ブラックホール磁気圏研究会@広島大学 2015/03/02-04

NEGATIVE ENERGY MHD INFLOWS と磁場形状

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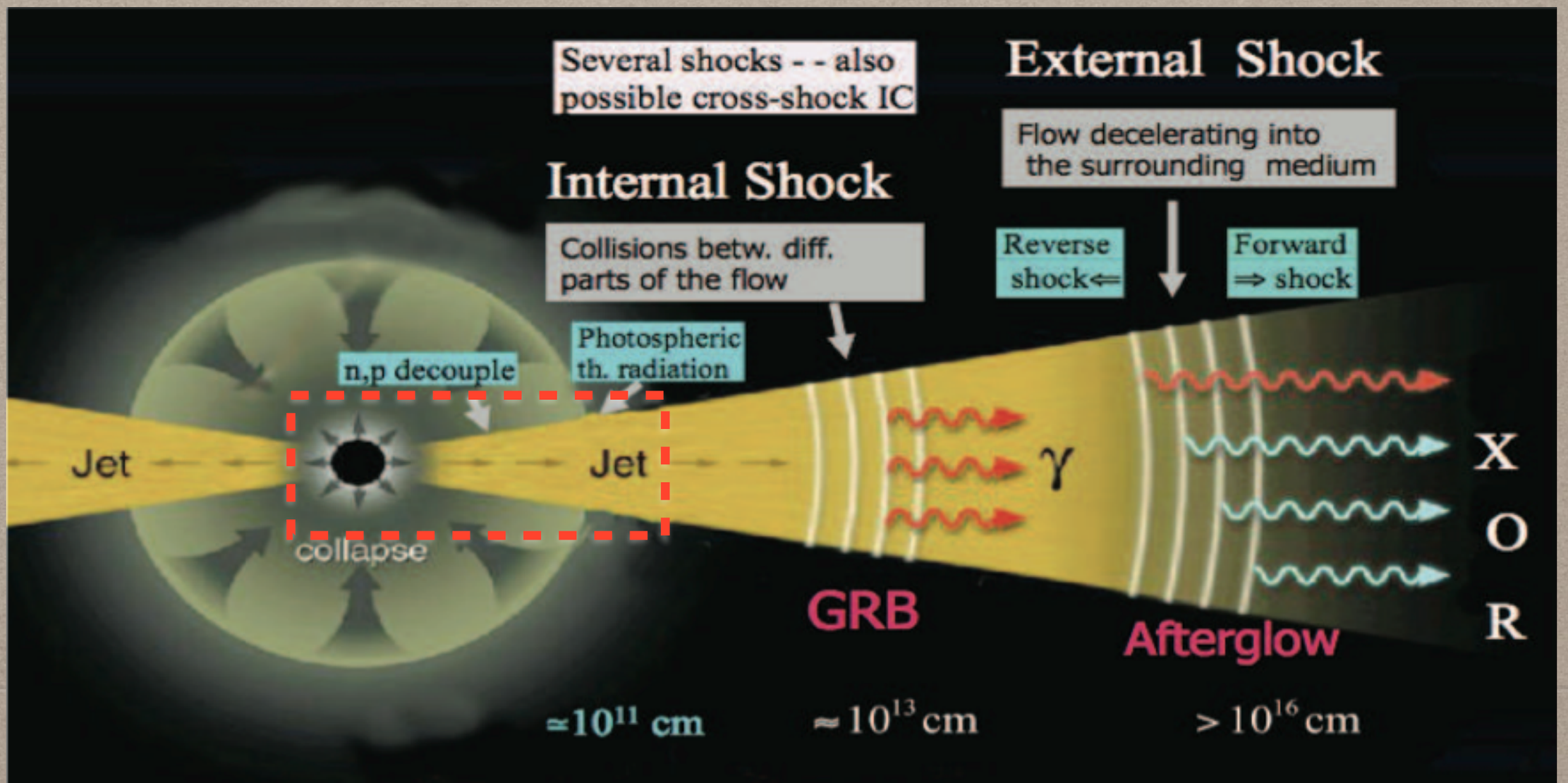
Meszaros & Rees 2014



MacFadyen and Woosley 1999

GRB-JET FORMATION

1. **Transient stage** ~ **2 sec** *free fall*

$$\bar{\rho} > 10^8 \ \mathrm{g/cm}^3$$

2. **Quasi-Steady stage** ~ **15 sec**

$$\bar{\rho} \sim 10^4 \ \mathrm{g/cm}^3 \qquad \dot{M} \sim 0.07 M_\odot/\mathrm{sec} \qquad M \sim 10 \ M_\odot$$

3. **Explosion of the star** ~ **数 10 sec**

MacFadyen and Woosley 1999

DISK FORMATION

1. Transient stage

Time ～ 0.751sec

降着流と衝撃波面　→　降着円盤の形成

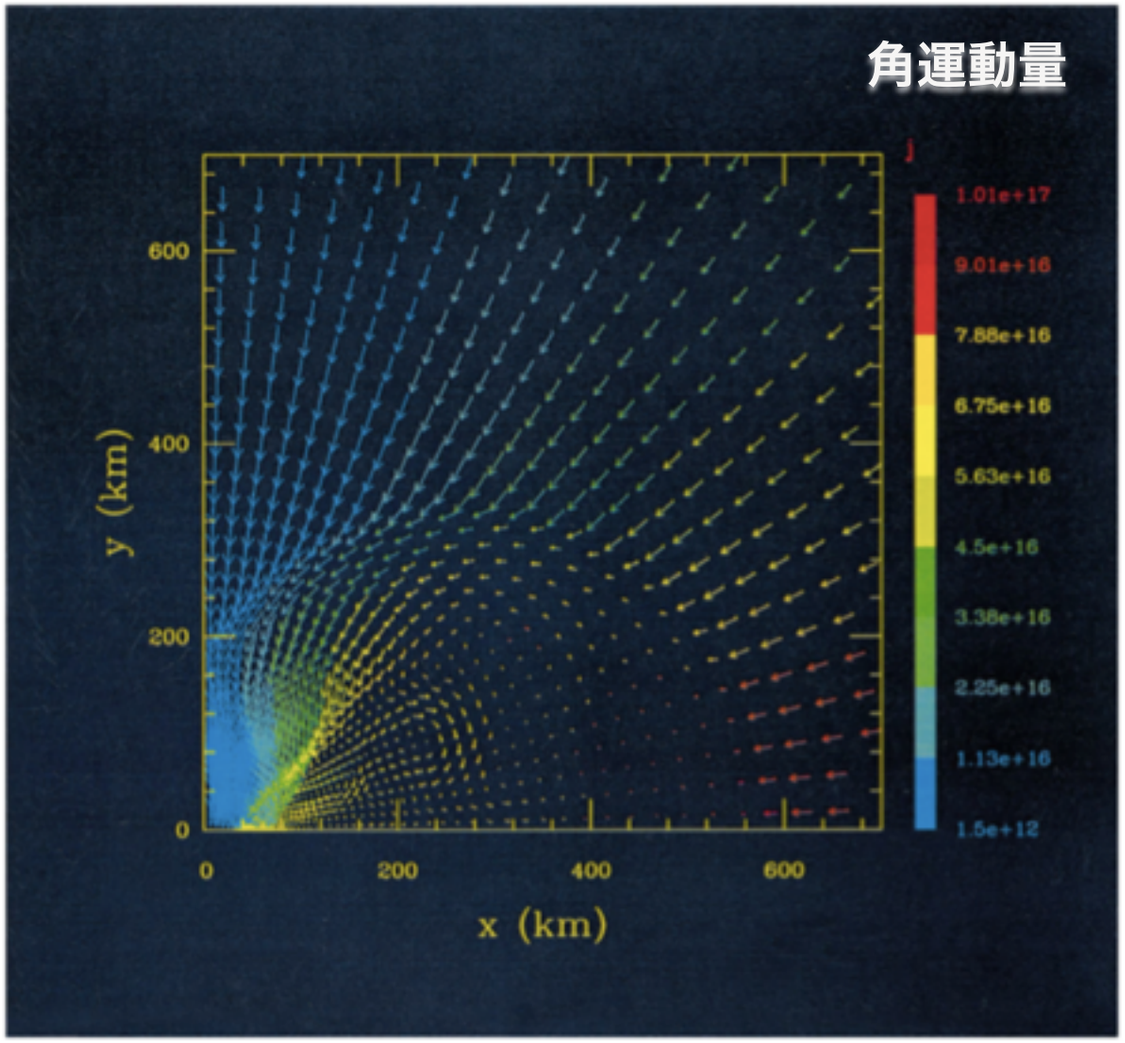


FIG. 3.—Velocity field in model 14A at $t = 0.751$ s showing early disk formation as centrifugal forces begin to halt accretion near the equator and a toroidal accretion shock forms at about 550 km. Meanwhile accretion along the polar axis proceeds relatively uninhibited. The largest infall velocities just outside the accretion shock in the equator are about 25,000 km s^{-1} and at the pole 55,000 km s^{-1}. Material impacting the disk at high latitude is channeled into the accretion column. Some circulation in the disk is apparent. The color coding is specific angular momentum (cm^2 s^{-1}). Because velocity arrows in this and subsequent figures have their tails in the zone they represent, the spherical inner boundary at 50 km is obscured.

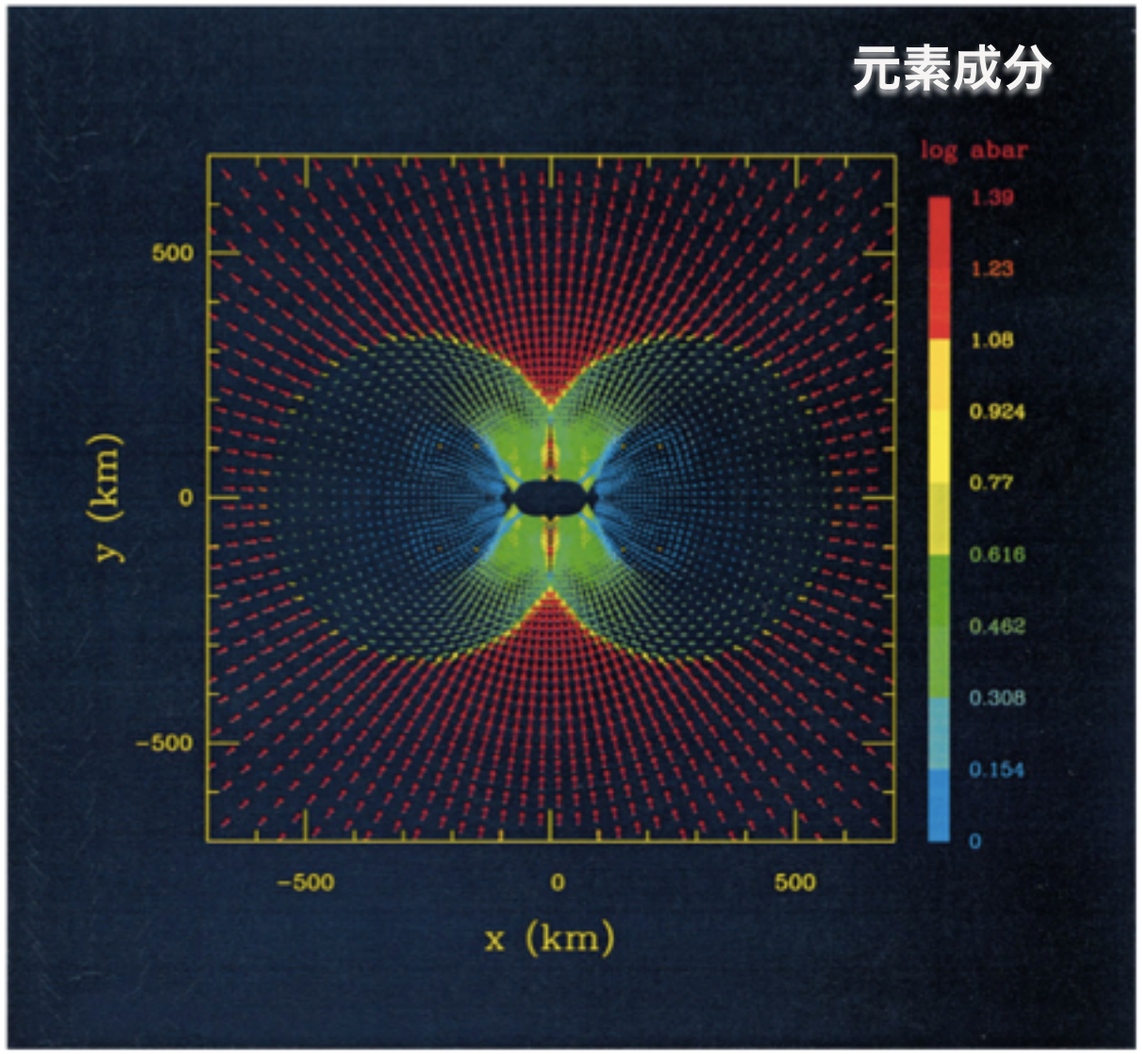


FIG. 4.—Velocity field at $t = 0.751$ s color coded by composition. The color shows the logarithm mean atomic weight of the nuclei in the accreting gas. Here, that weight ranges from 1 (nucleons) to 24 (oxygen and silicon). The shock heats the infalling gas to temperatures above 10^9 K, causing heavy nuclei to disintegrate. The dark blue inner torus is composed of free neutrons and protons.

MacFadyen and Woosley 1999

WIND FORMATION

2. Quasi-Steady stage

Time ～ 7.598sec

高緯度領域：密度の減少　→　*Wind* 発生

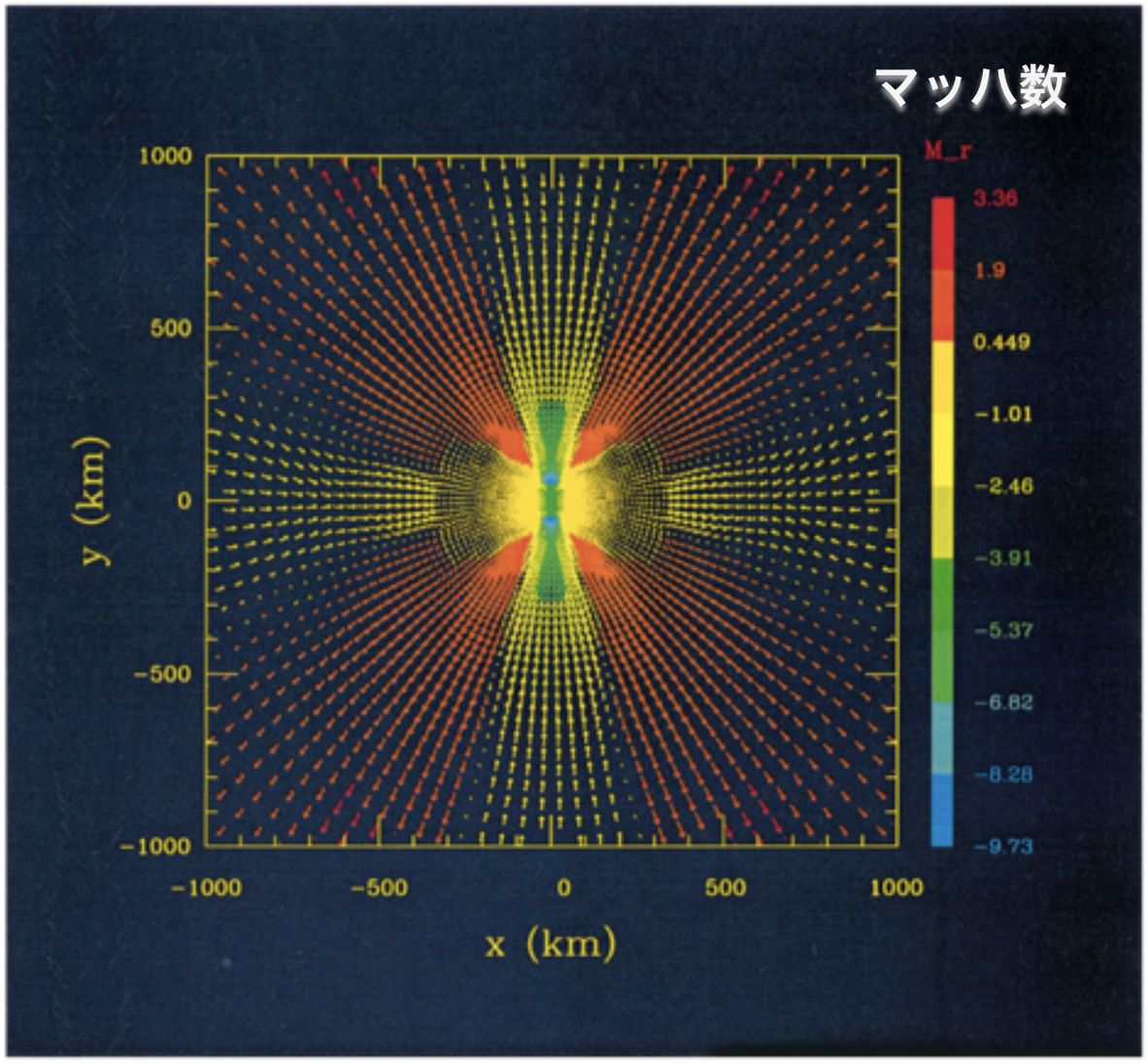


FIG. 8.—Radial Mach number, M_r, at 7.598 s in the standard run, model 14A. This figure highlights regions of supersonic flow and the location of shocks, especially the one at the edge of the accretion disk at 350 km. The yellow arrows near the equator represent the gas that feeds the disk (the tan toroidal region interior to 350 km). The orange and red arrows indicate outflow due to centrifugal bounce and viscous heating. The largest outward arrows represent outflow speeds of nearly 40,000 km s^{-1}. The largest inflow arrows (which are obscured because of overlap) indicate inflow speeds of $c/2$ along the pole at the inner boundary.

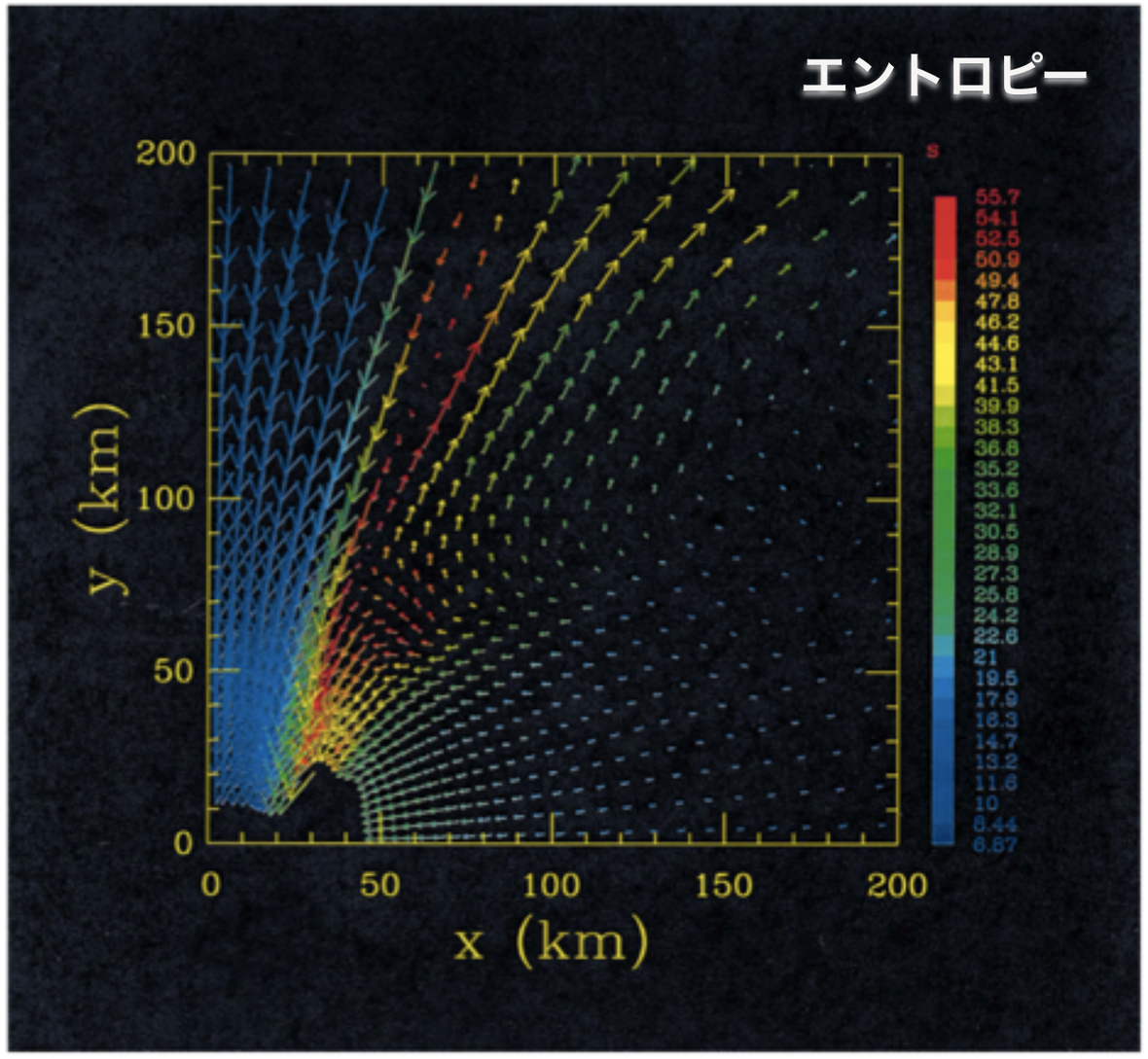


FIG. 17.—Origin of the viscosity driven "wind" shown in Fig. 16. Regions of high entropy (*red*; $S \approx 55$) are heated by viscous dissipation. Typical ratios of orbital velocity to radial velocity are about 10 in the high-entropy region and the orbital speed is about 10^{10} cm s^{-1}. Density in the high-entropy region is about 5×10^7 g cm^{-3} and the temperature, 1.5×10^{10} K. Viscous dissipation is releasing 5×10^{29} ergs cm^{-3} s^{-1}. All the material shown in the figure, except that along the accretion column, is composed of nucleons. When these reassemble the composition will be mostly ^{56}Ni and helium.

MacFadyen and Woosley 1999

エントロピー

s

55.7
54.1
52.5
50.9
49.4
47.8
46.2
44.6
43.1
41.5
39.9
38.3
36.8
35.2
33.6
32.1
30.5
28.9
27.3
25.8
24.2
22.6
21
19.5
17.9
16.3
14.7
13.2
11.6
10
8.44
6.87

200
150
100
50
0

y (km)

0
50
100
150
200

x (km)

Time ～ 7.598sec

ブラックホール磁気圏に拡張

MacFadyen and Woosley 1999 では磁場を考慮していないが、準定常状態の流線が「磁力線」を反映していると考えて、

この「磁気圏」における回転エネルギー引き抜き

を考察してみる。

Key words :

角運動量分布、 アルファン点、 *Negative Energy Inflows*

GRMHD

General Relativistic Magneto-hydrodynamic

Basic Equations

The *ideal MHD* condition	$u^\beta F_{\alpha\beta} = 0$
The particle conservation law	$(nu^\alpha)_{;\alpha} = 0$
Maxwell equations	$F^{\mu\nu}_{;\nu} = -4\pi j^\mu \ , \ \ F_{[\mu\nu;\sigma]} = 0$
Polytropic relation (*Tooper 1965*)	$P = K\rho_0^\Gamma$
The equation of motion	$T^{\alpha\beta}_{;\beta} = 0$

Field-aligned ``conserved quantities"

flow's parameters

1. Number flux per unit magnetic flux
2. Angular velocity of the field lines
3. Total energy of the magnetized flow
4. Total angular momentum
5. Entropy

$$\eta(\Psi) = \frac{nu^p}{B^p}$$
$$\Omega_F(\Psi) = -\frac{F_{tr}}{F_{\phi r}} = -\frac{F_{t\theta}}{F_{\phi\theta}}$$
$$E(\Psi) = \mu u_t - \frac{\Omega_F}{4\pi\eta} B_\phi$$
$$L(\Psi) = -\mu u_\phi - \frac{1}{4\pi\eta} B_\phi$$
$$S(\Psi)$$

GRMHD Flows

1. Relativistic Bernoulli equation

energy in corotation frame

gtavitational Lorentz factor

poloidal magnetic field

$$(E - \Omega_F L)^2 = \mu^2 \alpha + M^2(\alpha B_p^2 + B_\phi^2)$$

→ PLOT Solution

enthalpy
(rest mass energy + internal energy)

Alfven Mach number

$$M^2 \equiv \frac{u_p^2}{u_{\rm AW}^2}\alpha$$

toroidal magnetic field

2. total Energy of MHD Flow

$$E(\Psi) = \mu u_t - \frac{\Omega_F}{4\pi\eta} B_\phi$$

energy conversion
KE <--> ME

the fluid part of energy

Poynting flux per the particle number flux

Bernoulli equation （Newtonian）

- From equation of motion,

gravitational potential

Poynting flux

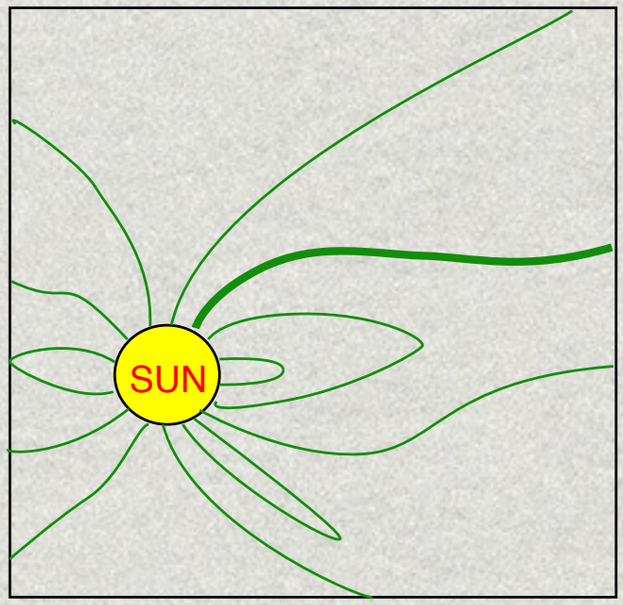
$$E = \left(\frac{1}{2} v_r^2 + \frac{1}{2} v_\phi^2 + \frac{\Gamma}{\Gamma - 1} \frac{P}{\rho} + \Psi(r) \right) - \frac{r B_\phi B_r \Omega_F}{4\pi \rho v_r}$$

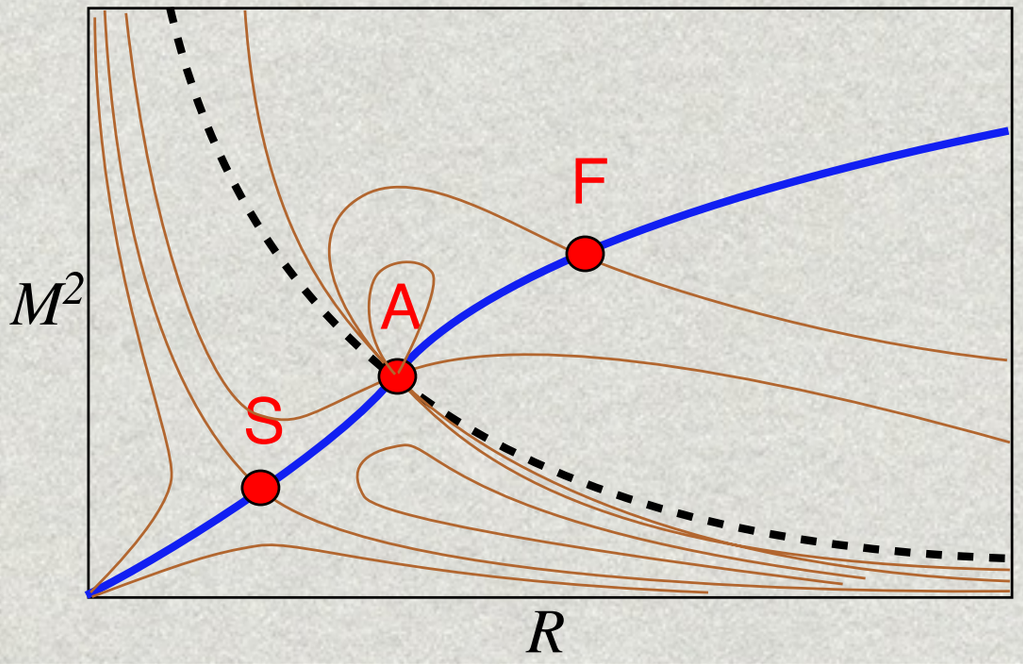
kinetic energy　　internal energy

OR

$$(E - \Omega_F L) = \frac{1}{2} v_r^2 + \frac{1}{2} v_\phi^2 + \frac{a_{\rm sw}^2}{\Gamma - 1} + \Psi(r) - r v_\phi \Omega_F$$

SOLAR WIND





GR-MHD ACCRETION SOLUTION

一般相対論版のベルヌーイ方程式より

2.2.1 Cold MHD flows

* the solution of the poloidal equation for cold MHD flows :

$$M_{\pm}^2[r;\Psi(r,\theta)] = \frac{1}{A}\left[(\hat{E}-\Omega_F\hat{L})^2 - \alpha \pm \sqrt{D}\right] \ , \tag{16}$$

where

$$A \equiv -(k+1) - (G_\phi\hat{E} + G_t\hat{L})^2 \frac{1}{\beta^2\rho_w^2} \ , \tag{17}$$

$$D \equiv [(\hat{E}-\Omega_F\hat{L})^2 - \alpha](G_\phi\hat{E} + G_t\hat{L})^2(\alpha+\beta^2)\frac{1}{\beta^2\rho_w^2} \ , \tag{18}$$

$$k \equiv (g_{\phi\phi}\hat{E}^2 + 2g_{t\phi}\hat{E}\hat{L} + g_{tt}\hat{L}^2)\frac{1}{\rho_w^2} \ , \tag{19}$$

and $\beta \equiv \mathcal{B}_\phi/\mathcal{B}_p$. For the trans-Alfvenic flows, we adopt $M^2 = M_-^2$ in the sub-Alfvénid region and $M^2 = M_+^2$ in the super-Alfvénid region From the definition of the Alfven Mach number $M^2 \equiv (4\pi\mu\eta^2)/n$, we obtain the number density of the flows,

$$n = \frac{4\pi\mu_c\eta^2}{M_\pm^2} \ , \qquad \text{or} \quad \tilde{n} \equiv \frac{n}{4\pi\mu_c\eta^2} = \frac{1}{M_\pm^2} \ . \tag{20}$$

一般相対論版のベルヌーイ方程式より

2.2 Funnel region in a Black Hole Magnetosphere

BZ-magnetic field model (paraboloidal / radial) + MHD accretion

* poloidal equation :

$$(\hat{E} - \Omega_F \hat{L})^2 = \hat{\mu}^2 \alpha + M^4 (\alpha \mathcal{B}_p^2 + \mathcal{B}_\phi^2) \ , \tag{14}$$

where $\hat{E} \equiv E/\mu_c$ and $\hat{L} \equiv L/\mu_c$ ($\mu_c = m_{\rm p} c^2$; $m_{\rm p}$ is the plasma particle's mass), and we define

$$\mathcal{B}_p \equiv \frac{B_p}{4\pi \mu_c \eta} \ , \quad \mathcal{B}_\phi \equiv \frac{1}{\rho_w} \frac{B_\phi}{4\pi \mu_c \eta} \ . \tag{15}$$

2.2.2 Hot MHD flows

The relativistic enthalpy can be expressed

$$\hat{\mu} = 1 + \hat{\mu}_1 \, n^{\Gamma - 1} \quad , \quad \hat{\mu}_1 \equiv \frac{\Gamma K}{\Gamma - 1} m_{\rm p}^{\Gamma - 2} \ , \tag{21}$$

where we assume the relativistic polytropic relation, $P = K(m_{\rm p} n)^\Gamma$. Here, we define the Temperature parameter Θ as

$$\Theta \equiv \frac{k_B T}{m_p c^2} = \frac{\hat{\mu} - 1}{1 + N} \ ,$$

where the polytropic index N is given by $\Gamma = 1 + 1/N$.

To obtain the distribution of the flow velocity, the number density and the temperature, we should solve the poloidal equation (14) with the relativistic enthalpy (21), numerically.

流線にそっての保存量とアルフェン点

- The total energy and angular momentum of MHD flows along a magnetic field line (TNTT90)

$$E(\psi) = (G_t/\alpha)_{\rm A}(\mu\sqrt{\alpha})_{\rm inj}\ , \qquad L(\psi) = -(G_\phi/\alpha)_{\rm A}(\mu\sqrt{\alpha})_{\rm inj}\ . \tag{2}$$

From the poloidal equation, at the injection point ($r = r_{\rm inj}$, $M^2 = M^2_{\rm inj} = 0$), we will define

$$e(\psi) \equiv E(\psi) - \Omega_F(\psi) L(\psi) = \mu(r_{\rm inj};\psi)\sqrt{\alpha(r_{\rm inj};\psi)}\ . \tag{3}$$

The value e is also conserved along the magnetic field line. Note that $\sqrt{g^{tt}}\,(E - \Omega_F L)$ is the locally measured total energy for a corotating observer with Ω_F. [1]

角運動量の θ 依存性

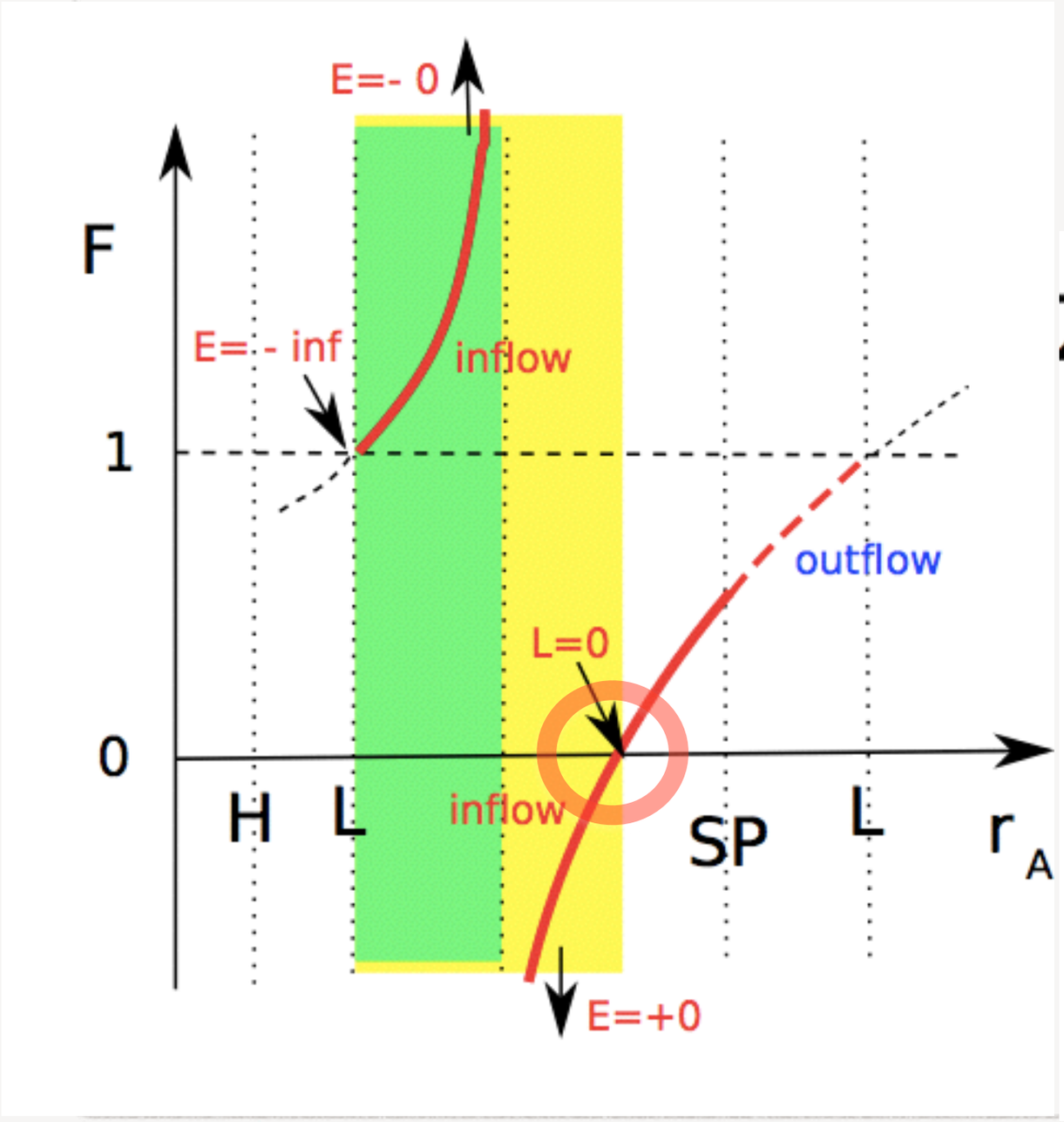
- At the Alfvén point, from the regularity condition, we obtain

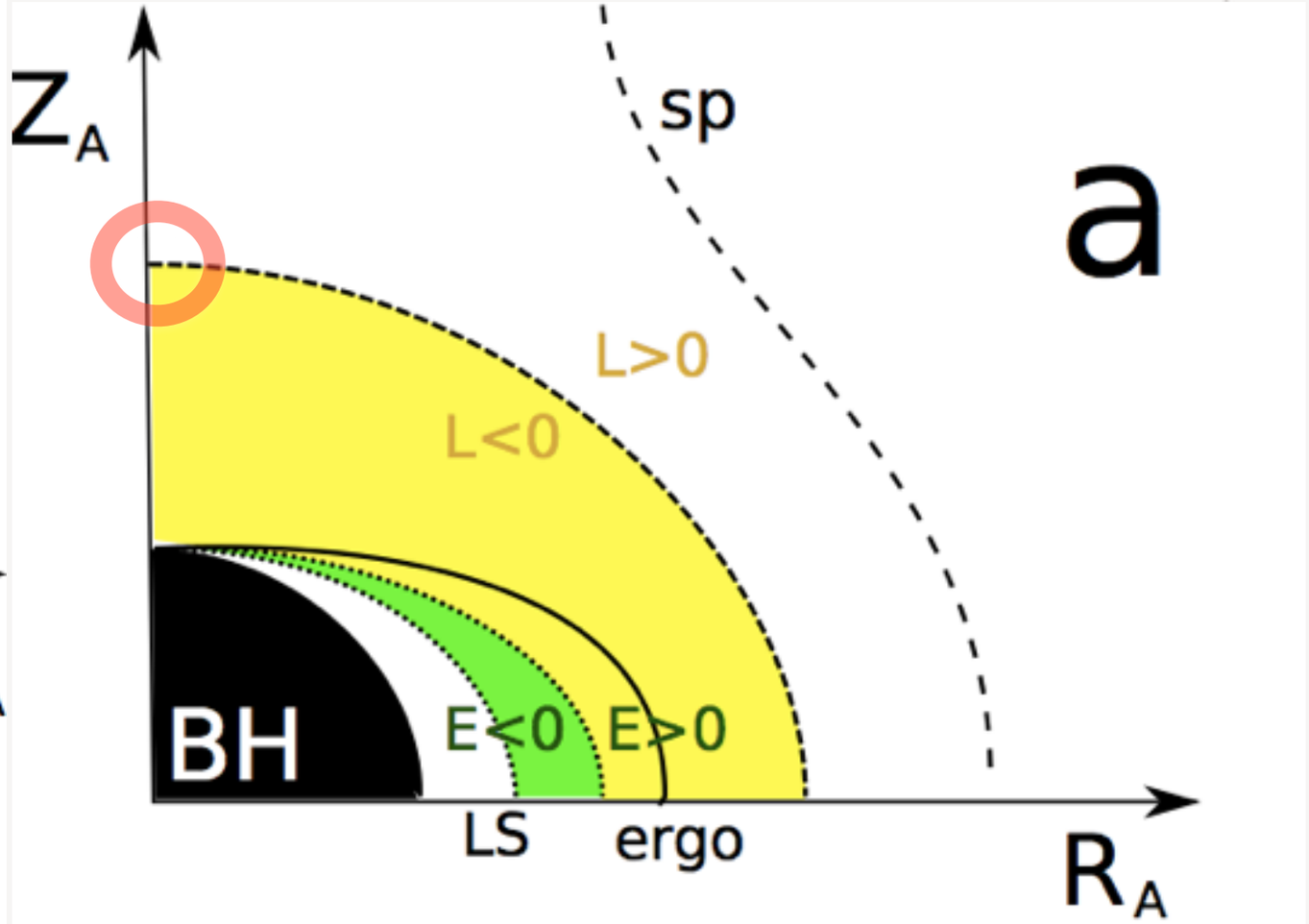
$$L[r_{\rm A};\psi(\theta)] = \left[-\frac{e}{\alpha_{\rm A}}\left(\frac{(r^2+a^2)^2 - \Delta a^2 \sin^2\theta}{\Sigma}\right)_{\rm A}(\omega_{\rm A} - \Omega_F)\right]\sin^2\theta\ . \tag{4}$$

Near the rotational axis ($\theta \ll 1$), $L \propto \sin^2\theta$ [also $\mu u_\phi \propto \sin^2\theta$, $B_\phi \propto \sin^2\theta$].

CRITICAL CONDITION @ALFVEN POINT

Angular Momentum of MHD flow
Energy of MHD flow
Angular velocity of MF





エネルギー・フラックス

- The outgoing energy flux ($\mathcal{E}^\mu \equiv T^{\mu\nu}k_\nu$) and angular momentum flux ($\mathcal{L}^\mu \equiv T^{\mu\nu}m_\nu$), which stream along the magnetic flux surface [$\psi(r,\theta)$=constant], are

$$\mathcal{E}^r = -\frac{\Omega_F B_\phi}{4\pi\sqrt{-g}}(A_{\phi,\theta}) = \Omega_F \mathcal{L}^r \ , \qquad \mathcal{E}^\theta = \frac{\Omega_F B_\phi}{4\pi\sqrt{-g}}(A_{\phi,r}) = \Omega_F \mathcal{L}^\theta \ , \tag{5}$$

where $k^\mu = (1,0,0,0)$ is the timelike Killing vector and $m^\mu = (0,0,0,1)$ is the axial Killing vector (BZ77).

事象の地平面での境界条件

- At the event horizon, the toroidal component of the magnetic field is

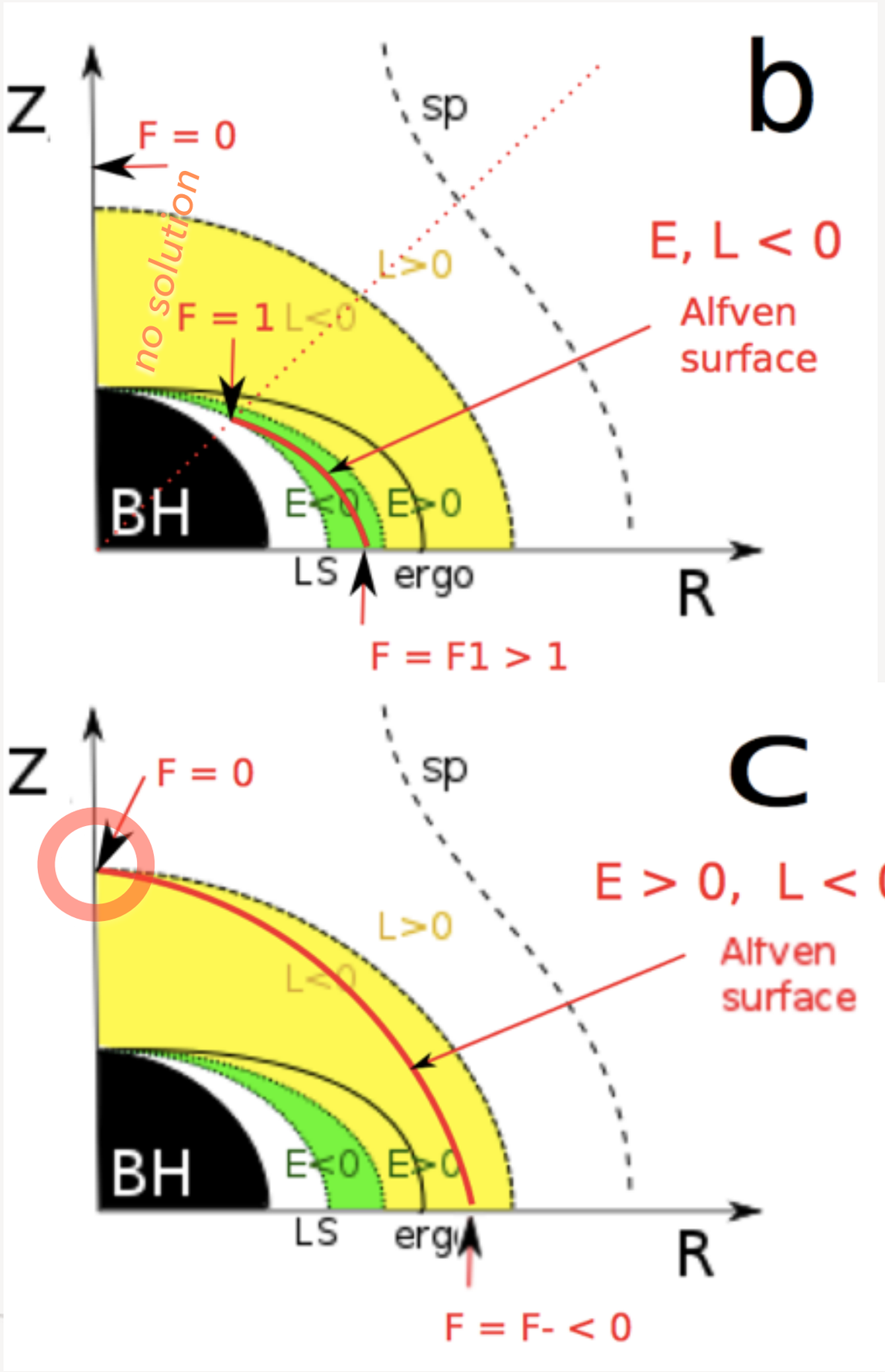
$$B_\phi^{\rm H}(\psi) = \frac{(r_{\rm H}^2 + a^2)\,\sin\theta}{\Sigma_{\rm H}}(\Omega_F - \omega_{\rm H})\,(A_{\phi,\theta})_{\rm H} \ , \tag{6}$$

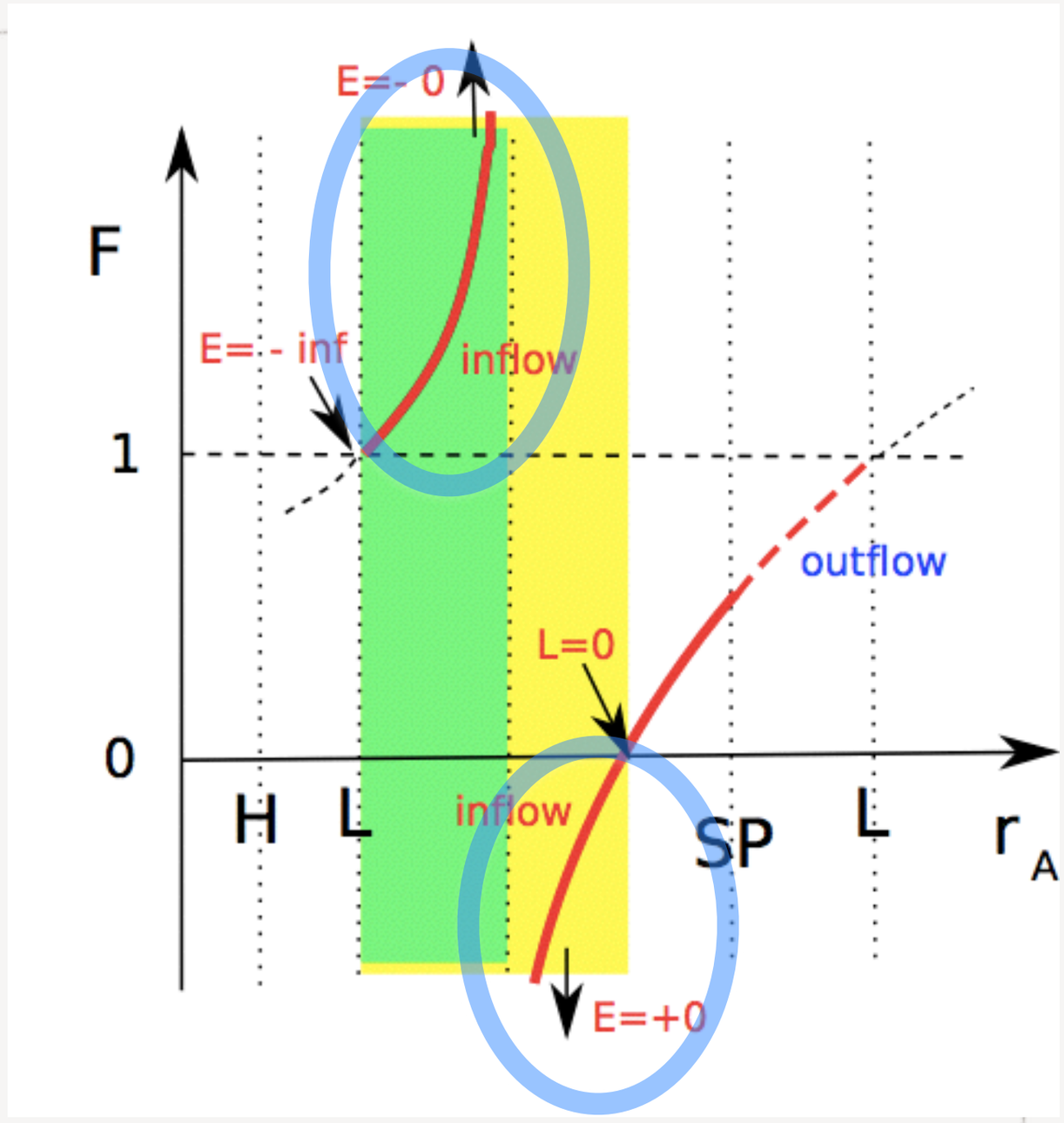
and then

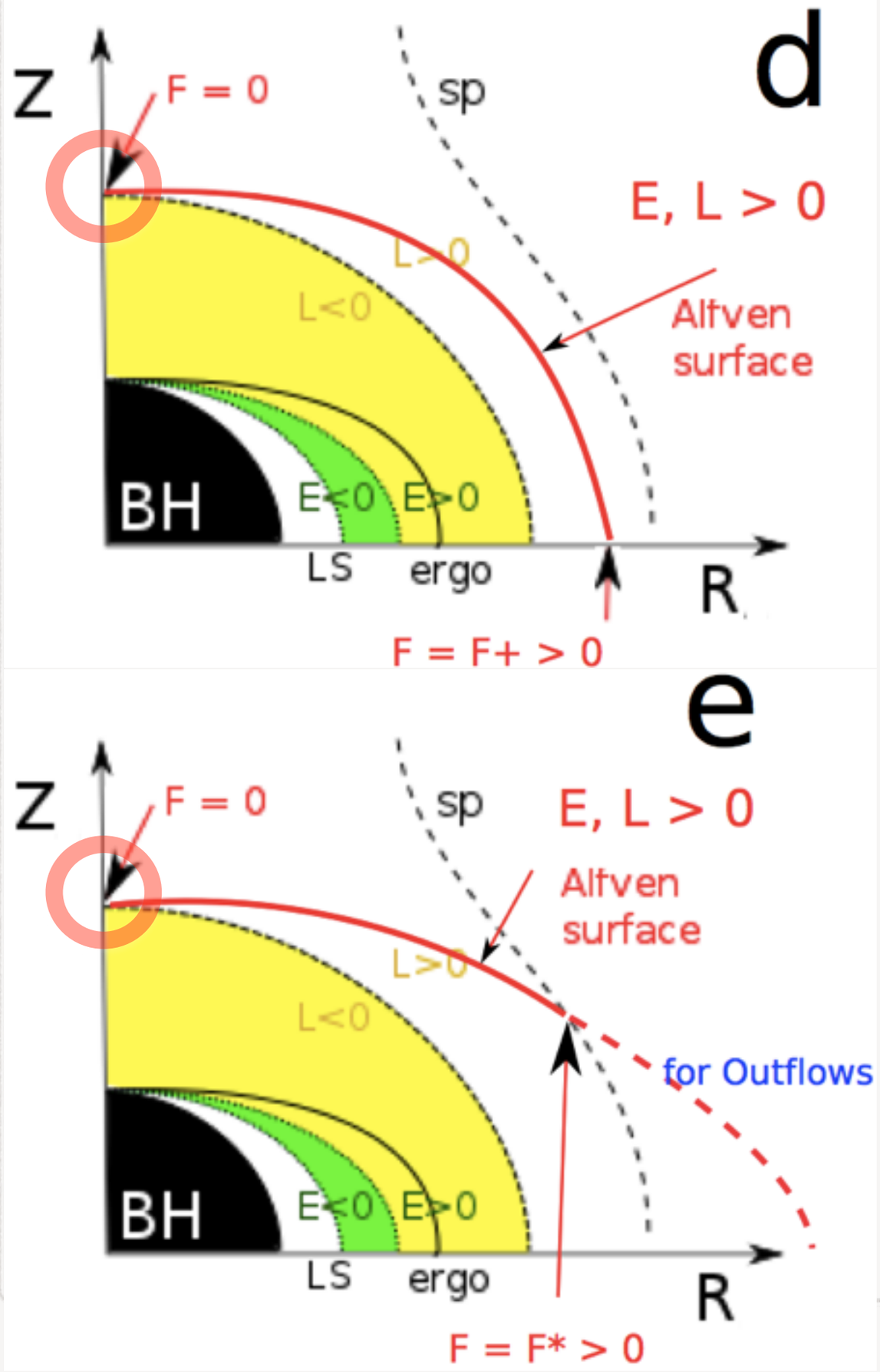
$$\mathcal{E}^r_{\rm H} = \Omega_F(\omega_{\rm H} - \Omega_F)\left(\frac{A_{\phi,\theta}}{\Sigma_{\rm H}}\right)^2 \frac{r_{\rm H}^2 + a^2}{4\pi} \ , \tag{7}$$

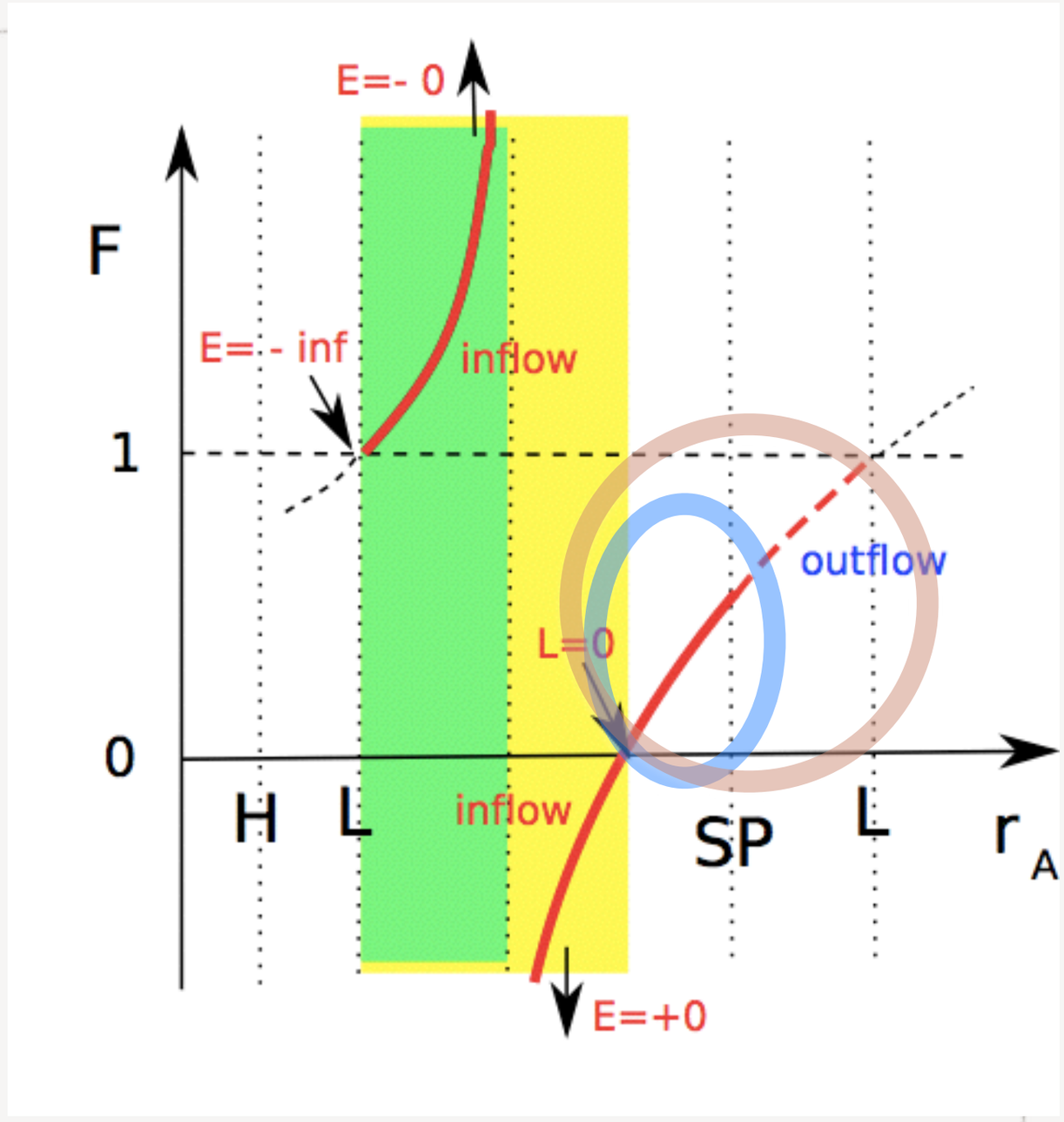
which is positive for a rapidly rotating black hole of $0 < \Omega_F < \omega_{\rm H}$ (BZ77) .

ANGULAR MOMENTUM OF MHD FLOWS


b
Z
R
F = 0
sp
no solution
L>0
F = 1
L<0
E, L < 0
Alfven
surface
BH
E<0
E>0
LS
ergo
F = F1 > 1
C
Z
R
F = 0
sp
E > 0, L < 0
L>0
L<0
Alfven
surface
BH
E<0
E>0
LS
ergo
F = F- < 0


E=- 0
F
E=- inf
inflow
1
outflow
L=0
0
H
L
inflow
SP
L
r A
E=+0


d
Z
F = 0
sp
E, L > 0
L>0
L<0
Alfven
surface
BH
E<0
E>0
LS
ergo
R
F = F+ > 0
e
Z
F = 0
sp
E, L > 0
Alfven
surface
L>0
L<0
for Outflows
BH
E<0
E>0
LS
ergo
R
F = F* > 0


E=- 0
F
E=- inf
inflow
1
outflow
L=0
0
H
L
inflow
SP
L
r
A
E=+0

MAGNETIC FIELD (0)

The force-balance equation is derived by Nitta et al.(1991)

GRMHD

$$\frac{\alpha - M^2}{4\pi}(\partial^r\partial_r\Psi + \partial^\theta\partial_\theta\Psi) + \frac{\hat{B}_p^2\rho_w^2}{4\pi\sqrt{-g}}\left[\frac{\sqrt{-g}}{\rho_w^2}(\alpha - M^2)\right]'$$
$$+ \quad \frac{4\pi\mu^2\rho_w^2}{M^2}\eta\eta' + \frac{2\pi}{M^2}\left[g_{\phi\phi}(E^2\eta^2)' + 2g_{t\phi}(EL\eta^2)' + g_{tt}(L^2\eta^2)'\right] \qquad (7.47)$$
$$- \quad \frac{4\pi\eta^2\rho_w^2}{M^4}(\mu^2 G_\phi - e\mu u_\phi)\Omega_F' + \frac{4\pi}{M^2}\left[G_\phi(E\eta)' + G_t(L\eta)'\right](E\eta)\hat{f} \;=\; 0$$

where the prime $(') \equiv -(1/\hat{B}_p^2)[(\partial^r\Psi)\partial_r + (\partial^\theta\Psi)\partial_\theta]$, $\hat{B}_p \equiv \rho_w B_p$ and $\hat{f} \equiv \rho_w f$.

Force-free

* slowly rotating black hole

- $\Omega_F = \frac{a}{M}W(r,\theta)$,
- $B_\phi = \frac{a}{M^2}Y(r,\theta) + O(a/M)^3$,
- $A_\phi = X(r,\theta) + \frac{a^2}{M^2}x(r,\theta) + O(a/M)^4$.

MAGNETIC FIELD (1)

2.1.1 Split-monopole magnetic field in Kerr geometry ($a \ll M$)

(Blandford & Znajek 1977, McKinney & Gammie 2004)
In Kerr-Shild coordinates, ... [8]

$$B^r_{\rm KS} = \left\{ \frac{C}{r^2} + a^2 \frac{C}{2r^4} \left[-2\cos\theta + r^2(1+3\cos 2\theta) f(r) \right] \right\} \cdot G_t \ , \tag{1}$$

$$B^\theta_{\rm KS} = -a^2 \frac{C}{r^2} \cos\theta \sin\theta f'(r) \cdot G_t \ , \tag{2}$$

$$B^\phi_{\rm KS} = -a \frac{C}{8r^2} \left(1 + \frac{4}{r} \right) \cdot g_{tt} \ , \tag{3}$$

where for $r > 2$

$$f(r) = \left[\mathrm{Li}_2\left(\frac{2}{r}\right) - \ln\left(1-\frac{2}{r}\right) \ln\frac{r}{2} \right] \frac{r^2(2r-3)}{8} + \frac{1+3r-6r^2}{12} \ln\frac{r}{2} + \frac{11}{72} + \frac{1}{3r} + \frac{r}{2} - \frac{r^2}{2} \ , \tag{4}$$

and Li_2 is the second polylogarithm function:

$$\mathrm{Li}_2(x) = -\int_0^1 \frac{\ln(1-tx)}{t}\, dt \ . \tag{5}$$

MAGNETIC FIELD (2)

2.1.2 Paraboloidal magnetic field in Kerr geometry ($a \ll M$)

From Blandford & Znajek (1977),

$$X(r,\theta) = \frac{C}{2}\left[r(1-\cos\theta) + 2M(1+\cos\theta)(1-\ln[1+\cos\theta])\ \right] \tag{6}$$

and the boundary conditions at the event horizon are given as

$$Y(2M,\theta) = CM\left(W - \frac{1}{4}\right)\sin^2\theta\,(1+\ln[1+\cos\theta])\ , \tag{7}$$

$$W(2M,\theta) = \frac{(1/4)\sin^2\theta[1+\ln(1+\cos\theta)]}{4\ln 2 + \sin^2\theta + [\sin^2\theta - 2(1+\cos\theta)]\ln(1+\cos\theta)}\ . \tag{8}$$

* Magnetic field $[\sim O(a/M)\ ]$

$$B^r(r,\theta) = \frac{G_t}{\sqrt{-g}}\frac{C}{2}\left[r\sin\theta + 2M\sin\theta\ln(1+\cos\theta)\right] + O(a/M)^2\ , \tag{9}$$

$$B^\theta(r,\theta) = -\frac{G_t}{\sqrt{-g}}\frac{C}{2}(1-\cos\theta) + O(a/M)^2\ , \tag{10}$$

$$B_p^2(r,\theta) = \frac{C^2}{4\Sigma}\left[\frac{1}{\Delta}\left\{r + 2M\ln(1+\cos\theta)\right\}^2 + \left(\frac{1-\cos\theta}{\sin\theta}\right)^2\right] + O(a/M)^2\ . \tag{11}$$

$$\begin{aligned} B_\phi(r,\theta) &= B_\phi(A_\phi[r,\theta]) = B_\phi(\theta_H[r,\theta]) \\ &= \frac{a}{M}\cdot C\left[W(\theta_H) - \frac{1}{4}\right]\sin^2\theta_H(1+\ln[1+\cos\theta_H]) + O(a/M)^3\ . \end{aligned} \tag{12}$$

引き抜かれたエネルギーの行き場は？

Models

- Cold MHD accretion / wind
- Rapidly rotating Black Hole
- Energy extraction by Negative energy inflows
- Split-monopole magnetic field lines

MacFadyen and Woosley 1999

Accretion

NE

Wind

NE

Accretion

s エントロピー

55.7
54.1
52.5
50.9
49.4
47.8
46.2
44.6
43.1
41.5
39.9
38.3
36.8
35.2
33.6
32.1
30.5
28.9
27.3
25.8
24.2
22.6
21
19.5
17.9
16.3
14.7
13.2
11.6
10
8.44
6.87

y (km)

200
150
100
50
0

0
50
100
150
200

x (km)

Time ～ 7.598sec

MacFadyen and Woosley 1999

NE

NE

Energy の流れ

エントロピー

s

55.7
54.1
52.5
50.9
49.4
47.8
46.2
44.6
43.1
41.5
39.9
38.3
36.8
35.2
33.6
32.1
30.5
28.9
27.3
25.8
24.2
22.6
21
19.5
17.9
16.3
14.7
13.2
11.6
10
8.44
6.87

200
150
100
50
0

y (km)

0
50
100
150
200

x (km)

Time ～ 7.598sec

MHD Shock case (1)

Wald's **Uniform** MF

+

Loop & Open MF

by Disk-current

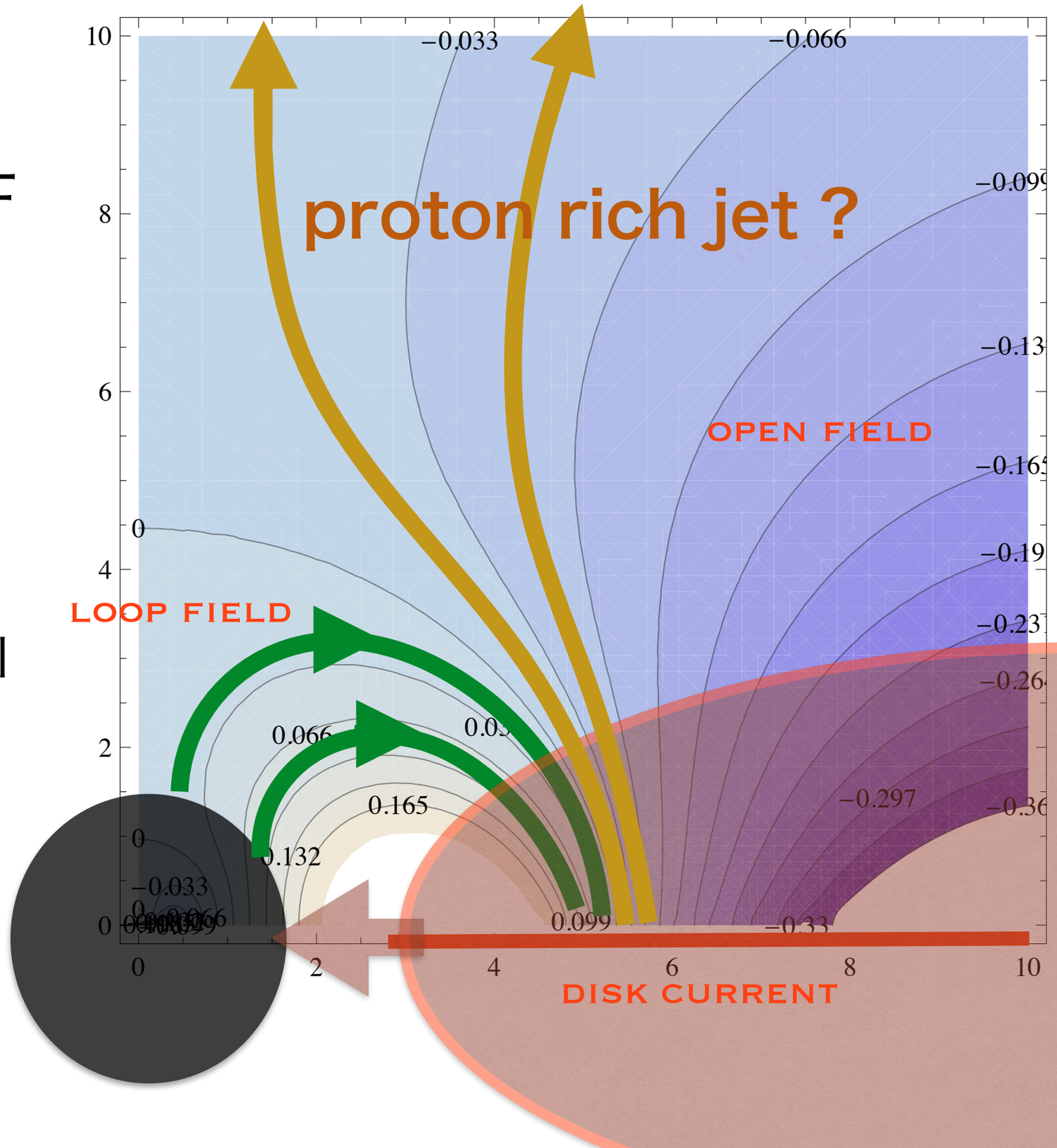
(superposition of

Loop MF by equatorial

ring-currents)

A solution of vacuum Maxwell eq.

$a = 0.8m,\ n = 20,\ r_0(r) > 3m$



Hot jet model by Shock

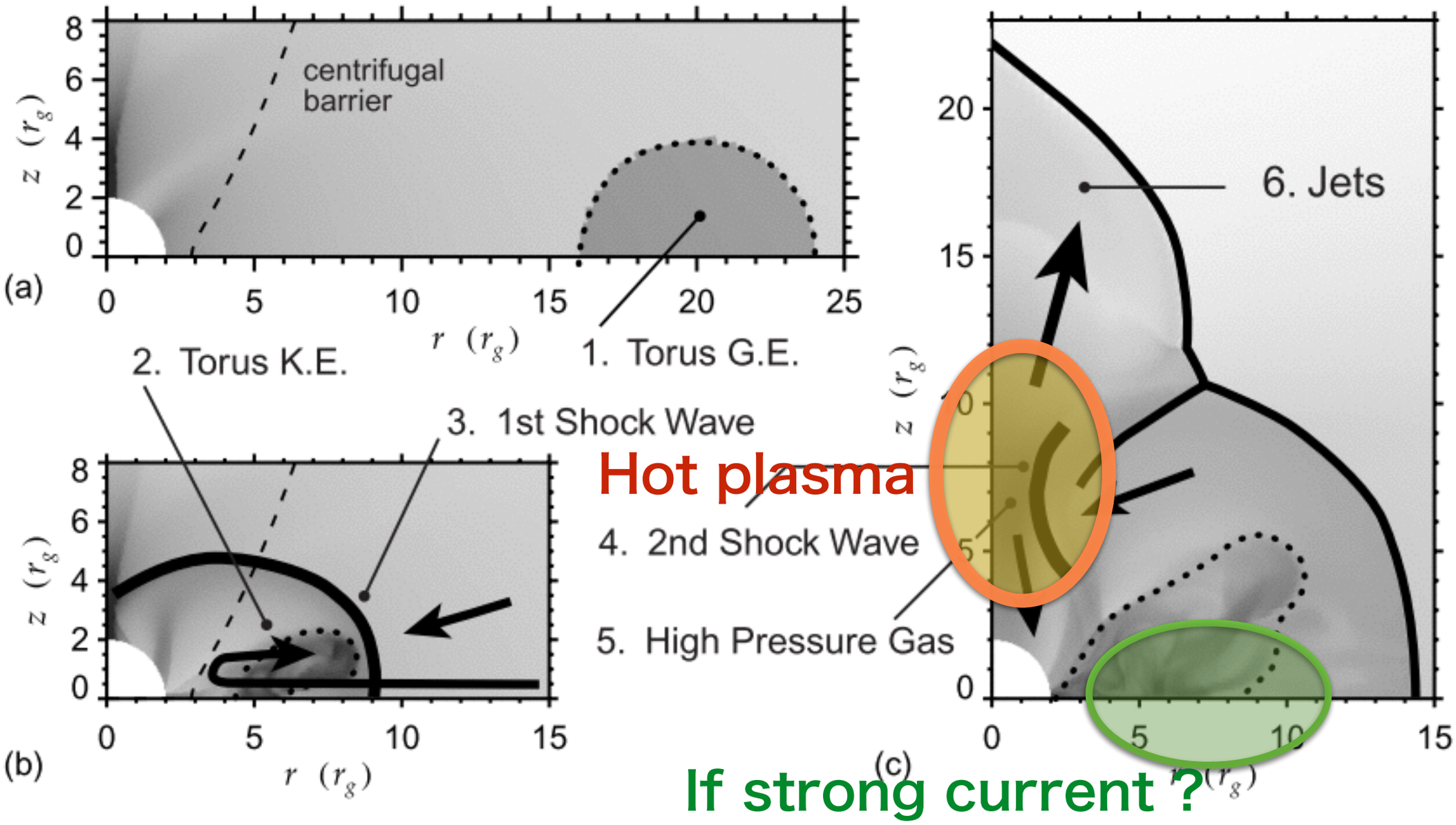
Collapse Torus

-> Shock formation (High pressure gas region)

-> Torus(near BH) + Jets

Nobta & Hanawa (1999) ApJ, 510, 614

post-Newton HD



MHD Shock case (2)

Hot-MHD Flow model

Wald's **Uniform** MF

+

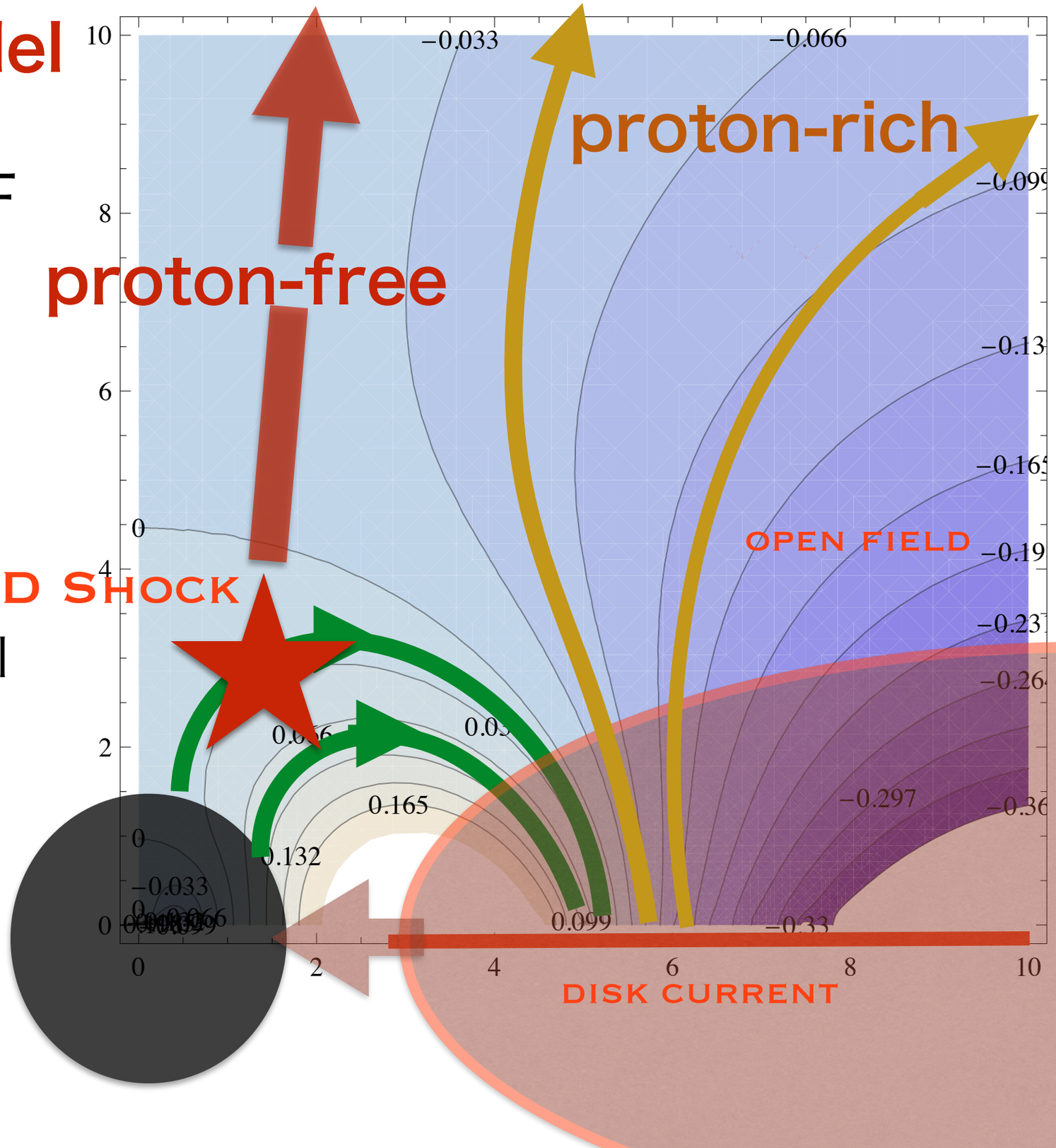
Loop & Open MF

by Disk-current

(superposition of **Loop** MF by equatorial ring-currents)

A solution of vacuum Maxwell eq.

$a = 0.8m,\ n = 20,\ r_0(r) > 3m$



角速度の θ 分布？

BZ Flux (I)

(1) Poynting-flux (r-compornant):

$$\mathcal{E}^r(r,\theta) \equiv T^r{}_t = \epsilon_0 \frac{E_\theta B_\phi}{\sqrt{-g}}$$

where $\sqrt{-g} = \Sigma \sin\theta$, $\Sigma = r^2 + a^2\cos^2\theta$ and $E_\theta = \Omega_F F_{\theta\phi} = \Omega_F A_{\phi,\theta}$.

(2) The boundary condition at the horizon : toroidal compornent of magnetic fields

$$B_\phi^H[\Psi(\theta)] = (\omega_H - \Omega_F)\frac{(r_H^2 + a^2)\sin\theta}{\Sigma_H}(A_{\phi,\theta})_H$$

(3) Blandford-Znajek Power (Poynting-flux at the horizon):

$$\mathcal{E}_H^r[\Psi(\theta)] = \epsilon_0 \Omega_F(\omega_H - \Omega_F)\frac{(r_H^2 + a^2)}{\Sigma_H^2}(A_{\phi,\theta})_H^2$$

Models for BZ-power

Strength of Mag. at EH
$B_p[A_\phi(\theta)]$

Distributions of $\Omega_F(\theta)$

A: Uniform magnetic field

$$\Omega_F(\theta) = \text{constant}$$

B: Diploe magnetic field

$$\Omega_F(\theta) = C\sin(\theta)$$

$$(i) \quad C > \omega_H$$

$$(ii) \quad C < \omega_H$$

C: Black Hole--Disk magnetic field

$$\Omega_F(\theta) = \Omega_{\rm Kepler}(r)$$

$$\theta = 0 \quad \leftrightarrow \quad r = r_{\rm max}$$

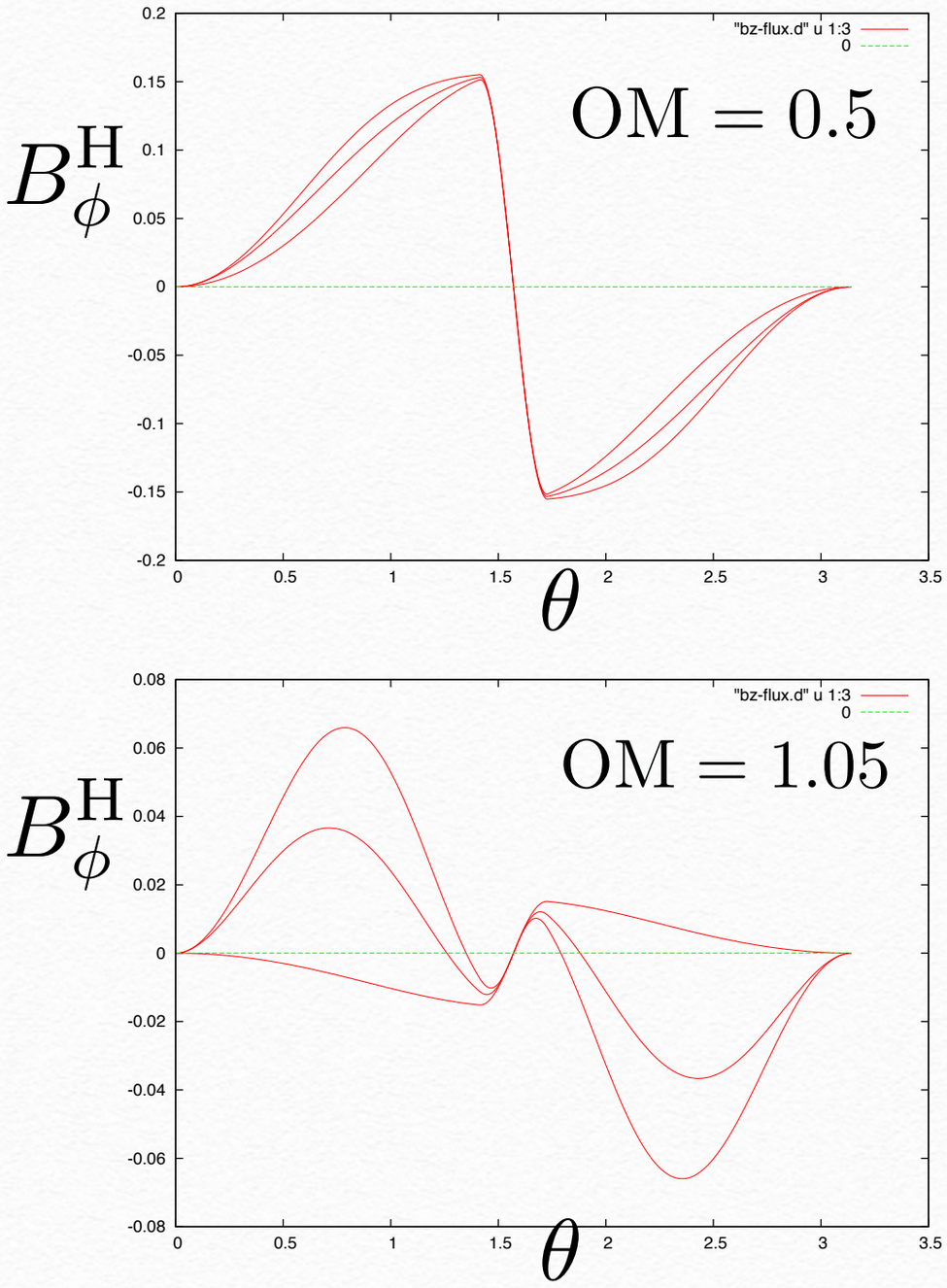
$$\theta = \pi/2 \quad \leftrightarrow \quad r = r_{\rm min}$$

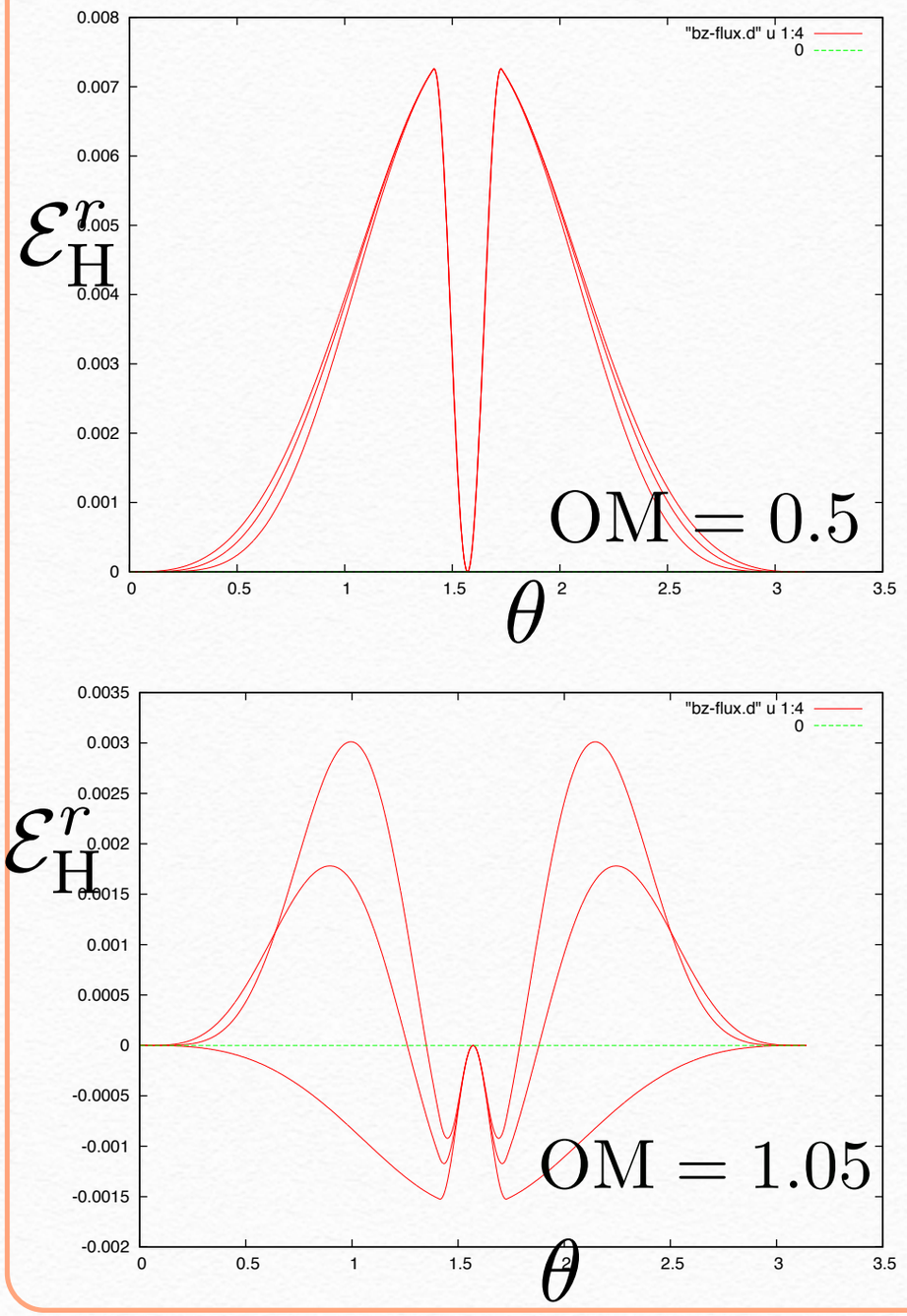
BZ Flux (3)

(i) split-monopole magnetic fields : $A_\phi(\theta) = B_0 \cos(\theta)$

$$(A_{\phi,\theta})_H/|B_0| = \sin\theta \quad (\ 0 < \theta < \theta_0 < 0.5\pi)$$
$$(A_{\phi,\theta})_H/|B_0| = \sin\theta_0 \cos[(\theta-\theta_0)/(1-2\theta_0/\pi)] \quad (\ \theta_0 < \theta < \pi - \theta_0)$$
$$(A_{\phi,\theta})_H/|B_0| = -\sin\theta \quad (\ \pi - \theta_0 < \theta < \pi)$$

$a = 0.8m$





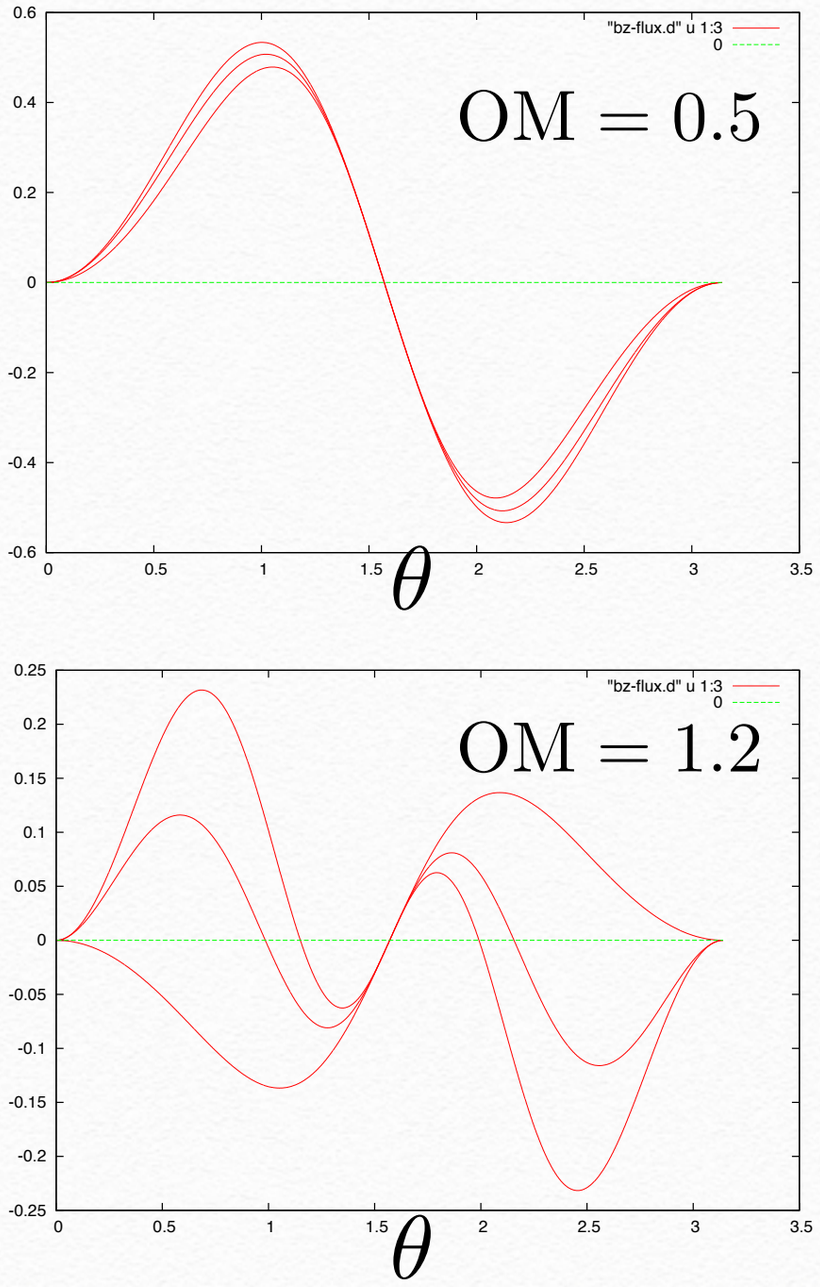
(ii) uniform magnetic fields : $A_\phi(\theta) = \frac{B_0}{2}(g_{\phi\phi} + 2a g_{t\phi}) - \frac{Q}{2M} g_{t\phi}$

$$(A_{\phi,\theta})_H = \frac{B_0}{2}(g_{\phi\phi,\theta})_H + \left(aB_0 - \frac{Q}{2M}\right)(g_{t\phi,\theta})_H$$
$$(g_{\phi\phi,\theta})_H = \frac{2}{\Sigma_H^2}(r_H^2 + a^2)^3 \sin\theta\cos\theta,$$
$$(g_{t\phi,\theta})_H = \frac{4Mar_H}{\Sigma_H^2}(r_H^2 + a^2)\sin\theta\cos\theta$$

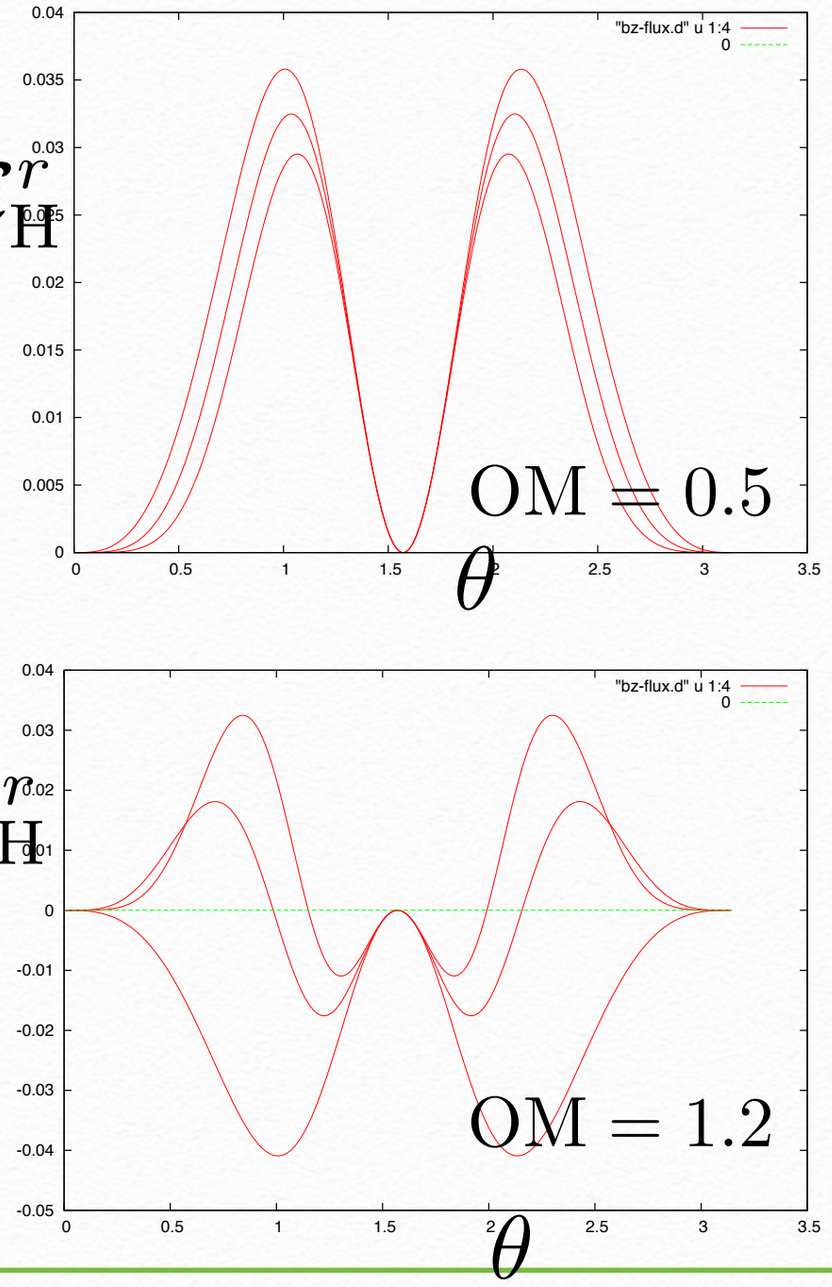
BZ Flux (4)

$a = 0.8m$

$B_\phi^{\rm H}$



$\mathcal{E}^r_{\rm H}$



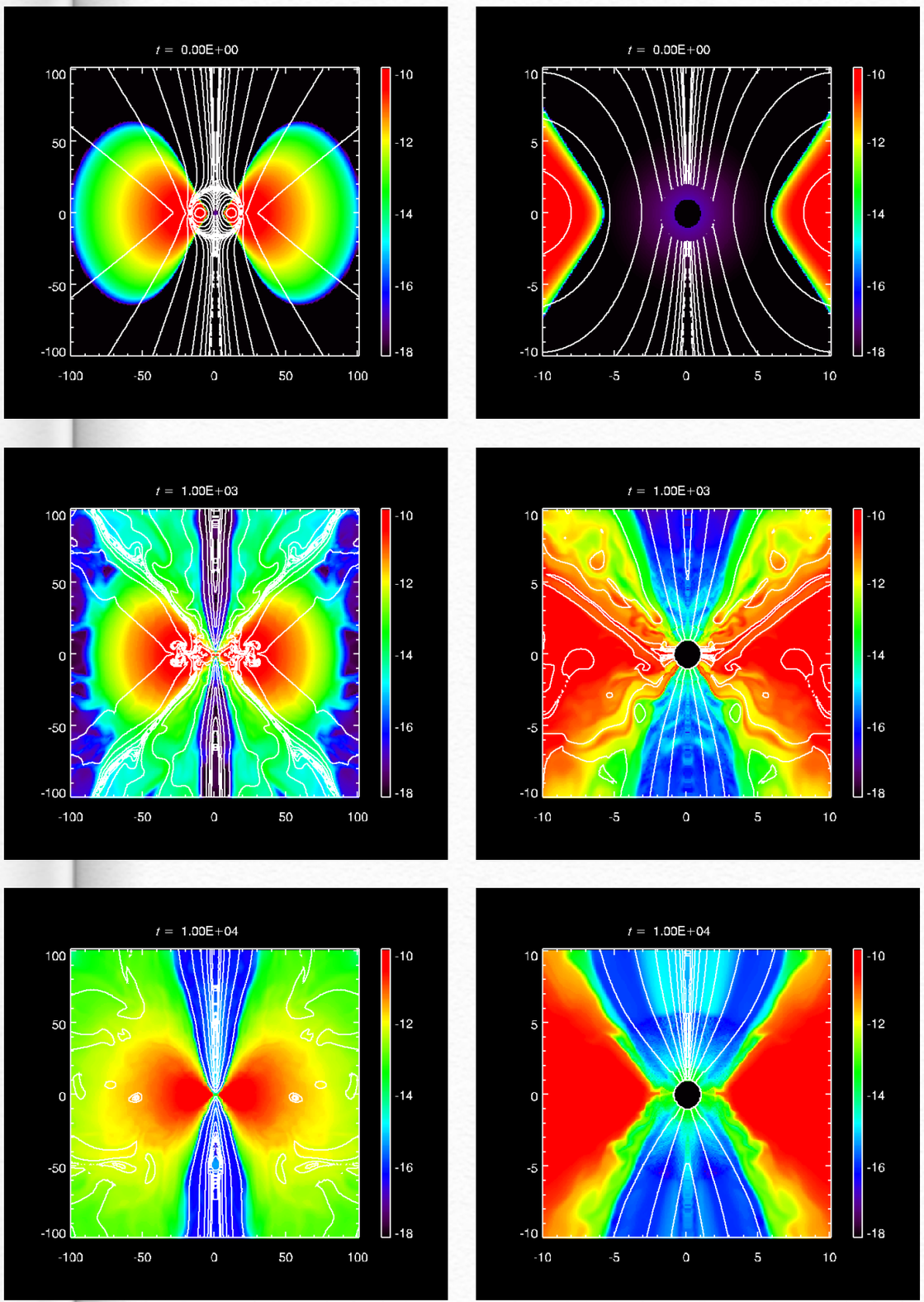
$B_\phi^{\rm H}$

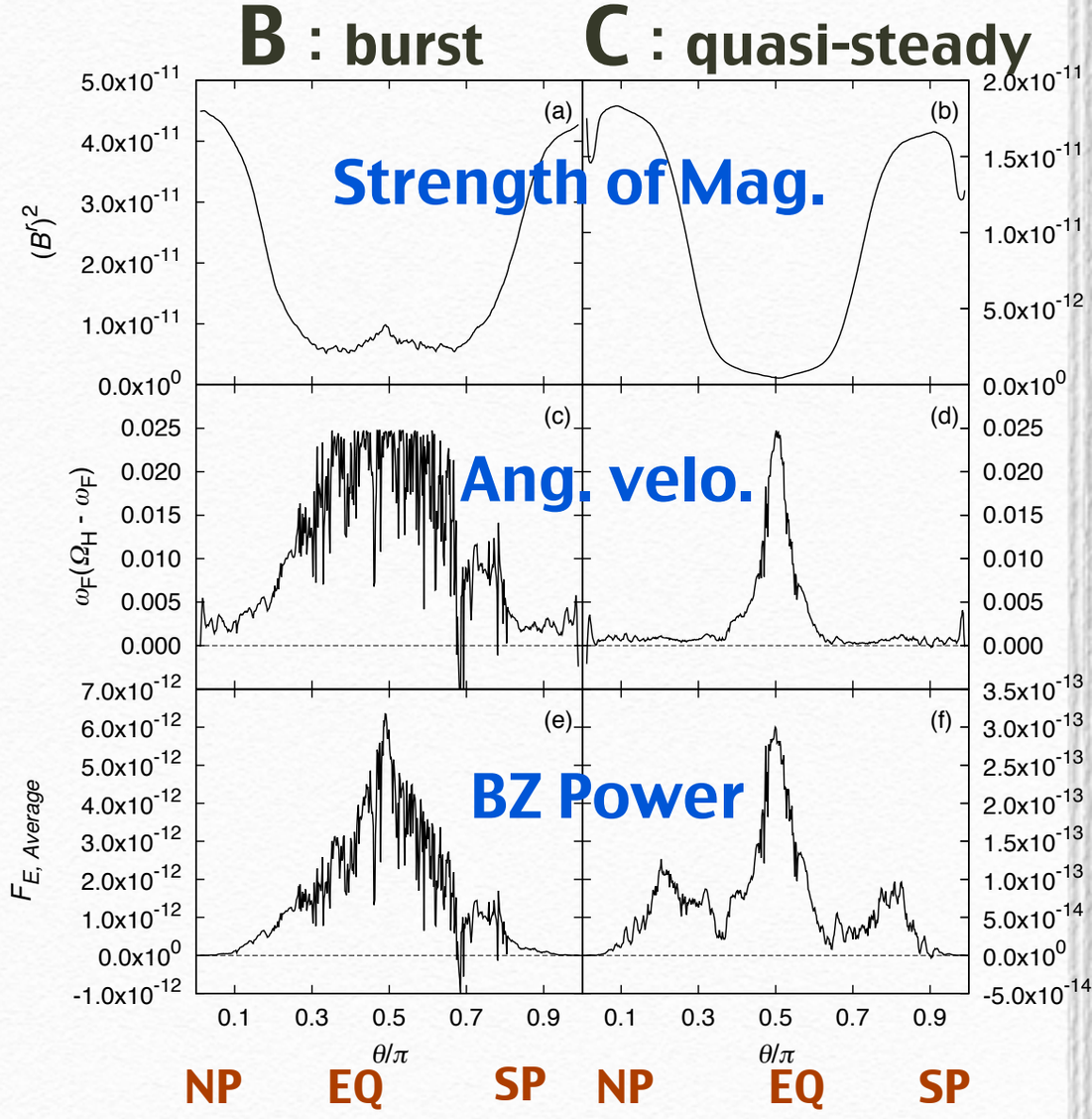
$\mathcal{E}^r_{\rm H}$

GRMHD Simulation

Maeda+ 2014

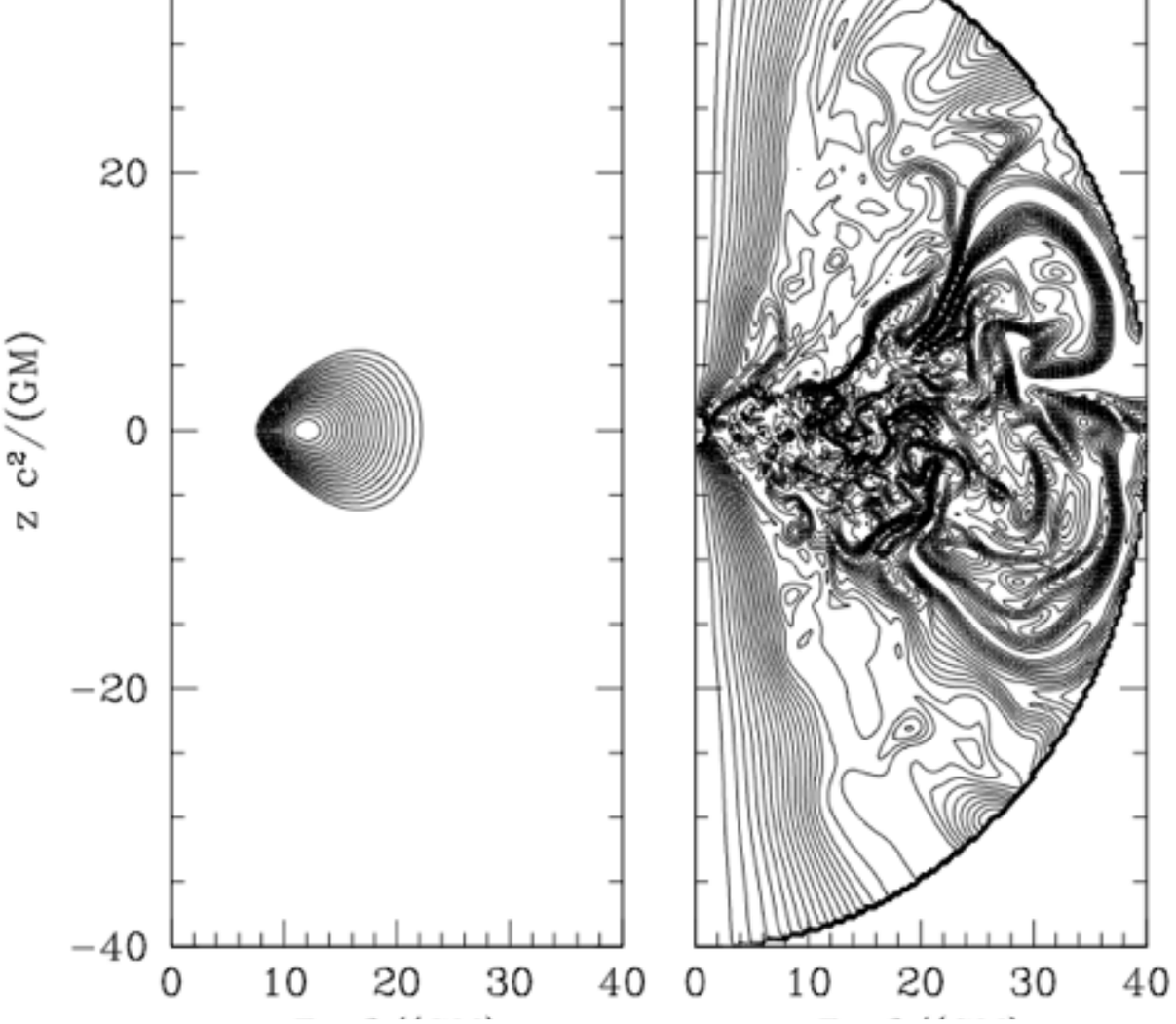
BH+Acc.Disk+Magnetosphere





Angular Distribution of EM-Flux
We can see the Max. in equatorial region **!**

BZ-Power => Effect on Disk activity !



GRMHD Simulation

McKinney & Gammie 2004

Maeda et al. とは異なる結果

磁場強度分布と角速度分布

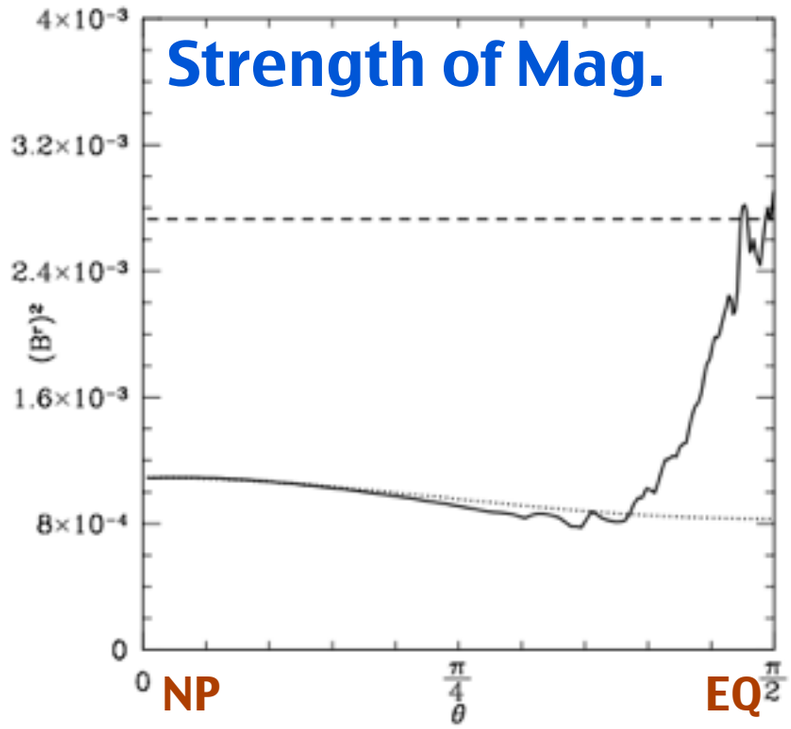


FIG. 9*a*

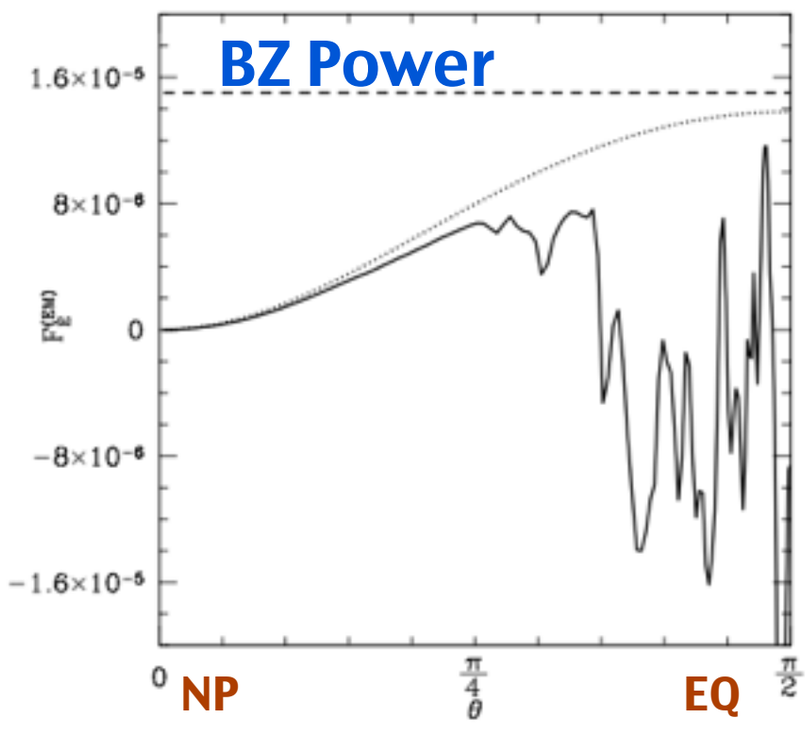
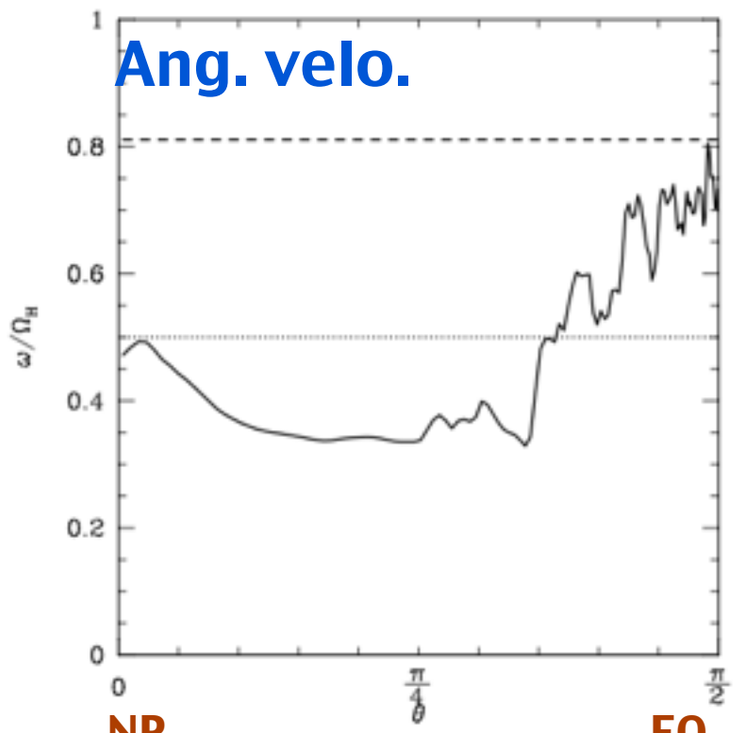
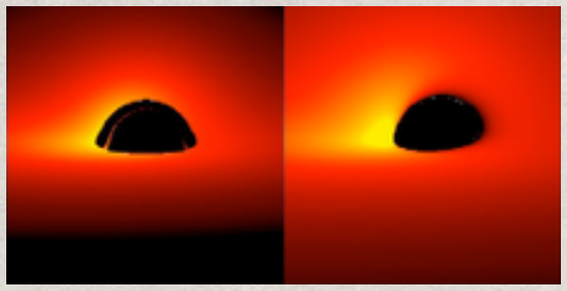


FIG. 9*b*



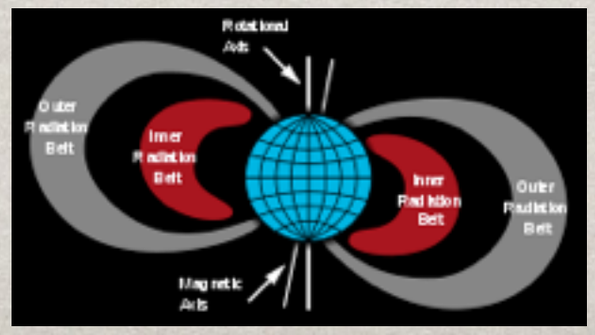
Information of the black hole space-time

Accretion Disk --- The radiation is emitted from outside of ISCO.
(radio ~ X-ray, gamma-ray)



BH - Aurora --- The **HE emission** from very close to the Event Horizon is possible.

BH - Van Allen radiation belt --- The plasma can be trapped in this zone, which may be related to a **cosmic ray** .



Appendix

Table 1: Typical values for GRB jets (Kevin's report 2014.Dec.09 ; see also, MacFadyen & Woosley 1999)

Total Luminosity of GRB (MW99, sec.7)	$\mathcal{L}_{\rm S} \sim 10^{52}$ erg/sec
Mass accretion rate	$\dot{M} \sim 1 M_\odot$/sec
Mass of BH	$M_{\rm BH} \sim 3 M_\odot$ ($r_{\rm H} \sim 10$km)
Radius of separation point	$r_{\rm sp} \sim 50$ km
Density near the rot. axis (MW99 fig.7)	$\rho \sim 10^{6-7}$g/cm^3
Number flux (MW99, figs.12 & 15)	$nv^r \sim 10^{17}$cm^{-2}sec^{-1}
$\rightarrow$ Number density	$n \sim 10^{30}$cm^{-3}
$\rightarrow$ Velocity of flow	$v^r \sim 10^{10}$cm/sec ($\sim c$)
Magnetic field strength	$\lvert B \rvert \sim 10^{15}$ G
Lorentz factor at distant region	$\gamma_\infty \sim 50$

* * *

GRMHD Flows

under given magnetic field

1. Relativistic Bernoulli equation

energy in corotation frame

gtavitational Lorentz factor

poloidal magnetic field

$$(E - \Omega_F L)^2 = \mu^2 \alpha + M^2(\alpha B_p^2 + B_\phi^2)$$

→ PLOT Solution

enthalpy
(rest mass energy + internal energy)

Alfven Mach number

$$M^2 \equiv \frac{u_p^2}{u_{\rm AW}^2}\alpha$$

toroidal magnetic field

2. total Energy of MHD Flow

$$E(\Psi) = \mu u_t - \frac{\Omega_F}{4\pi\eta} B_\phi$$

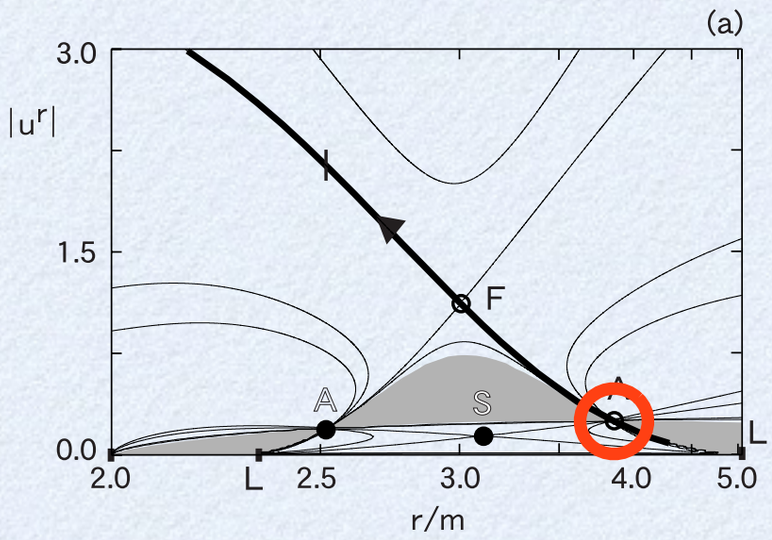
energy conversion
KE <--> ME

the fluid part of energy

Poynting flux per the particle number flux

MHD Accretion onto Black Holes

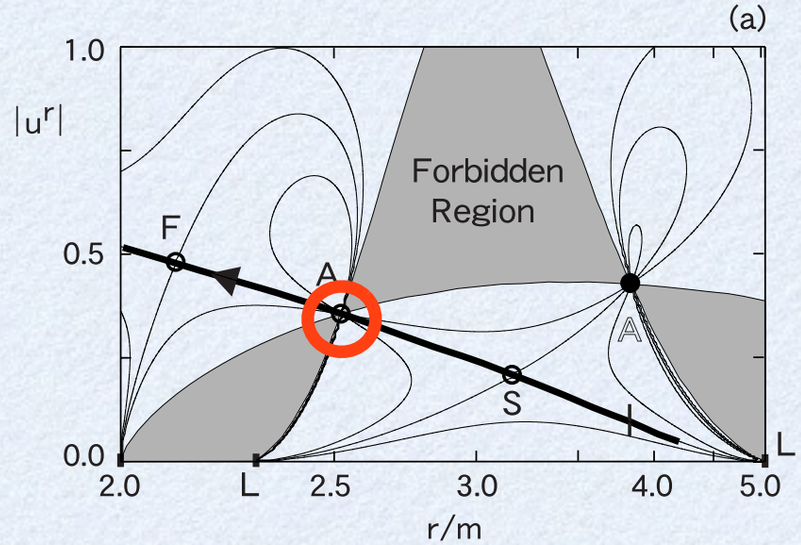
BH: slowly rotating



hydro-like

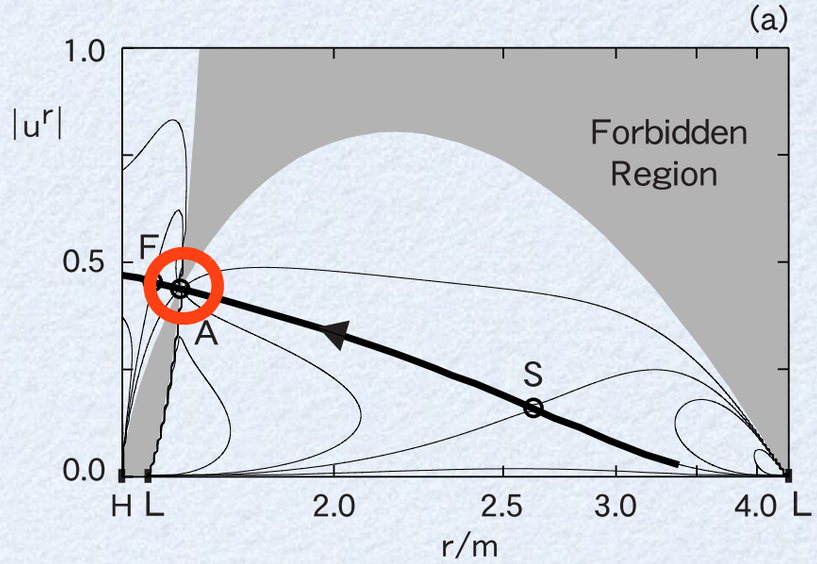
weak-magnetic field limit
=> Hydro Dynamical Flow

BH: slowly rotating



magneto-like

BH: rapidly rotating



magneto-like

strong-magnetic field limit
=> Force-free magnetosphere

Some critical points make regularity conditions.

Plasma source => Slow Point => Alfven P. => Fast P. => Event Horizon
(boundary conditions)

When the Alfven point locates inside the Ergosphere,

Energy Extraction from a Rotating BH by MHD Inflows is possible.

Takahashi + 1990

fluid part EM part

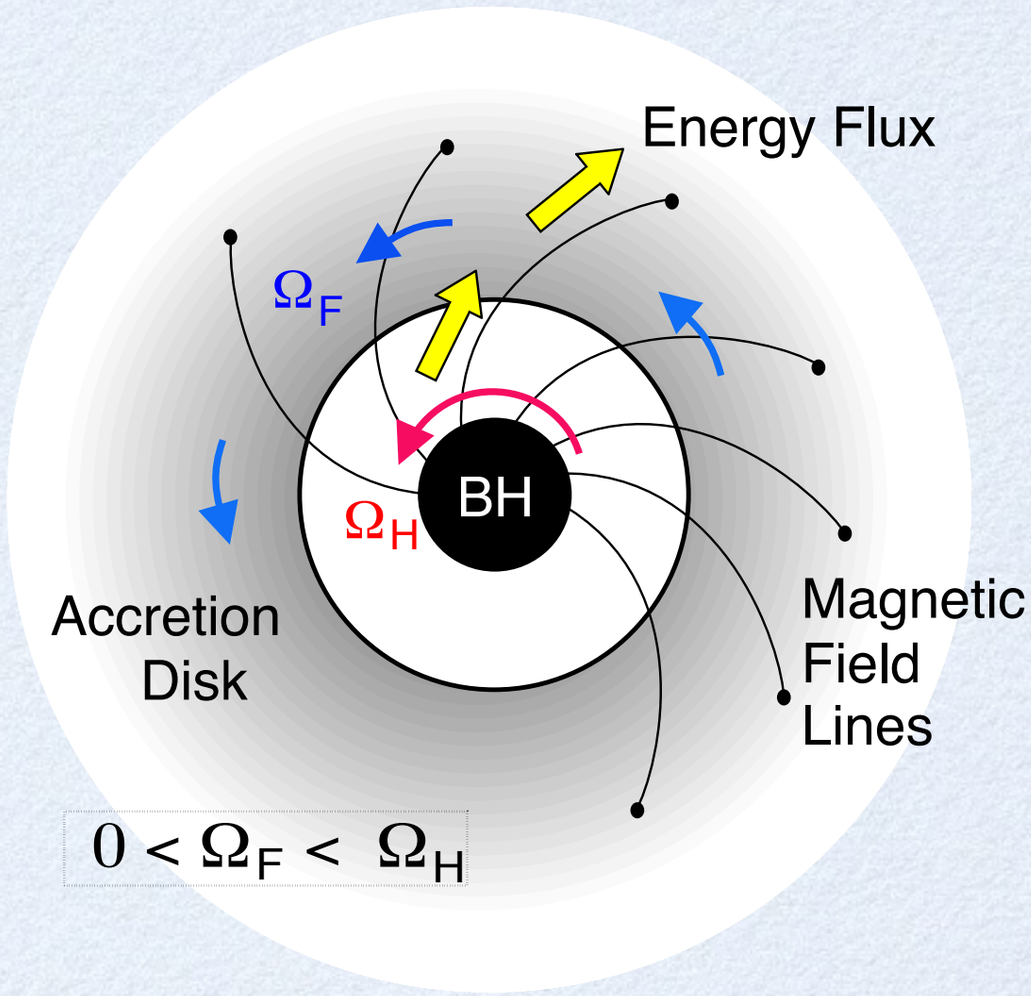
$$E(\Psi) = \mu u_t - \frac{\Omega_F}{4\pi\eta} B_\phi$$

$-$ $+/-$ $-$

Negative energy inflow

gas accretion

BZ process



This situation would be possible in the magnetically-dominated BH magnetosphere .

B : The Collimation of Jets in a trans-magnetosonic region

3. Trans-field equation

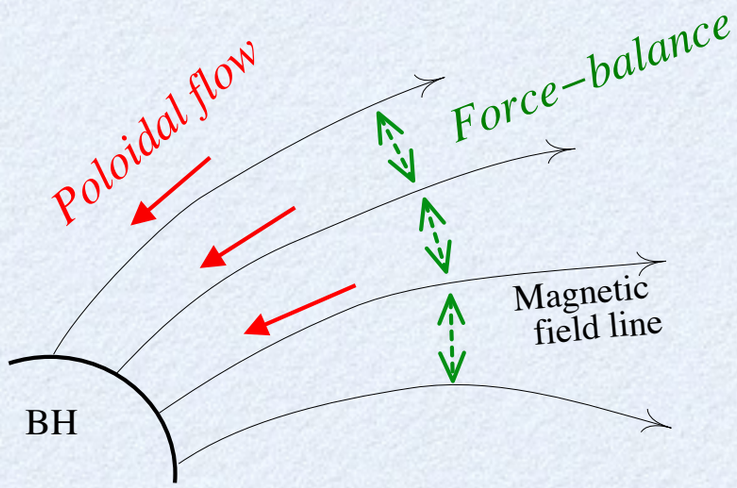
The force-balance equation is derived by Nitta et al.(1991) GRMHD

$$\frac{\alpha - M^2}{4\pi}(\partial^r\partial_r\Psi + \partial^\theta\partial_\theta\Psi) + \frac{\hat{B}_p^2\rho_w^2}{4\pi\sqrt{-g}}\left[\frac{\sqrt{-g}}{\rho_w^2}(\alpha - M^2)\right]'$$

$$+ \quad \frac{4\pi\mu^2\rho_w^2}{M^2}\eta\eta' + \frac{2\pi}{M^2}\left[g_{\phi\phi}(E^2\eta^2)' + 2g_{t\phi}(EL\eta^2)' + g_{tt}(L^2\eta^2)'\right] \qquad (7.47)$$

$$- \quad \frac{4\pi\eta^2\rho_w^2}{M^4}(\mu^2 G_\phi - e\mu u_\phi)\Omega_F' + \frac{4\pi}{M^2}\left[G_\phi(E\eta)' + G_t(L\eta)'\right](E\eta)\hat{f} \;=\; 0$$

where the prime $(') \equiv -(1/\hat{B}_p^2)[(\partial^r\Psi)\partial_r + (\partial^\theta\Psi)\partial_\theta]$, $\hat{B}_p \equiv \rho_w B_p$ and $\hat{f} \equiv \rho_w f$.



The ideal MHD plasmas stream along magnetic field lines.

The configuration of field lines is determined by the force–balance between the field.

β - model and GS equation

Takahashi & Tomimatsu (2008)

- The ratio of toroidal and poloidal magnetic field components are **assumed**.

$$\beta(r,\theta) = B_\phi / B_p$$

- Then, we can always obtain a trans-magneto-sonic flow solutions without the regularity conditions.

- It is easy to discuss accretion flows onto a black hole, which must become super-magnetosonic at the event horizon.

Approximated GS eq. & a solution

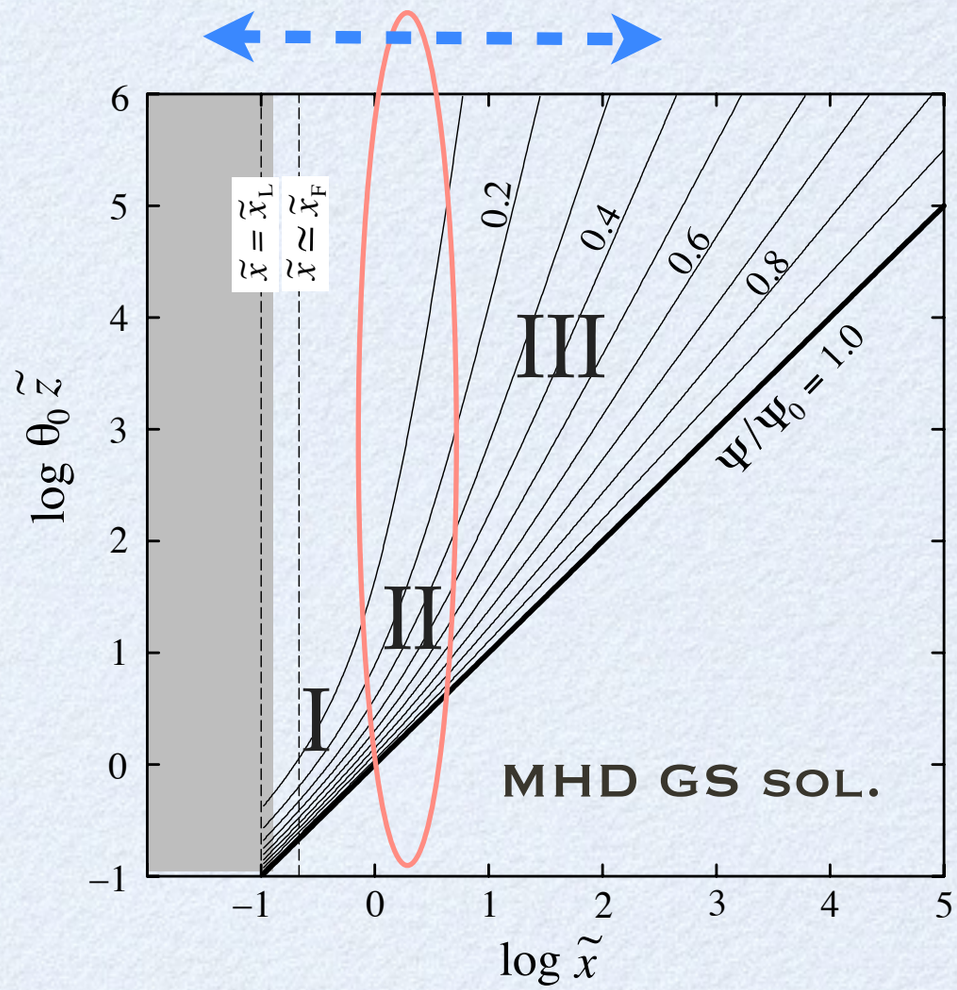
Assumptions:

- trans-magnetosonic region
  (by β-model)
- highly relativistic outflows

$$R_L \ll R < R_L E \qquad (E \gg mc^2)$$

- narrow opening angle

$$B^R/B^Z \ll 1$$



I. $E_{\rm k} \ll E_{\rm m}$ → **radial**

II. $E_{\rm k} \sim E_{\rm m}$ (equipatition)

III. $E_{\rm k} \gg E_{\rm m}$ → **jet bending**

AGN Jet

Lorentz factor $\gamma \sim 10$

Jet radius

$R_{\rm jet} \equiv \gamma R_L \sim 10 R_L$ (where $E_k \sim E_m$)

→ hydrodynamical phenomena

(e.g., shock, some kinds of plasma instability)

Light cylinder

$R_L \sim 1/\Omega_F$, $\Omega_F(\Psi) \sim \Omega_{\rm Kepler}(r_{\rm inner})$

$R_{\rm jet} \sim 100 r_s \sim 0.01{\rm pc}$, for $M_{\rm BH} \sim 10^9 M_\odot$

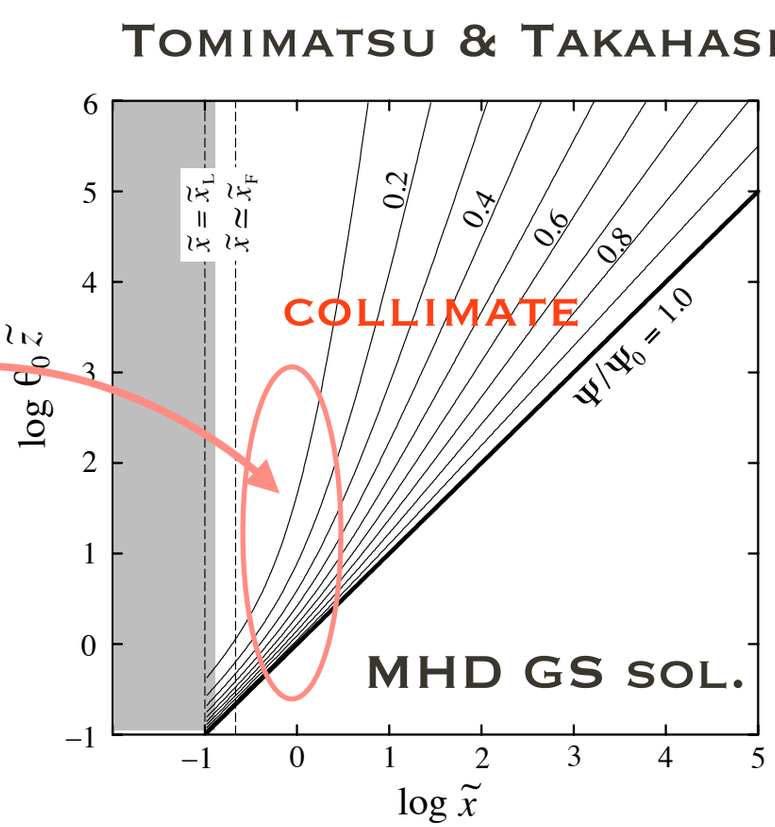
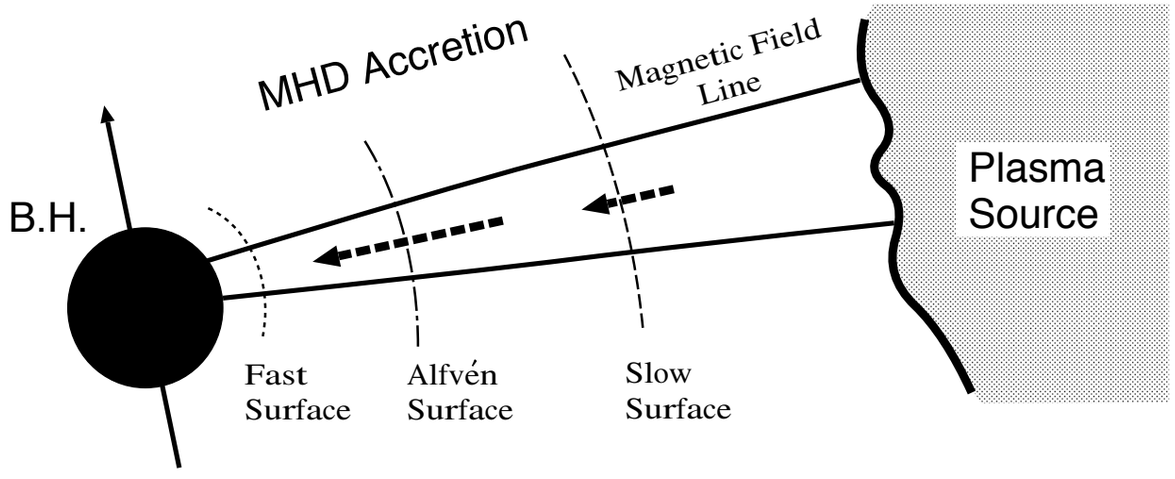
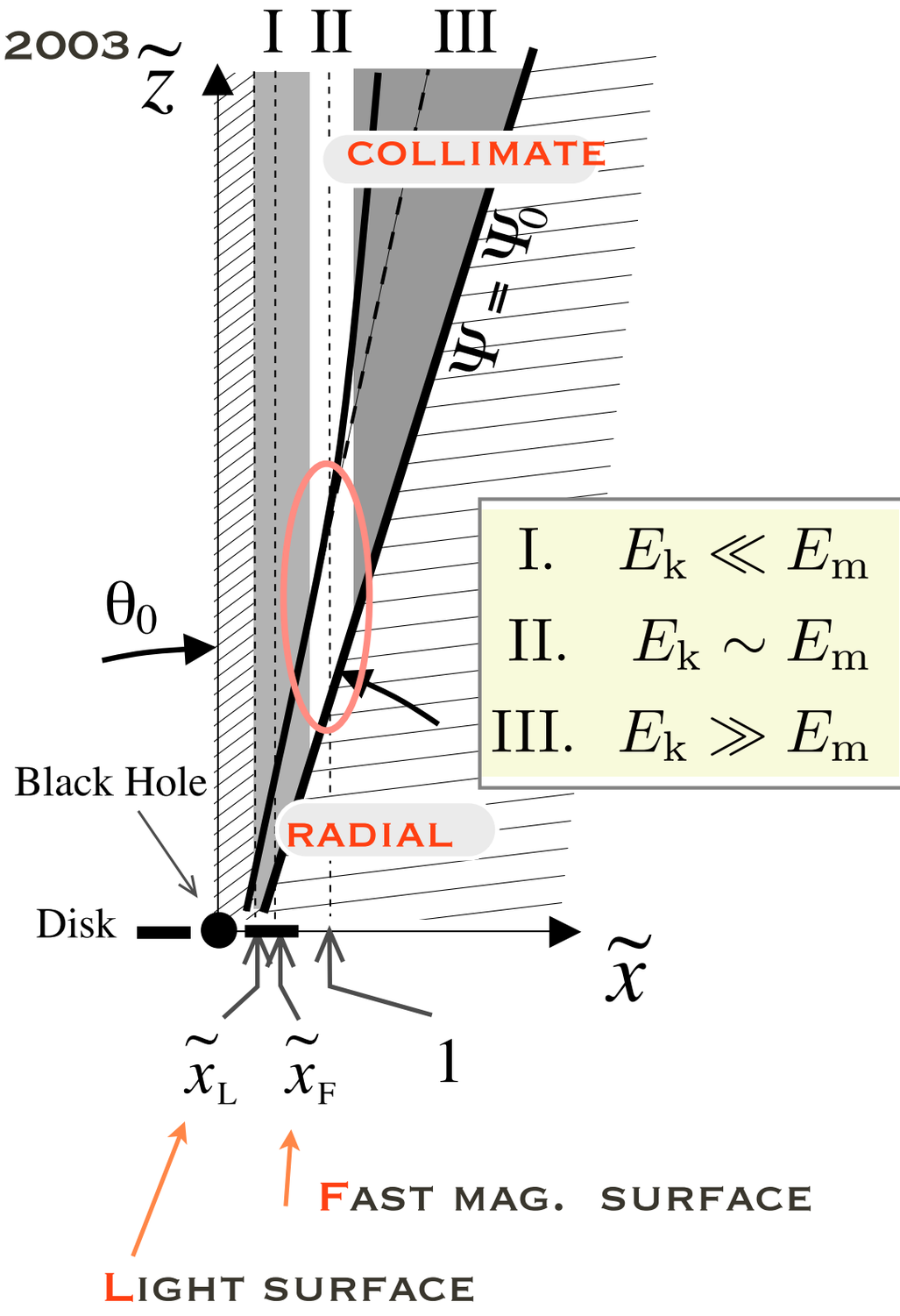
Analytical models of BH magnetosphere

trans-magnetosonic solutions

- **OUTER REGION**

at $E_{\rm k} \sim E_{\rm m}$ (equipatition)

→ **jet bending**



- **INNER REGION**