Black hole vicinity: collisional Penrose process photon sphere/sonic point correspondence

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Y. Koga and T. Harada, arXiv:1601.07290
K. Ogasawara, T. Harada and U. Miyamoto, arXiv:1511.00110
M. Patil and T. Harada, arXiv:1510.08205

Outline

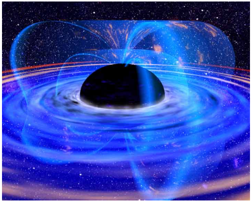
Outline

BH vicinity

- Gravity near BHs is highly general relativistic.
- Phenomena unique to BHs may be observed.
- Not completely understood yet.
- We focus on the following three phenomena:
  - High-energy particle collision (rapidly rotating BHs)
  - Collisional Penrose process (rapidly rotating BHs)
  - Photon sphere/sonic point correspondence (static spherically symmetric BH)

Outline

Rotating BHs as particle accelerators

 as 

- Kerr BHs act as particle accelerators (Bañados, Silk & West 2009, Piran, Shaham & Katz 1975): The CM energy of colliding particles can be unboundedly high near the horizon.
- Not only microscopic particles but also macroscopic objects: compact BHs and stars are accelerated by SMBHs.

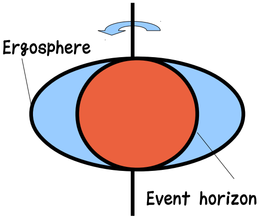
Kerr spacetime

- Kerr metric

$$ds^2 = -\left(1-\frac{2Mr}{\rho^2}\right)dt^2 - \frac{4Mar\sin^2\theta}{\rho^2}d\phi dt + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \left(r^2+a^2+\frac{2Mra^2\sin^2\theta}{\rho^2}\right)\sin^2\theta d\phi^2,$$

where $\rho^2 = r^2 + a^2\cos^2\theta$ and $\Delta = r^2 - 2Mr + a^2$.

- Horizon: $r_\pm = M \pm \sqrt{M^2-a^2}$
- Ergosphere: $r_E = M + \sqrt{M^2 - a^2\cos^2\theta}$
- Angular velocity: $\Omega_H = a/(r_H^2 + a^2)$
- Extremal: $a_* := a/M = 1$: $r_H = M$, $r_E = M(1+\sin^2\theta)$, $\Omega_H = 1/(2M)$



Geodesic motion in the equatorial plane

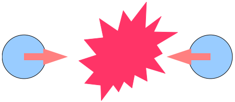
- Conserved quantities: $E = -p_t = -(\partial_t)^a p_a$, $L = p_\phi = (\partial_\phi)^a p_a$, where p^a is the four-momentum with $m^2 = -p_a p^a$.
- The geodesic eqs are reduced to a 1D potential problem

$$\frac{1}{2}\dot{r}^2 + V(r) = 0,$$
$$V(r) = -\frac{m^2 M}{r} + \frac{L^2 - a^2(E^2 - m^2)}{2r^2} - \frac{M(L - aE)^2}{r^3} - \frac{1}{2}(E^2 - m^2),$$

where the dot is the derivative w.r.t. the affine parameter.

- The condition $\dot{t} > 0$ near the horizon is reduced to $E - \Omega_H L \geq 0$.
- We call particles with $E - \Omega_H L = 0$ *critical particles* and $L_c := E/\Omega_H$ *critical angular momentum*.

CM energy of colliding particles



- CM energy: the total energy of two particles at the same spacetime point observed in the centre-of-mass frame

$$p^a_{\rm tot} = p^a_1 + p^a_2, \quad E^2_{\rm cm} = -p^a_{\rm tot} p_{{\rm tot}\,a}.$$

- For the Kerr BH in the equatorial plane

$$\begin{aligned} E^2_{\rm cm} &= m_1^2 + m_2^2 \\ &\quad + \frac{2}{r^2}\left[\frac{P_1 P_2 - \sigma_1\sigma_2\sqrt{R_1}\sqrt{R_2}}{\Delta} - (L_1 - aE_1)(L_2 - aE_2)\right], \\ P_i(r) &= (r^2 + a^2)E_i - L_i, \\ R_i(r) &= P_i^2(r) - \Delta(r)[m_i^2 r^2 + (L_i - aE_i)^2]. \end{aligned}$$

CM energy for near-horizon collision

- $E_{\rm cm}$ in the limit to $r \to r_H$ for noncritical particles
  - "Rear-end" collision (most likely to occur): $\sigma_1\sigma_2 = 1$

$$\begin{aligned} E_{\rm cm}^2 &= m_1^2 + m_2^2 - 2\frac{(L_1 - aE_1)(L_2 - aE_2)}{r_H^2} \\ &\quad + \frac{m_1^2 r_H^2 + (L_1 - aE_1)^2}{r_H^2}\frac{E_2 - \Omega_H L_2}{E_1 - \Omega_H L_1} + (1 \leftrightarrow 2). \end{aligned}$$

  Finite except in the limit $E_i - \Omega_H L_i \to 0$ (critical condition).
  - "Head-on": $\sigma_1\sigma_2 = -1$, "Side": $\sigma_1\sigma_2 = 0$

$$E_{\rm cm} \propto \frac{1}{\sqrt{\Delta}} \propto \begin{cases} (r - r_H)^{-1/2} & (|a| < M) \\ (r - r_H)^{-1} & (|a| = M). \end{cases}$$

- Collision of critical and subcritical particles

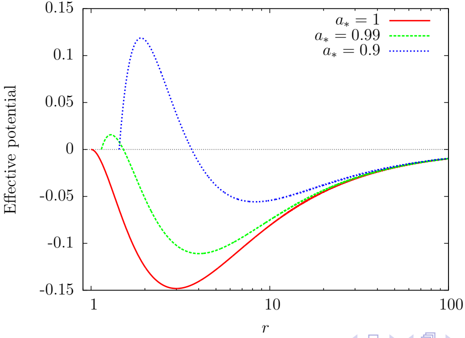
$$E_{\rm cm} \propto \frac{1}{\sqrt{\Delta}} \propto (r - r_H)^{-1/2} \quad (|a| = M).$$

Motion of critical particles

- A massive particle which was at rest at infinity can reach the horizon if $l = L/(mM)$ satisfies

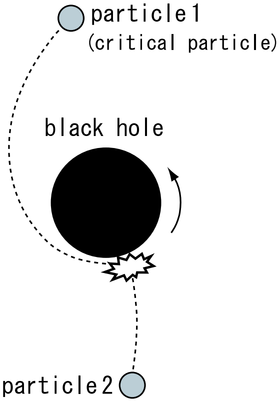
$$-2(1+\sqrt{1+a_*}) = l_L < l < l_R = 2(1+\sqrt{1-a_*}).$$

- $l_R \leq l_c$, where $l_R = l_c(=2)$ only for $a_* = 1$; a critical particle reaches the horizon only for the extremal BH.



Banados-Silk-West process

- A particle with $l_L < l < l_R = l_c = 2$ can reach the horizon of an extremal Kerr BH from infinity.
- $E_{\rm cm}$ of particles 1 and 2 for the near-horizon collision diverges in the limit $l_1 \to 2$ (or $l_2 \to 2$). (Bañados, Silk & West 2009).
- For $l_1 = 2$ and $l_L < l_2 < 2$, $E_{\rm cm} \propto (r - r_H)^{-1/2}$ (Grib & Pavlov 2010).
- Necessary to finetune the angular momentum. Natural finetuning by the ISCO (Harada & Kimura 2011).



A physical explanation

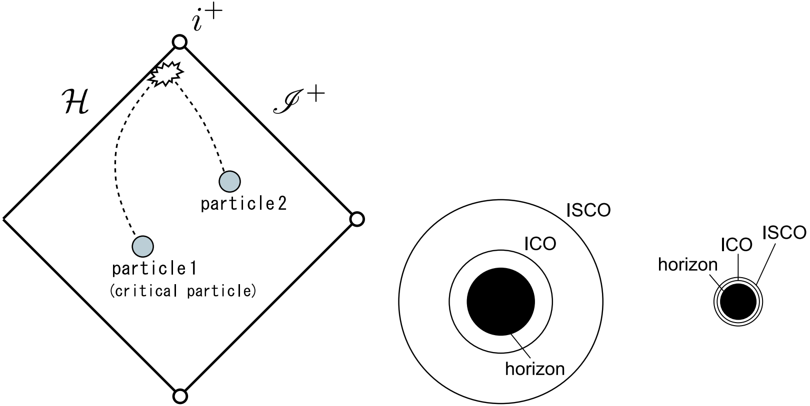
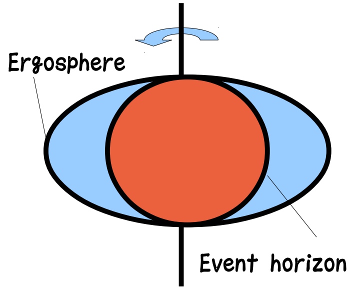


Figure: An infalling subcritical particle is accelerated to the light speed. If an observer can stay at a constant radius near the horizon, he or she will see the particle falling with almost the speed of light. (Harada & Kimura 2014, Zaslavskii 2011)

Outline

Ergoregion in the Kerr spacetime



- The Killing vector $(\partial_t)^a$, which is future-pointing timelike and normalised in the asymptotic region, becomes spacelike in the ergoregion, where

$$M+\sqrt{M^2-a^2}=r_H<r<r_E=M+\sqrt{M^2-a^2\cos^2\theta}.$$

- The conserved energy $E:=-(\partial_t)^a p_a$, which is positive if $(\partial_t)^a$ is future-pointing timelike, can be negative in the ergoregion.

Collisional Penrose process

- A negative energy particle is possible in the ergoregion. This enables us to extract energy from the BH.
- High-energy collision may produce superheavy and/or superenergetic particles.
- Collisional Penrose process (Piran, Shaham & Katz 1975)
  $\eta = E_3/(E_1 + E_2)$ can be larger than unity.

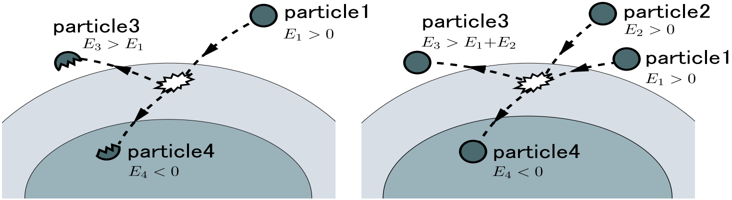


Figure: Left: Penrose process, Right: CPP

Conservation laws and escape to infinity

- $E_1 + E_2 = E_3 + E_4, \quad L_1 + L_2 = L_3 + L_4$
- $p_1^r + p_2^r = p_3^r + p_4^r$
- Can the ejecta espape to infinity?
- We will focus on the extreme rotation $a = M$.

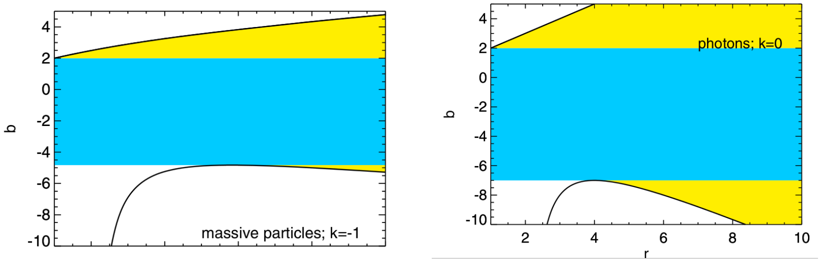
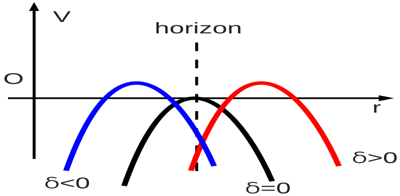


Figure: Turning points for an extremal Kerr BH for the impact parameter b (Taken from Schnittman 2014)

Near-critical particle near the horizon

- A sequence of particles: $L = 2ME(1+\delta)$, where $\delta = \delta_{(1)}\epsilon + O(\epsilon^2)$, at $r = M/(1-\epsilon)$. $\dot{t} > 0$ at $r = M/(1-\epsilon)$ yields $\delta < \epsilon + O(\epsilon^2)$.
- Turning points of the potential

$$r_{t,\pm}(e) = M\left(1 + \frac{2e}{2e \mp \sqrt{e^2+1}}\delta_{(1)}\epsilon\right) + O(\epsilon^2), \quad \text{where} \quad e = E/m.$$



- To escape to infinity from $r = M/(1-\epsilon)$, we need
  (a) $e \geq 1$, $\delta_{(1)} < 0$ and $\sigma(:= \mathrm{sign}\, u^r) = 1$
  (b) $e \geq 1$, $\delta_{(1)} > 0$ and $r \geq r_{t,+}(e)$, i.e., $\delta_{(1)} \leq \delta_{(1)\max}(e)$

Upper limits on the efficiency

(a) Ingoing critical+ ingoing subcritical: $\eta_{\rm max} \simeq 1$ (Jacobson & Sotiriou 2010)

(b) Ingoing critical+ ingoing subcritical: $\eta_{\rm max} \simeq 1.4$ (Bejger et al. 2012, Harada, Nemoto & Miyamoto 2012). The ejecta is bounced back.

(c) Outgoing critical + ingoing subcritical: $\eta_{\rm max} = (2+\sqrt{3})^2 \simeq 14$ (Schnittman 2014, Leiderschneider & Piran 2016)

- $\eta \simeq 14$ requires the production of a heavy particle ($m_4 \sim (r - r_H)^{-1/2}$). (Ogasawara, Harada & Miyamoto 2016)

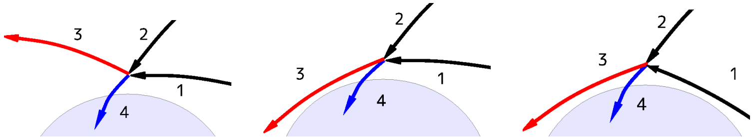


Figure: Left: (a), Middle: (b), Right: (c)

Super Penrose process

(d) Outgoing subcritical+ ingoing subcritical: $\eta_{\max} = \infty$ (Berti, Brito & Cardoso 2015)

- Is an outgoing subcritical particle physically motivated? If we consider a preceding collision to produce an outgoing subcritical particle, the total efficiency is less than 14. (Leiderschneider & Piran 2015)

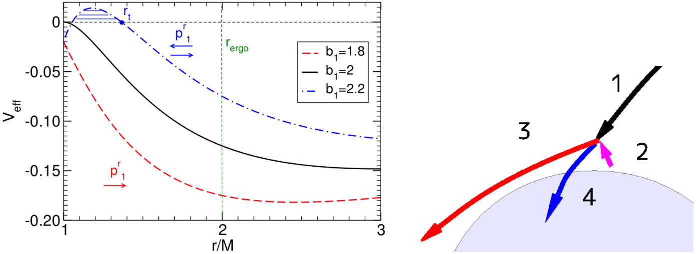
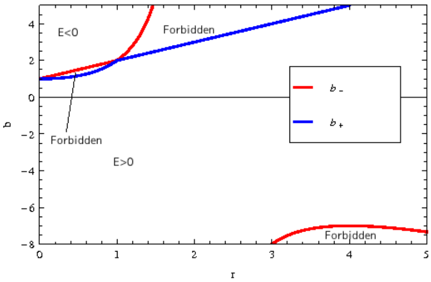


Figure: Left: Effective potential (Taken from Berti, Brito & Cardoso 2015), Right: Super-Penrose process

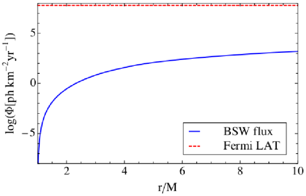
CPP for $a > M$ (overspinning Kerr)



- Outgoing subcritical particles arise after the bounce inside $r = M$. Thus, a head-on collision at $r = M$ naturally occurs with $E_{\rm cm} \propto (a - M)^{-1/2}$. (Patil & Joshi 2011)
- Particles with positive energy at $r = M$ eventually escape to infinity, enabling high efficiency, i.e., $\eta \propto (a - M)^{-1/2}$. (Patil & Harada 2016)

Implications to observation

- Near-extremal BH spins are "measured" ($a_* > 0.98$, see McClintock et al. 2011).
- The BSW flux from dark matter annihilation is too low for the Fermi satellite detection (McWilliams 2013).



- The observational implications of Schnittman's process and super Penrose process are unclear.
- The CPP around the near-exremal overspinning Kerr is observationally interesting if it exists.

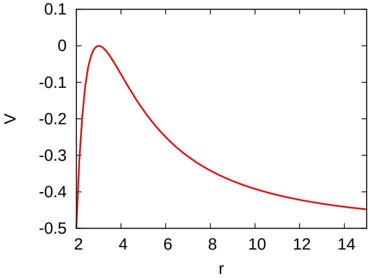
Outline

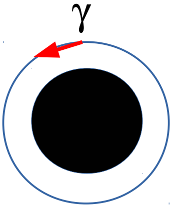
Photon sphere in Schwarzschild

- Schwarzschild spacetime

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2.$$

- Photon sphere: There is a unique circular orbit at $r = r_{ph} = 3M$, which is unstable.

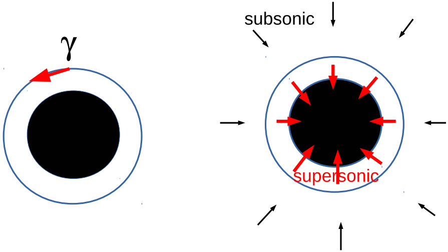




- $r_{ph} = 3M$ is also the case for the Sch-dS.

Sonic point in Schwarzschild

- Stationary spherical flow (Michel(1972), Bondi (1952))
  A sonic point lies on the sphere of $r_s = (1+3v_s^2)/(2v_s^2)M$, where $v_s = \sqrt{(dp/d\rho)_s}$ is the sound speed. For a radiation fluid $p = \rho/3$, for which $v_s = 1/\sqrt{3}$, we obtain $r_s = 3M$.



- $r_s = 3M$ is also the case for the Sch-dS (cf. Mach, Malec & Karkowski (2013)).
- None has ever pointed out the correspondence $r_{ph} = r_s$ in the Sch and the Sch-dS in literature.

Generalisation

- Static spherically symmetric spacetime in arbitrary dimensions

$$ds^2 = -f(r)dt^2 + g(r)dr^2 + r^2 d\Omega_{D-2}^2,$$

where we assume $f(r) > 0$ and $g(r) > 0$.

- Application
  - Electrically charged BHs
  - BHs with cosmological constant
  - Nonvacuum BHs
  - Hairy BHs
  - Higher dimensional BHs in GR or supergravity
  - Nonvacuum exterior of an isolated star
  - BHs or isolated stars in modified gravity
  - BH mimickers with a photon sphere
  - ...

Theorem

Theorem (Correspondence of a sonic point with a photon sphere)

For any stationary and spherically symmetric physical transonic accretion flow of an ideal photon gas, its sonic point is located at (one of) the unstable photon sphere(s).

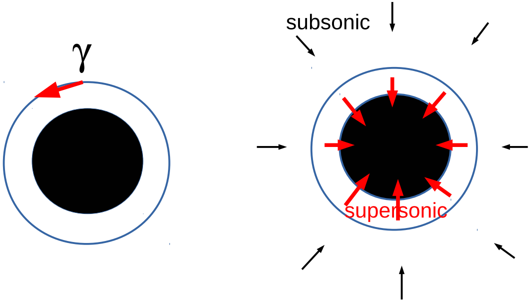


Figure: The theorem tells $r_{ph} = r_s$.

Photon sphere

- Effective potential for a free massless particle

$$\frac{1}{2}\dot{r}^2 + V(r) = 0, \quad V(r) := -\frac{1}{2gf}\left[E^2 - \frac{L^2 f}{r^2}\right]$$

- Photon sphere: $r = r_{ph}$, where $V(r) = V'(r) = 0$.

$$(r^{-2}f)' = 0 \quad \text{and} \quad b^2 \left(:= \frac{L^2}{E^2}\right) = \frac{r_{ph}^2}{f(r_{ph})}.$$

- Stability: $V''(r_{ph}) > 0 \Rightarrow$ Stable, $V''(r_{ph}) < 0 \Rightarrow$ Unstable

$$(r^{-2}f)'' > 0 \Rightarrow \text{Stable}, \quad (r^{-2}f)'' < 0 \Rightarrow \text{Unstable}$$

Stationary spherical flow

- Basic equation
  - Perfect fluid $T^{ab} = nhu^a u^b + pg^{ab}$
  - First law: $dh = Tds + dp/n$
  - Conservation law: $\nabla_a T^{ab} = 0$
  - Number conservation: $\nabla_a (nu^a) = 0$
  - Stationarity: $\partial_t Q = 0$
- Constants of integration (cf. Chaverra & Sarbach (2015))
  - Number flux: $j_n(r, n) =: 4\pi\mu$
  - Energy flux: $j_\epsilon(r, n)$
  - Energy square per particle in place of $j_\epsilon(r, n)$

$$F_\mu(r, n) := \left(\frac{j_\epsilon(r, n)}{j_n(r, n)}\right)^2 = h^2(n)\left[f(r) + \frac{\mu^2}{r^{2(D-2)}n^2}\right]$$

  - An accretion flow with $j_n = 4\pi\mu$ is given by a contour of $F_\mu(r, n)$.

Critical point

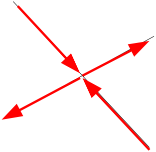
- A contour of $F_\mu(r,n)$ can be recasted in the system of a Hamiltonian flow of $F_\mu(r,n)$:

$$\frac{d}{d\lambda}\begin{pmatrix} r \\ n \end{pmatrix} = \begin{pmatrix} \partial_n \\ -\partial_r \end{pmatrix} F_\mu(r,n).$$

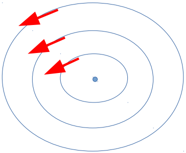
- Critical point $r = r_c$: $\partial_n F_\mu = \partial_r F_\mu = 0$
- Linearisation around the critical point

$$\frac{d}{d\lambda}\begin{pmatrix} \delta r \\ \delta n \end{pmatrix} = \begin{pmatrix} \partial_r\partial_n F_\mu & \partial_n^2 F_\mu \\ -\partial_r^2 F_\mu & -\partial_r\partial_n F_\mu \end{pmatrix}\begin{pmatrix} \delta r \\ \delta n \end{pmatrix}$$

- The critical points are saddle or extremum.



Saddle



Extremum

Ideal photon gas = radiation fluid

Lemma (EOS of an ideal photon gas)

The EOS for an ideal photon gas is given by $p = \rho/(D-1)$, where ρ is the energy density. Then, the enthalpy h as a function of the number density n is given by $h = (k\gamma/(\gamma-1))n^{\gamma-1}$, where k is a positive constant and $\gamma = D/(D-1)$.

- A radiation fluid is one of the basic matter fields, where local thermal equilibrium is realised for a gas of photons with weak interactions.

Sonic point

- Fluid velocity with respect to the static observer:

$$u^\mu = \frac{1}{\sqrt{1-v^2}}(e^\mu_{(0)} + v e^\mu_{(1)}),$$

where

$$e_{(0)} := f^{-1/2}\frac{\partial}{\partial t}, \quad e_{(1)} := g^{-1/2}\frac{\partial}{\partial r}.$$

- Transonic flow and sonic point

$$\begin{aligned} v &< v_s \quad (r > r_s), \\ v &= v_s \quad (r = r_s), \\ v &> v_s \quad (r < r_s). \end{aligned}$$

Sketch of the proof for the correspondence theorem

1. A sonic point in the physical (= with finite density gradient) transonic flow corresponds to a critical point which is saddle.
2. A critical point of the flow of an ideal photon gas satisfies $(r^{-2}f)' = 0$ and is saddle (extremum) if $(r^{-2}f)'' < 0$ (> 0).
3. A critical point of the flow of an ideal photon gas is located on the photon sphere. A saddle (extremum) point lies on the unstable (stable) photon sphere.

Discussion

- Physical mechanism
  - Just a coincidence? If not, what is a physical reason?
  - Is there any correspondence between a geodesic motion of a photon and a sound wave of radiation fluid.
- Application
  - Does the present accretion flow describe the accretion of radiation in astrophysics or CMB?
  - Hairy BHs: coloured BHs, BBMB BHs, Non-Abelian BHs, ...
- Generalisation
  - Hyperbolic symmetry, cylindrical symmetry, axial symmetry
  - If $\gamma \neq D/(D-1)$? Massive particles?
  - Accretion disks?

Outline

Summary

1. **High-energy particle collision near a near-extremal Kerr is robust as it is founded on the basic properties of geodesic orbits. No fine-tuning is needed for the overspinning Kerr.**
2. **The upper limit of the energy efficiency of the CPP has been revised from $\simeq 1$ to $\simeq 1.4$, $\simeq 14$, and even ∞. It is unbounded for the overspinning Kerr.**
3. **The correspondence of a sonic point of a radiation accretion flow with a photon sphere is seen not only in the 4D Sch but also in general static spherically symmetric spacetime in arbitrary dimensions.**