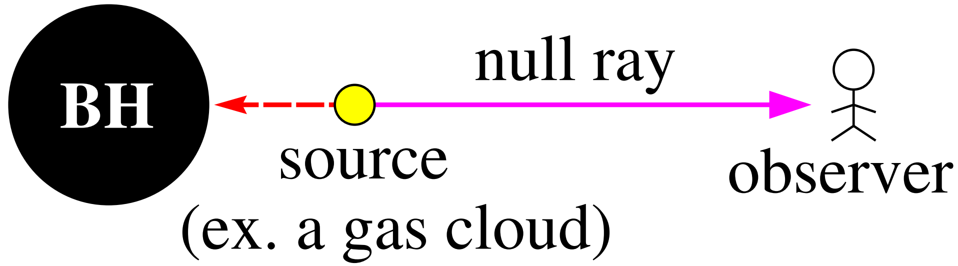
time–radial graph

(schematic diagram)



Any matter does necessarily fall to the center in BH region.

- **How to verify the existence of BH horizon**



◇ A source is falling radially into BH

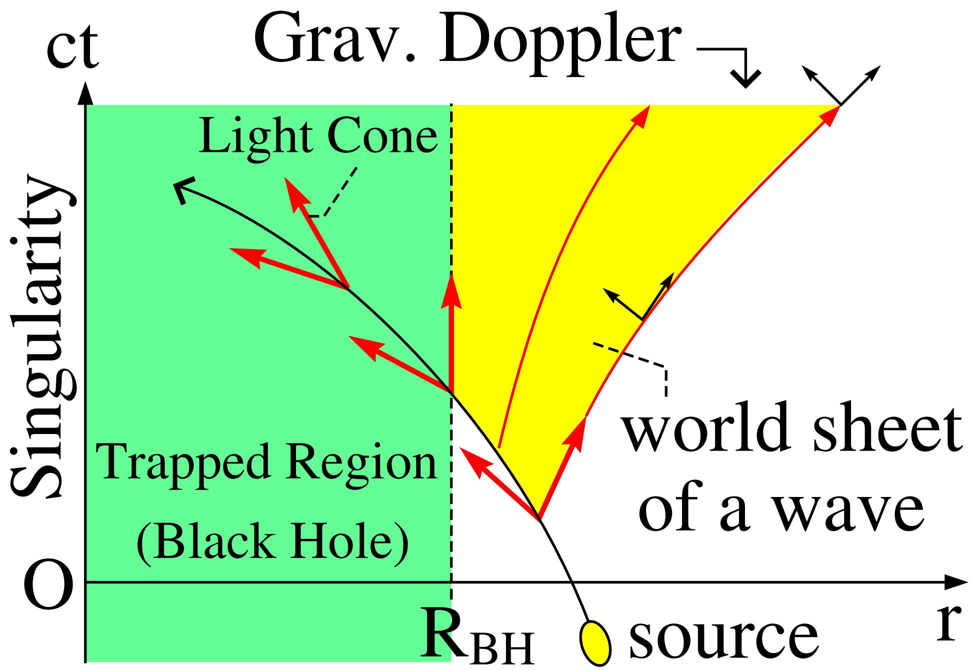
→ The wave length of wave emitted by this source is prolonged infinitely by the Grav. Doppler (redshift) due to BH horizon.

→ The infinite prolongation of wave length can verify the existence of BH horizon.

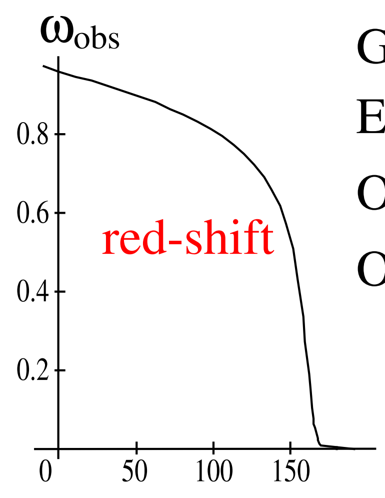
⇓

The freezing of wave should be observed.

(Numerical example is shown next)



◇ Ex. Freezing oscillation of observed wave (↓ "Gas's" = "Source's" ↓)



Gas's initial place $r_0 = 20\,R_{BH}$

Emission Freq. $\omega_{emit} = 1$

Observed red-shift ω_{obs}

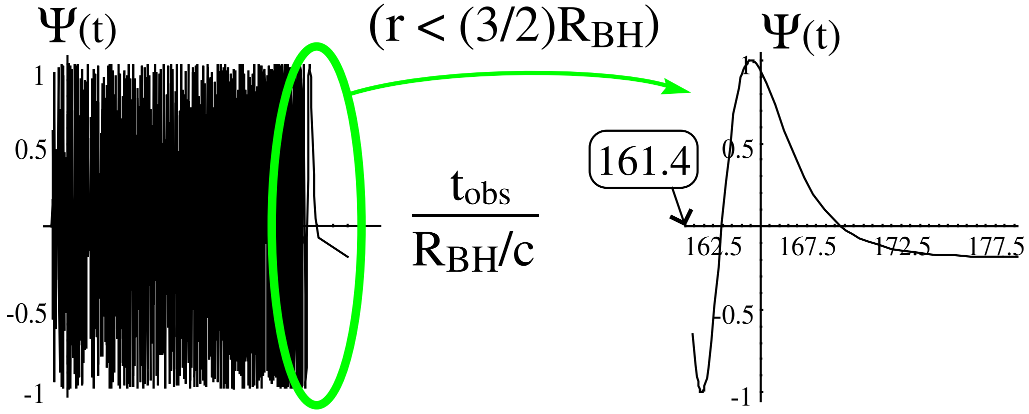
Observed wave (model)

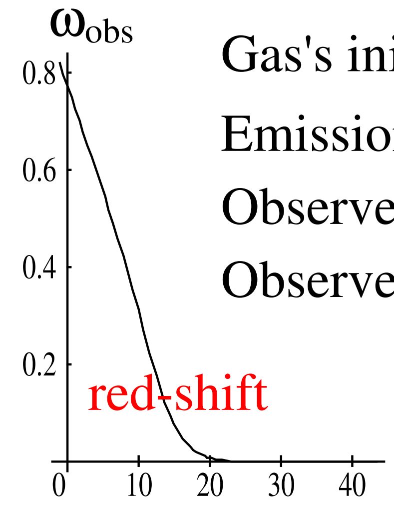
$$\Psi(t) = \mathrm{Sin}\left[\int \omega_{obs}\, d(t_{obs})\right]$$

$$\frac{t_{obs}}{R_{BH}/c}$$

Near Horizon Behavior

$(r < (3/2)R_{BH})$





Gas's initial place $r_0 = 3\,R_{BH}$ (ISCO)

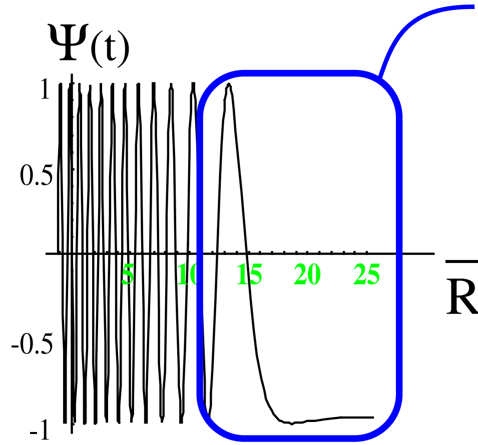
Emission Freq. $\omega_{emit} = 1$

Observed red-shift ω_{obs}

Observed wave (model)

$$\Psi(t) = \mathrm{Sin}\left[\int \omega_{obs}\, d(t_{obs})\right]$$

$$\frac{t_{obs}}{R_{BH}/c}$$



Near Horizon Behavior

$r < (3/2)R_{BH}$

$t_{obs} > 11.23\,R_{BH}/c$

$$\frac{t_{obs}}{R_{BH}/c}$$

→ **The freezing oscillation appears in observation,**

when the source approaches BH horizon.

◇ The gravitational redshift due to BH horizon is so strong that ⋯

The freezing of observed wave is a universal phenomenon

$$\textbf{independent of} \begin{cases} \textbf{details of BH environment} \\ \textbf{emission freq. by source} \end{cases} .$$

→ This freezing is a **direct observable of BH horizon**.

→ In order to detect the existence of BH horizon, it is important to

monitor the time evolution of frequency (time dpendent doppler effect)

$$\begin{cases} \text{within some transit time } (\sim 10\, R_{\rm BH}/c) \\ \text{for a wide freq. range } (0 < \omega < \text{finite}) \end{cases} \quad \cdots \quad \underline{\text{quick \& wide monitroing}}$$

4. The observable giving a clear relation of the mass and angular momentum of BH horizon

- By theoretical analysis of the freezing ocsillation of observed wave:

Time eovlution of the phase of observed oscillation (wave), as $t_{\rm obs} \to \infty$

$$\Theta(t_{\rm obs}) = \int \omega_{\rm obs}\, dt_{\rm obs} \to \omega_0 \exp\left[-\kappa\, c\, t_{\rm obs}\right] \; , \quad \begin{cases} c = \text{speed of light} \\ \omega_0 = \text{const.} \\ t_{\rm obs} = \text{observer's time} \end{cases}$$

$$\text{where} \quad \kappa = \begin{cases} \dfrac{1}{2\, R_{\rm BH}} & \text{for Schwarzschild BH} \\ \dfrac{\sqrt{M^2 - a^2}}{2\left(M^2 + M\sqrt{M^2 - a^2}\right)} & \text{for Kerr BH} \end{cases}$$

$$\left(\; M = \frac{G M_{\rm BH}}{c^2}\ [\text{cm}] \; , \; a = \frac{J_{\rm BH}}{M_{\rm BH}\, c}\ [\text{cm}] \; \to \; \left| \frac{a}{M} \right| < 1 \; \right)$$

◇ People in Gen.Rel. call κ the **surface gravity** of BH horizon.

- From the time evolution, **exponential dumping**, of observed frequency (Grav. Redshift), we can measure the surface gravity, κ, of BH horizon.

$$\kappa = \frac{\sqrt{M^2 - a^2}}{2\left(M^2 + M\sqrt{M^2 - a^2}\right)}$$

$\rightarrow$ **This gives a relation of M and a**

indepndent of black hole environment.

Note: This is purely a relation about BH horizon obtained from

only the time evolution of frequency of observed oscillation.

- **Comment 1 :** Appealing analysis to people in Gen.Rel. theory

Oscillation of observed wave: $\Psi_{\mathrm{R}}(t_{\mathrm{obs}}) \;:=\; A(\omega_0)\,\cos\Theta(t_{\mathrm{obs}})$

→ Fourier trans.: $F_{\mathrm{R}}(\Omega,\omega_0) \;=\; \int_{-\infty}^{\infty} e^{-i\,\Omega\,t_{\mathrm{obs}}}\,\Psi_{\mathrm{R}}(t_{\mathrm{obs}})\;\mathrm{d}t_{\mathrm{obs}}$

→ Power spectrum of time variation of freezing wave

mimics the Hawking radiation:

$$P_{\mathrm{R}}(\Omega) \;:=\; \left|F_{\mathrm{R}}(\Omega,\omega_0)\right|^2 \sim \frac{2\pi A(\omega_0)^2}{\kappa c}\,\frac{h(\Omega)}{\Omega}\,\frac{1}{\exp\left[2\pi\Omega/(\kappa\, c)\right]-1}$$

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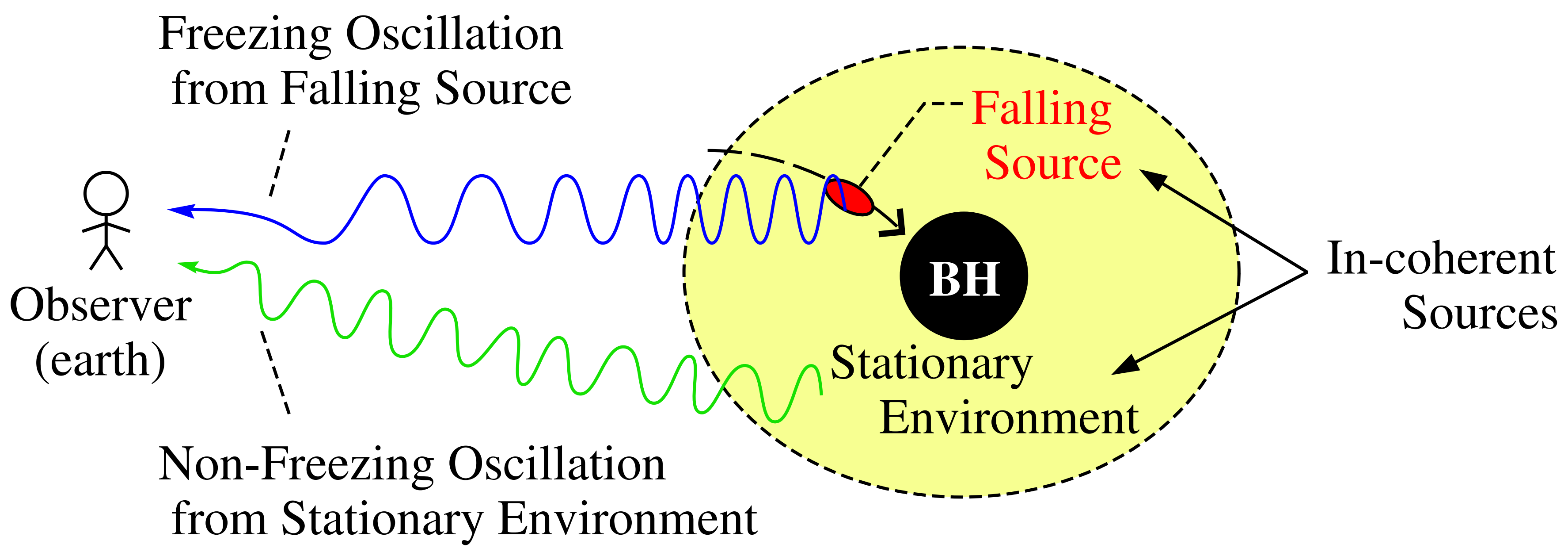
where $$\begin{cases} h(\Omega) \;=\; e^{2\pi\Omega/(\kappa c)} + 2\,e^{\Omega/(\kappa c)}\cos\Theta_\infty + 1 \\ \Theta_\infty \;=\; \Theta(t_{\mathrm{obs}}\to\infty) \;:\; \text{“frozen” phase} \end{cases}$$

Note: $A(\omega_0)$ is a constant depending only on the source,

not on the mass and angular momentum of BH horizon.

• **Comment 2 :** Radio waves coming from BH environment



→ Radio waves coming from incoherent sources are observed incoherently.

→ Can we see the freezing oscillation of radio coming from a falling Source?

→ No !? ··· since the radio of such long wave length is

absorbed by the plasma gas arround BH !?

→ Is there a 2nd best method for detecting BH horizon ?

5. The other approach to BH detection

- We search for **Strong Gravitational Lensing (SGL)** by BH horizon as a 2nd best method for detecting BH

→ What can we read from BH's strong gravitational lensing ?

◇ Spatial information — viewing image

→ ex. BH Shadow

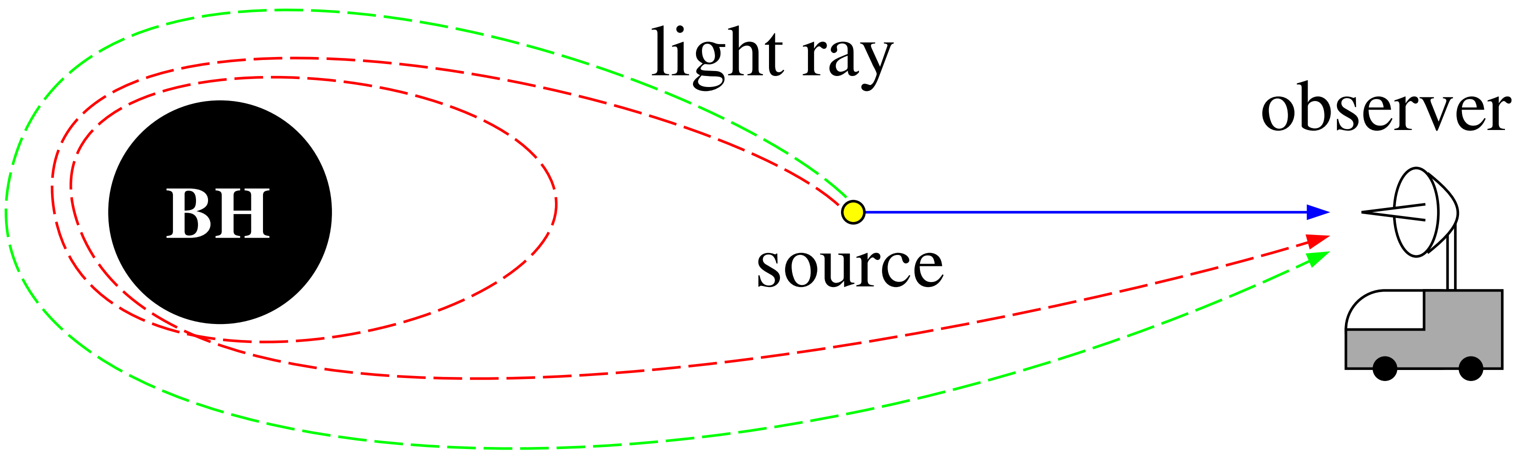
◇ Temporal information — time series of radio oscillation

→ **Time Delay Self-interferometry** ··· Next topic of this talk

——— Preliminary, under consideration/calculation ———

• **Time Delay Self-interferometry** (TDS $\neq$ Tokyo Disney Sea)

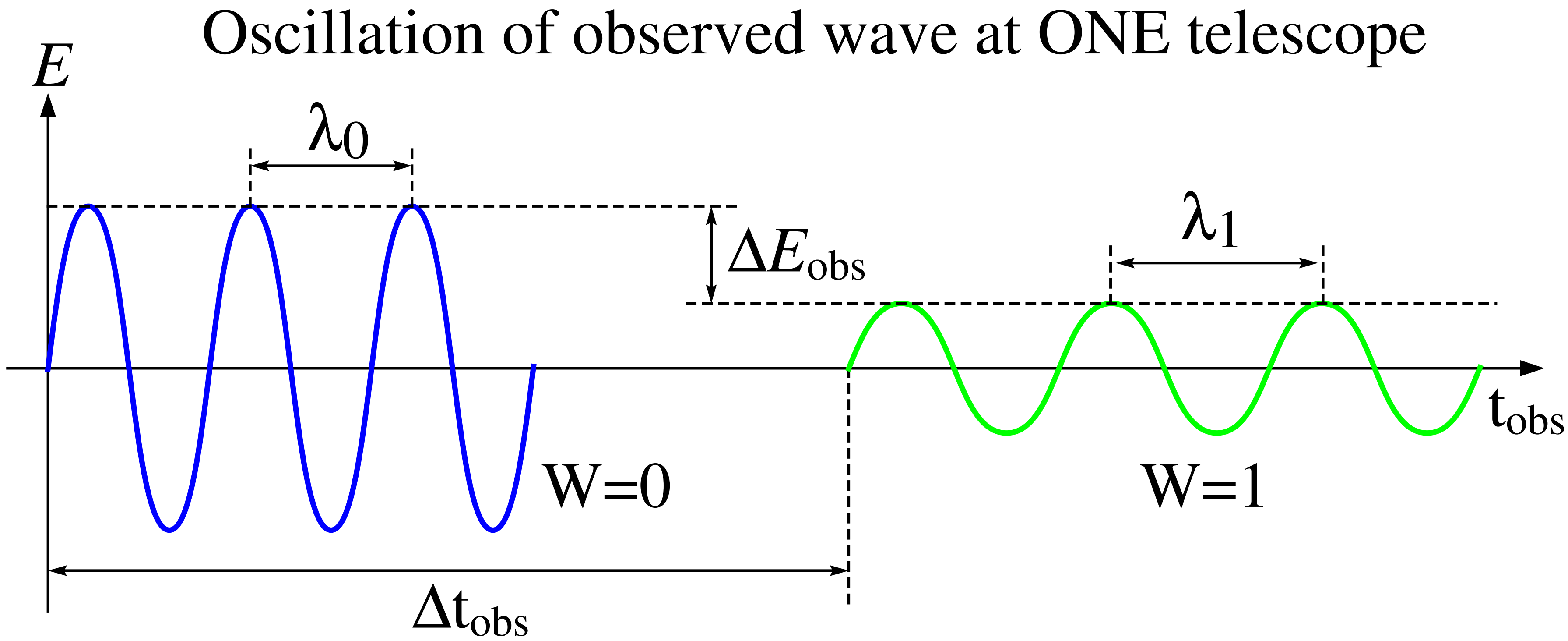
◇ Situation : $\begin{cases} \text{Observe rays emitted by a source at the same time near BH} \\ \text{Palse-like (a few wave length) emission with the same intensity.} \end{cases}$



→ **Comparing rays of winding number** $\begin{cases} W = 0 \text{ (“direct” ray)} \\ W = 1 \end{cases}$ **,**

the difference of $\begin{cases} \text{arrival time} \\ \text{intensity} \end{cases}$ **may let us observe BH !?**

$\diamond$ How do the rays (waves) of $W = 0$ and $W = 1$ appear in one telescope ?



$$\rightarrow \begin{cases} \Delta t_{\rm obs} & : \text{ due to Strong Grav. Lensing by BH} \\ \Delta E_{\rm obs} & : \text{ due to Strong Grav. Lensing by BH} \\ \lambda_0 \neq \lambda_1 & : \text{ due to Kinematic Doppler Effect by source's velocity} \end{cases}$$

$*$ Exactly, $\Delta t_{\rm obs}$ and $\Delta E_{\rm obs}$ depend on distance, BH $\leftrightarrow$ source. $\cdots$ 2nd best

◇ Task of Observation : How to identify the waves of $W = 0$ and $W = 1$.

→ The waves of $W = 0$ and $W = 1$ seem to be coherent,

since these waves are emitted by the same source at the same time.

→ "Self-interference" may let us identify these waves, when $\lambda_0 \simeq \lambda_1$.

(a suggestion by M.Tsuboi)

→ Time Delay Self-interferometry

◇ Task of Theory : When the waves of $W = 0$ and $W = 1$ are found $\cdots$,

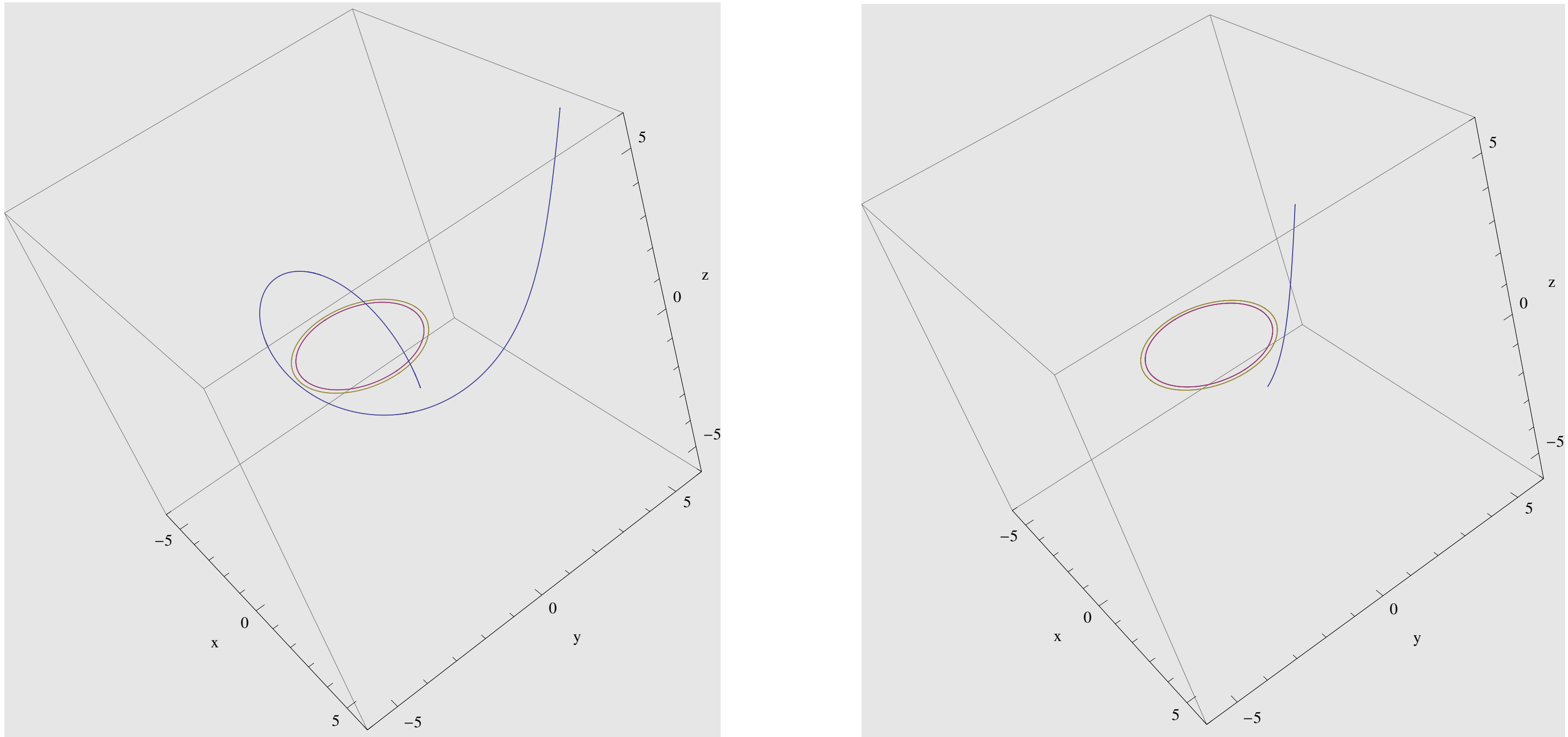
→ Under consideration/calculation :

∗ Estimation of $\dfrac{\lambda_0}{\lambda_1}$ → How is the case $\lambda_0 \simeq \lambda_1$ natural ?

∗ Formulas $\begin{cases} \text{BH mass} & , \ M(\Delta t_{\text{obs}}, \Delta E_{\text{obs}}, \Delta \lambda_{\text{obs}}) \\ \text{BH ang. mom.} & , \ J(\Delta t_{\text{obs}}, \Delta E_{\text{obs}}, \Delta \lambda_{\text{obs}}) \end{cases}$

→ How to extract M and J from observed data.

◇ A trial calculation : $\Delta t_{\rm obs}$, $\Delta E_{\rm obs}$ and λ_0/λ_1 for given M, J and source



$*$ Prepare rays : $W = -0.94$ $W = 0.06$ (winding about z-axis)

$\rightarrow$
- Parameters : $M = 1$, (with $c = 1$, $G = 1$)
- BH spin : $J = (1/2)(GM^2/c)$
- emission at : $\big(0\,,\,3\,(GM/c^2)\,,\,\pi/2\,,\,0\big)$ in Boyer-Lindquist coord.
- source vel. : $\big(1.70\,,\,0\,,\,0\,,\,0.0603\big) \;\rightarrow\; u_{(\phi)}/u_{(\mathrm{t})} \simeq 0.03$ (ZAMO)

$*$ The other paremeters :

- Observer's position : $\begin{cases} r = 3.65 \times 10^7\,(GM/c^2) \\ \theta = 0.300 \text{ rad } (17^\circ) \\ \phi = 0.405 \text{ rad} \end{cases}$
- Freq. at emission : $\omega_{\text{source}} = \dfrac{2\pi}{10}$ (trial value)

$*$ Results :

- $\Delta t_{\text{obs}} \simeq 30\,\dfrac{GM}{c^3} \rightarrow$ Sgr.A* : about 10 min. / Cyg.X-1 : about 0.001 sec.

$$\left(\frac{E_1}{E_0}\right)^2 = \frac{[\text{intensity of W=1}]}{[\text{intensity of W=0}]} \simeq 0.00868 \;\sim\; O(10^{-2})$$

$$\frac{\lambda_0}{\lambda_1} = \frac{[\text{freq. of W=-0.94}]}{[\text{freq. of W=0.06}]} \simeq 0.956871$$

$\rightarrow$ Is it possible to detect these values ?

(These values may change for different values of parameters.)

6. Summary

- The observable verifying the existence of BH horizon

  → **The freezing oscillation found in time evolution of observed wave**

- The observable relating the mass with angular momentum of BH horizon

  → **The exponential prolongation (time evolution of grav. redshift) of the frequency of freezing oscillation**

- The second best observables ⋯ Strong Grav. Lensing by BH

  ◇ BH shadow (Why 2nd best ? ⋯ Next slide, if I have enough time)

  ◇ **TDS** → How to extract M and J from TDS ?

  ⋯ under consideration

App. Does BH shodow denote BH directly ?

- Obvious relation : BH Shadow = IUCOP ($\neq$ ISCO)

$\rightarrow$ It is not sure : BH Shadow = BH horizon $\cdots$???

$\rightarrow$ A task for gen. rel. : To what extent does the IUCOP imply BH horizon ?