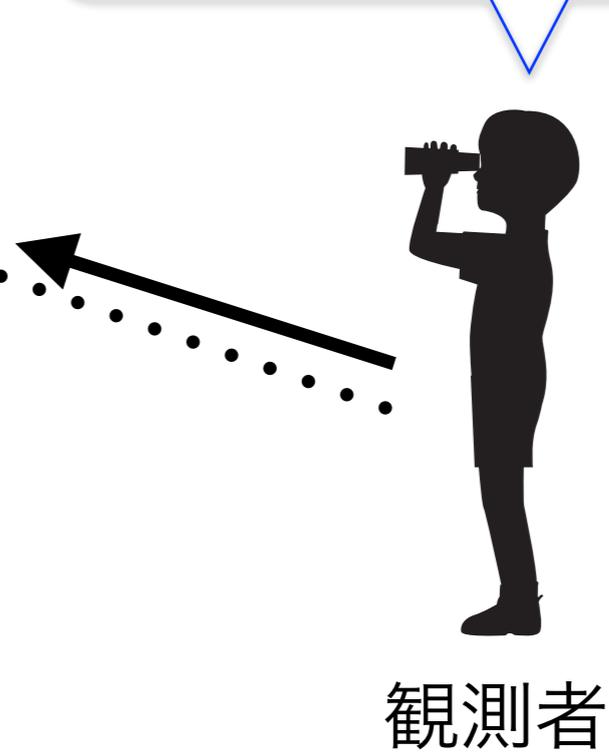
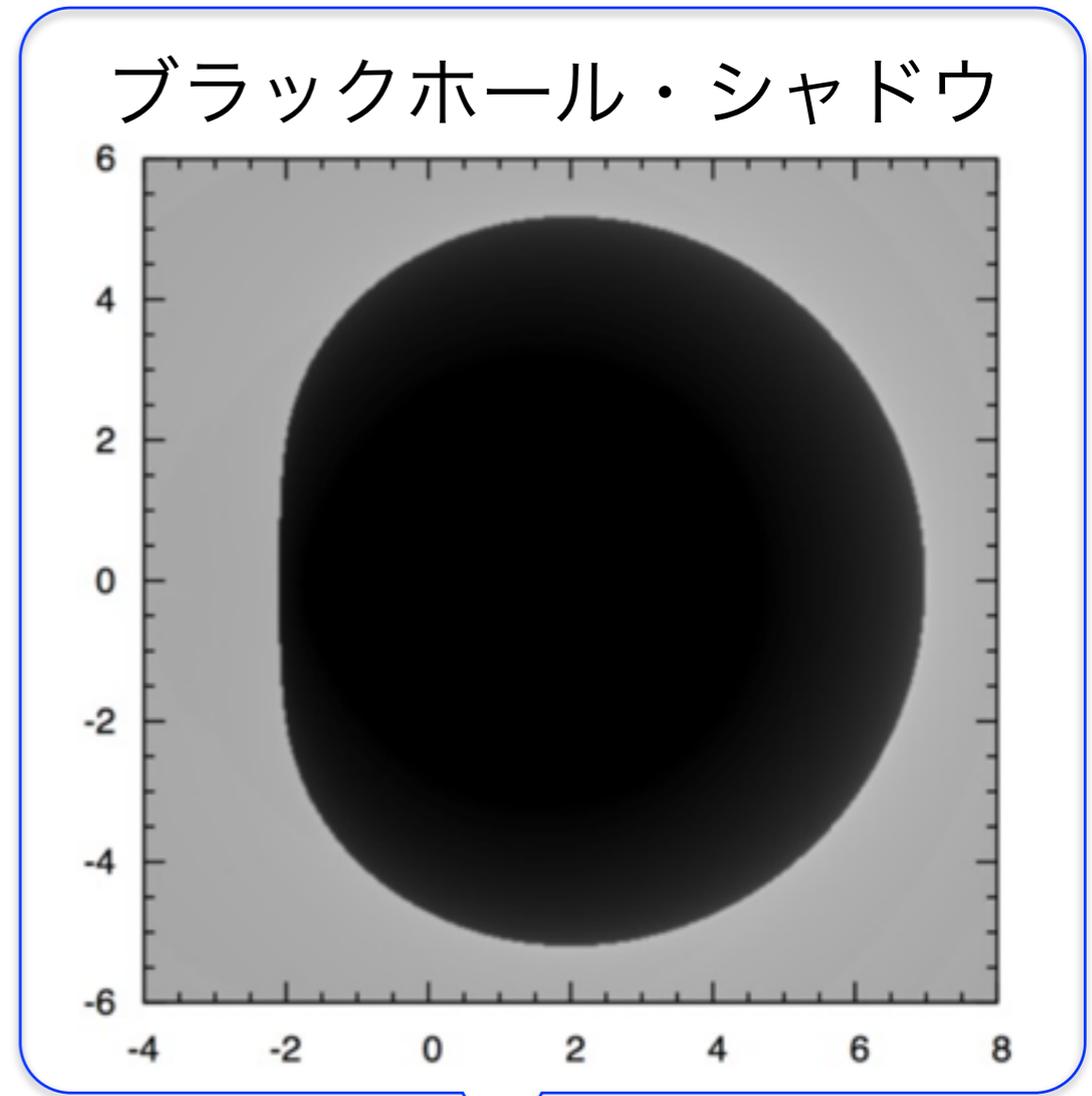
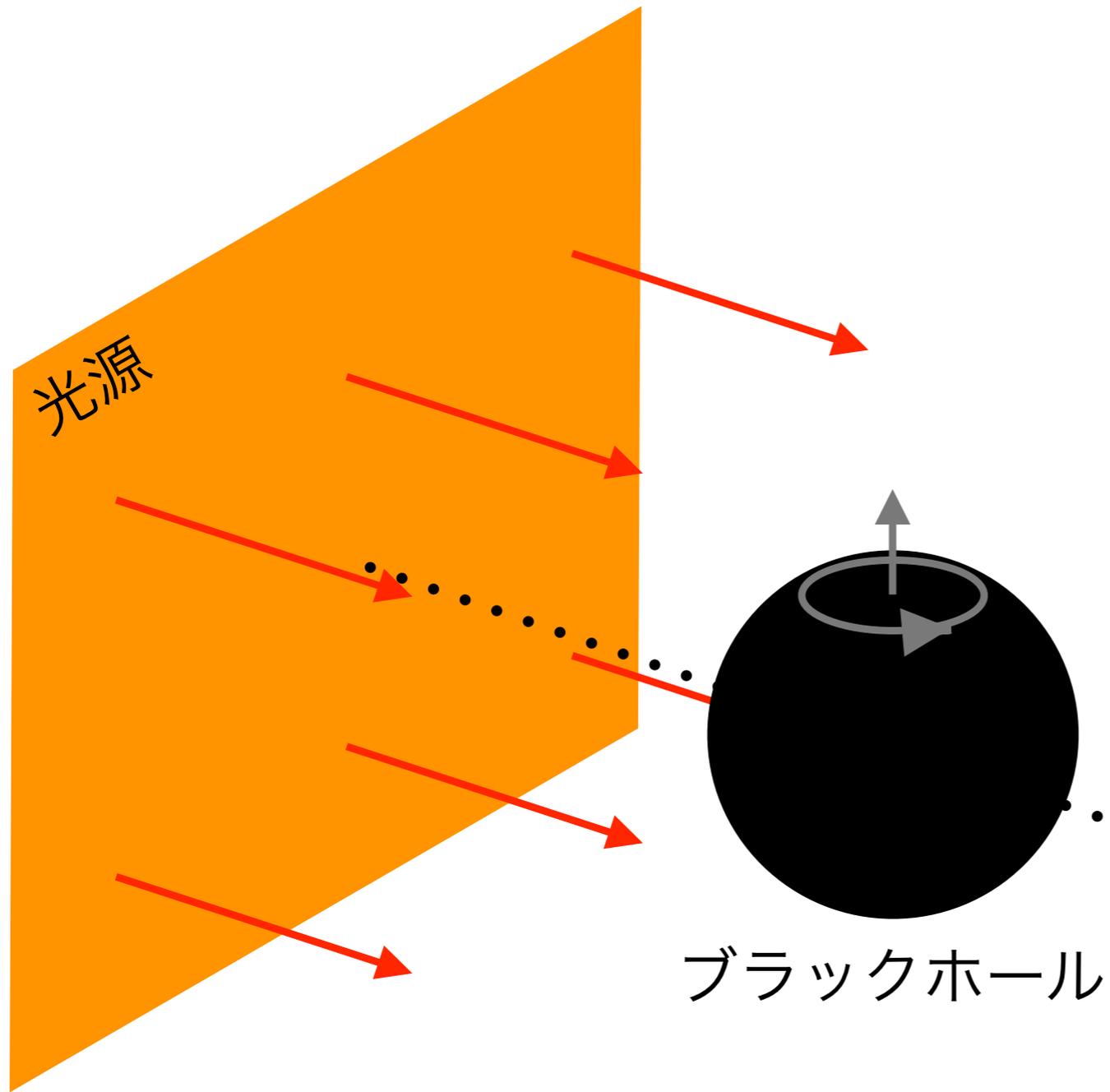


# Black Hole Shadowの物理

: 輪郭から分かる相対論効果

高橋 勇太 (国立高専機構 苫小牧高専)

# ブラックホール・シャドウ





光に照らされた闇夜のカラス

👉👉👉 「影絵」 として見える

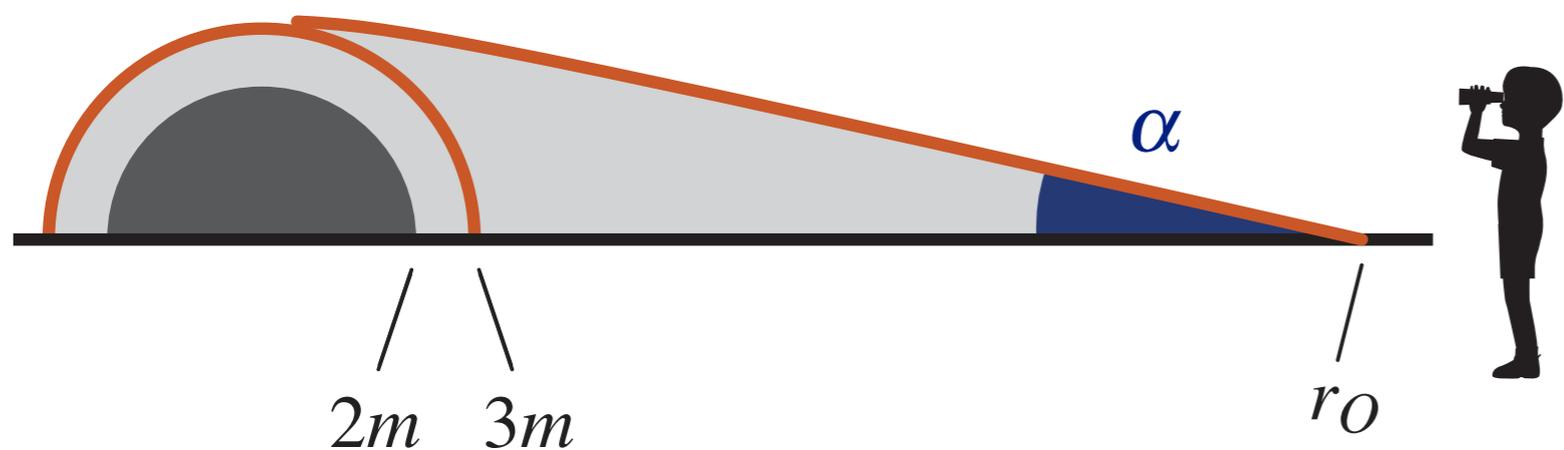
# ブラックホール・シャドウの大きさ

J. L. Synge, Mon. Not. R. astro. Soc., 131, 463-466 (1966)

“The Escape Photons from Gravitationally Intense Stars”

シュバルツシルト・ブラックホール

Syngeの公式

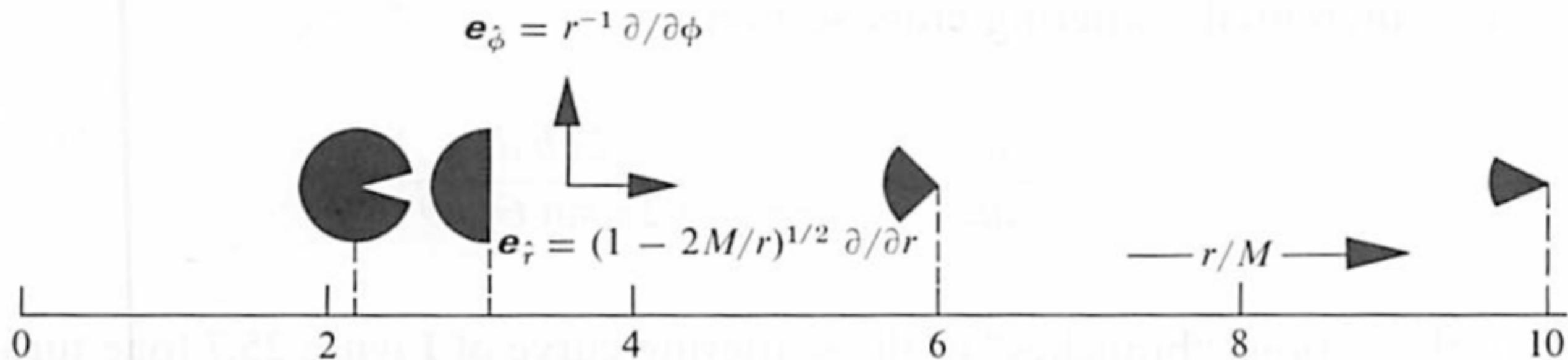


$$\sin^2 \alpha = \frac{27}{4} \frac{(\rho_0 - 1)}{\rho_0^3}$$

$$\rho_0 \equiv \frac{r}{2GM/c^2}$$

Grenzebach, Perlick & Lammerzahl (2014, PRD, 89, 124004)

Synge (1966, MNRAS, 131, 463)



Misner, Thorne & Wheeler (1973, Gravitation)

# Hagiwara 1931

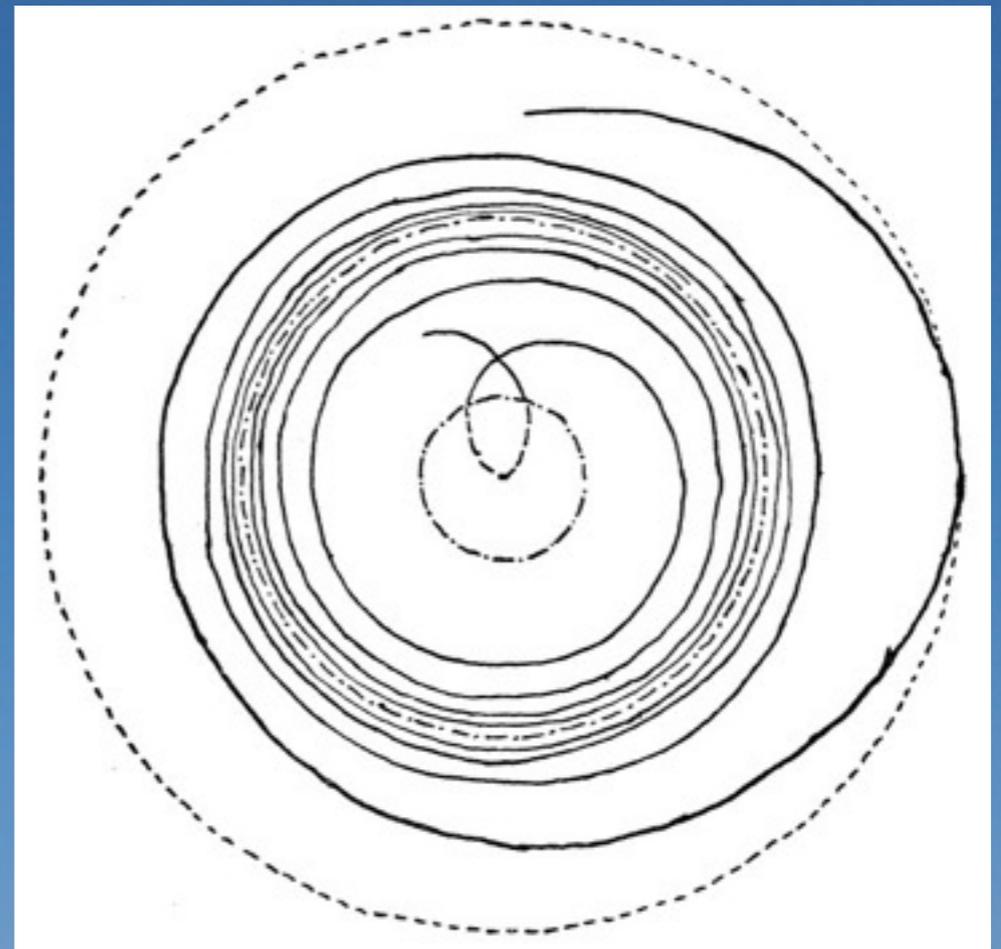
Yusuke Hagiwara, Jap. J. Astr. Geophys., 8, 67-176 (1931)

“Theory of the Relativistic Trajectories in a Gravitational Field of Schwarzschild”

\* Schwarzschild時空の測地線の解析解、正準方程式、Hamilton-Jacobi方程式、変数分離、軌道の分類



萩原雄祐 (1897-1979)



Chapter IX, Trajectory of a Light Ray

“... an observer in any part of the space can see every star ... ” (p79)

# Darwin 1958

C. Darwin, Proc. R. Soc., A, 249, 180-194 (1958)

“The gravity field of a particle”

\* Schwarzschild時空の測地線

$$\left(\frac{du}{d\theta}\right)^2 = 2mu^3 - u^2 + \frac{1}{l^2}.$$

\* bending angle

$P/m$	3.2	3.4	3.6	3.8	4	5	6	7	8	9	10	11	12
$\mu^\circ$	273	205	162	143	125	79	58	46	38	32	28	25	23
$l/m$	5.23	5.30	5.40	5.53	5.66	6.46	7.35	8.28	9.24	10.22	11.20	12.17	13.15

$P$  : perihelion distance,  $\mu$  : bending angle,  $l$  : constant

\* impact parameterとbending angleの関係式

(photon orbit近傍で成立する近似式)

$$l = (5.19 + 3.48 e^{-\mu}) m.$$

$$(\approx \sqrt{27})$$

\* relativistic imagesの明るさの近似式も計算

\* Hagiwara (1930) 引用されていない

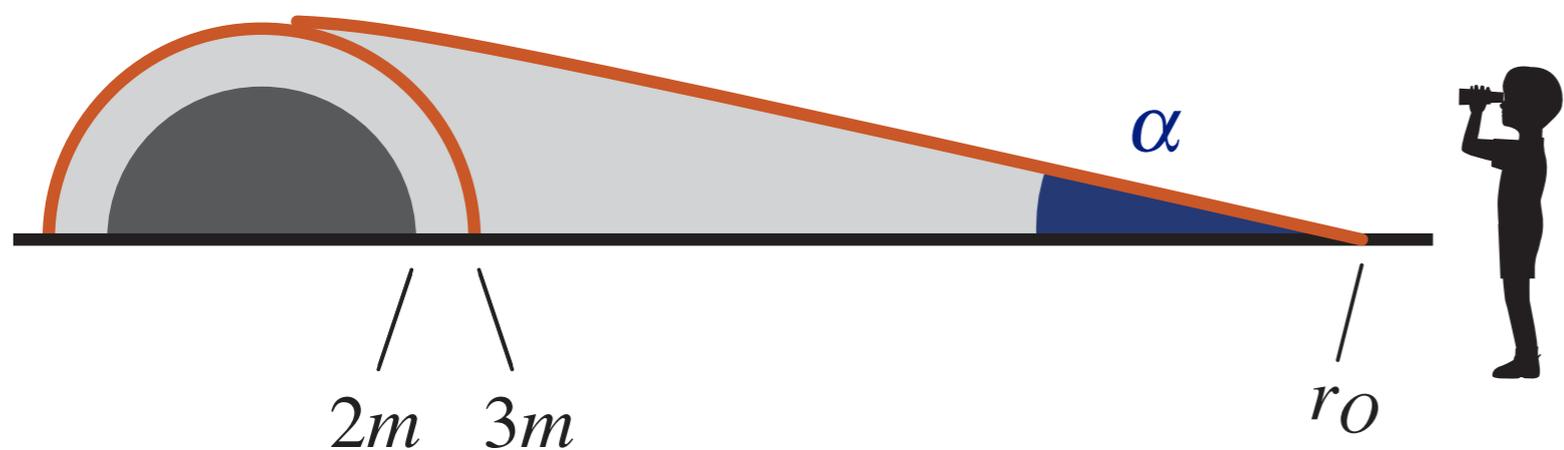
# ブラックホール・シャドウの大きさ

J. L. Synge, Mon. Not. R. astro. Soc., 131, 463-466 (1966)

“The Escape Photons from Gravitationally Intense Stars”

シュバルツシルト・ブラックホール

Syngeの公式

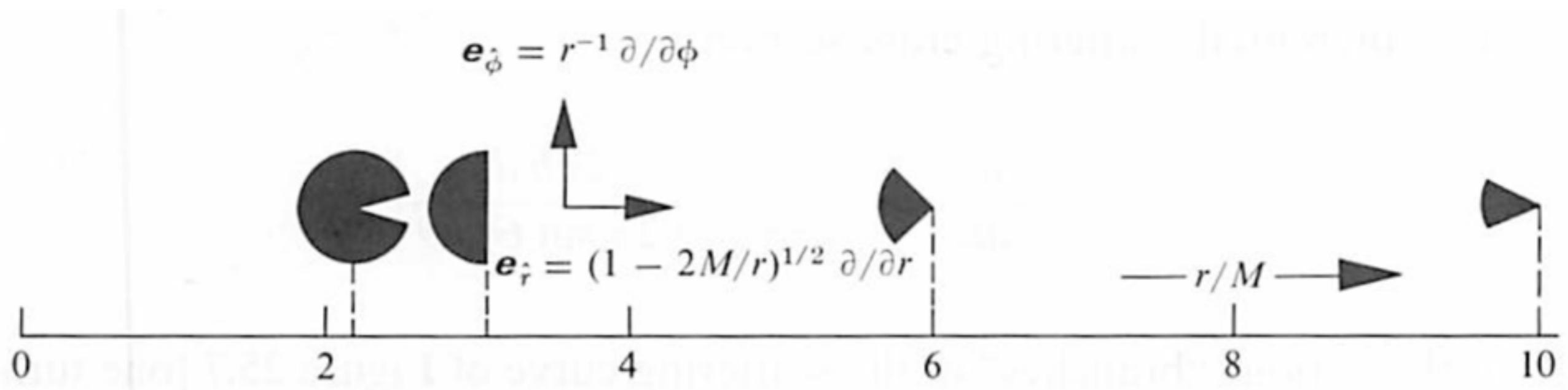


$$\sin^2 \alpha = \frac{27}{4} \frac{(\rho_0 - 1)}{\rho_0^3}$$

$$\rho_0 \equiv \frac{r}{2GM/c^2}$$

Grenzebach, Perlick & Lammerzahl (2014, PRD, 89, 124004)

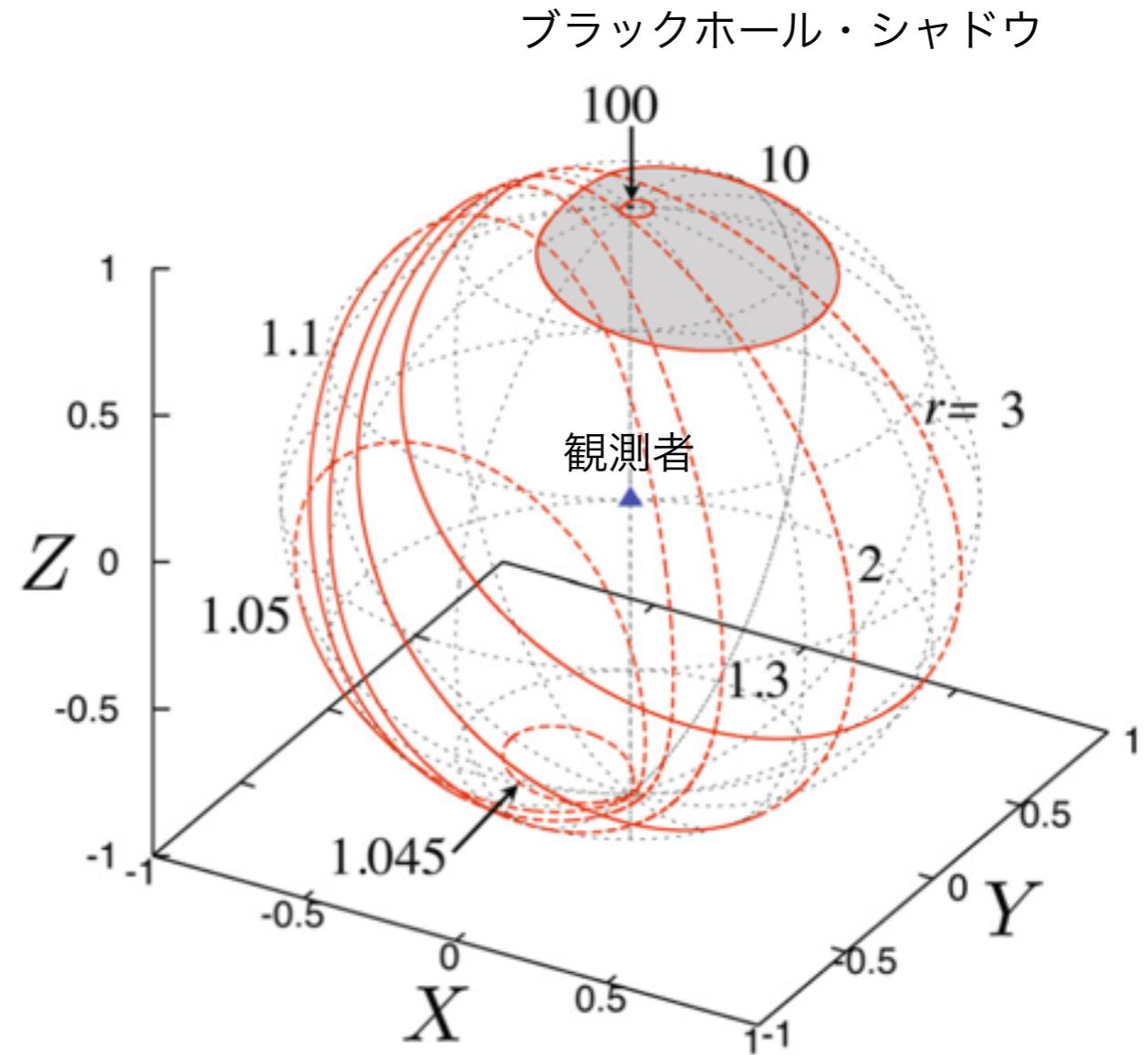
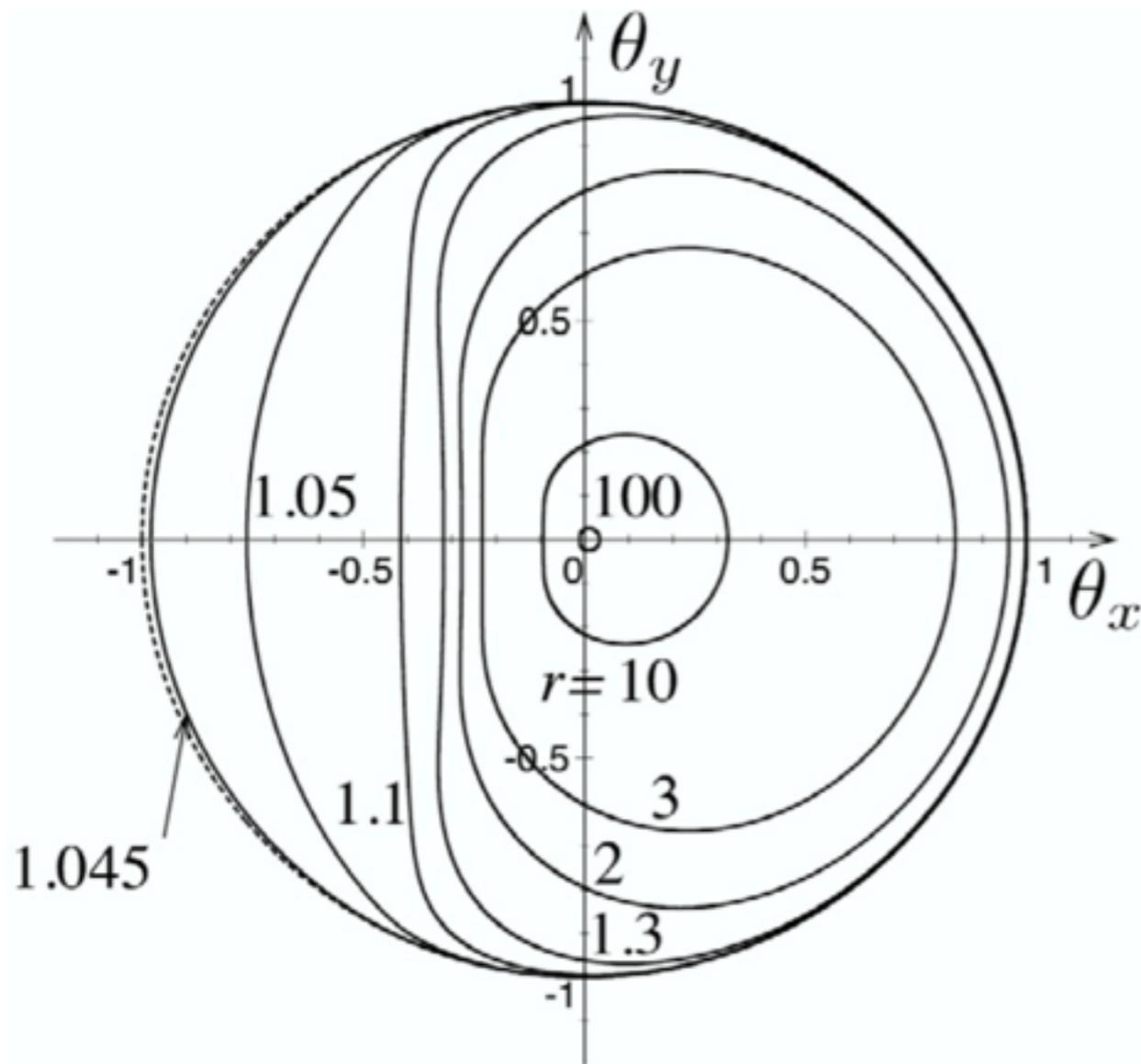
Synge (1966, MNRAS, 131, 463)



Misner, Thorne & Wheeler (1973, Gravitation)

# 観測者の空に見えるブラックホール・シャドウ

カー・ブラックホール（自転するブラックホール）の場合

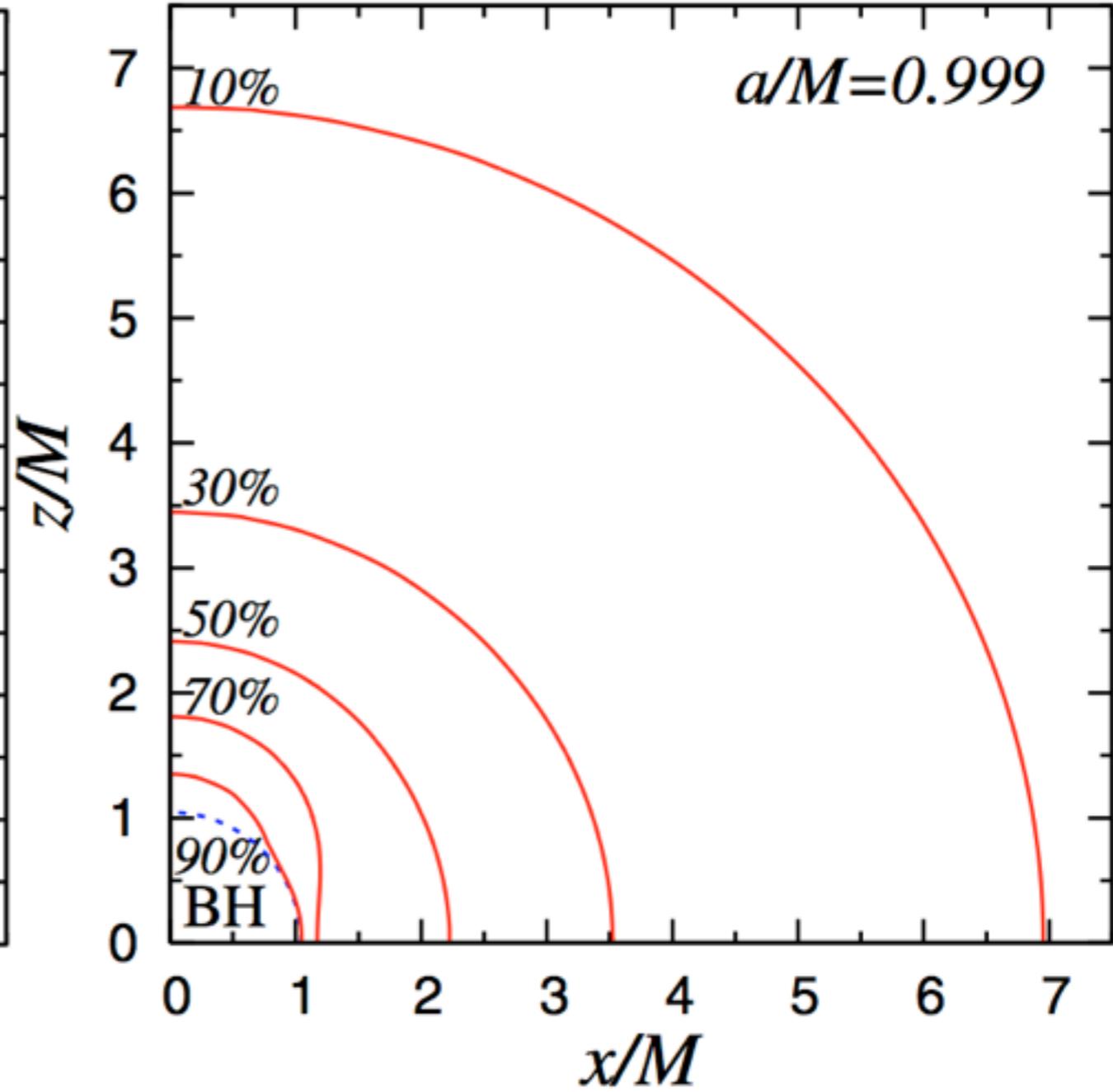
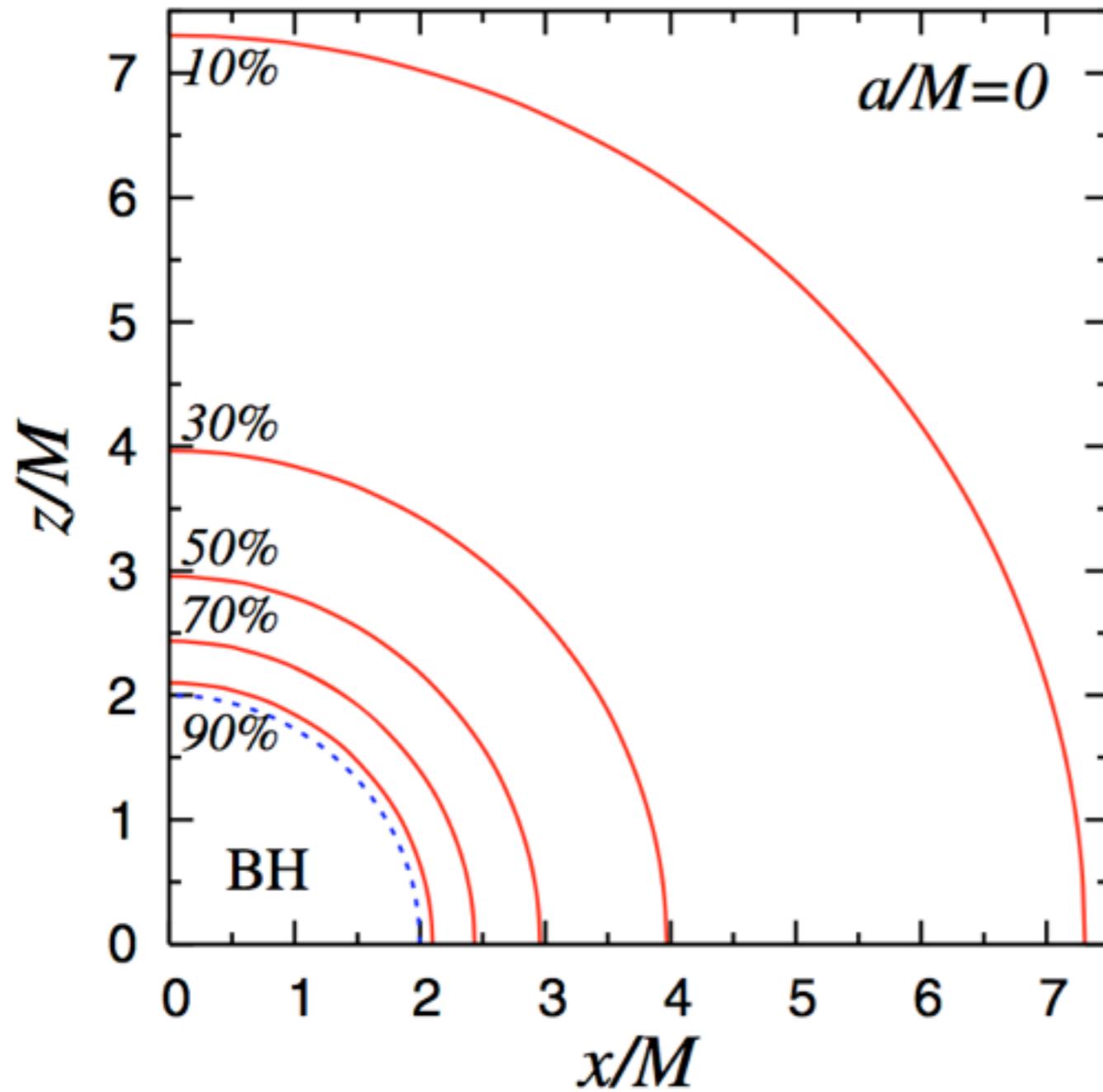


R.Takahashi & M.Takahashi (2010, A&A, 513, A77)

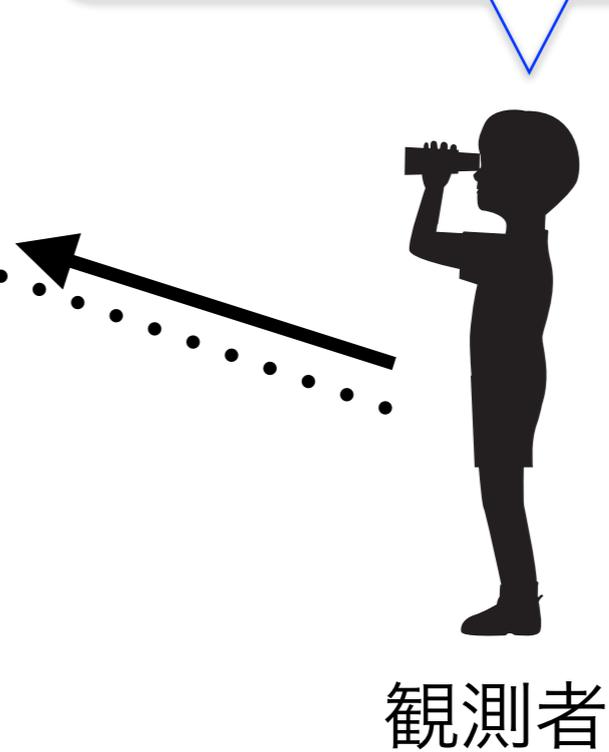
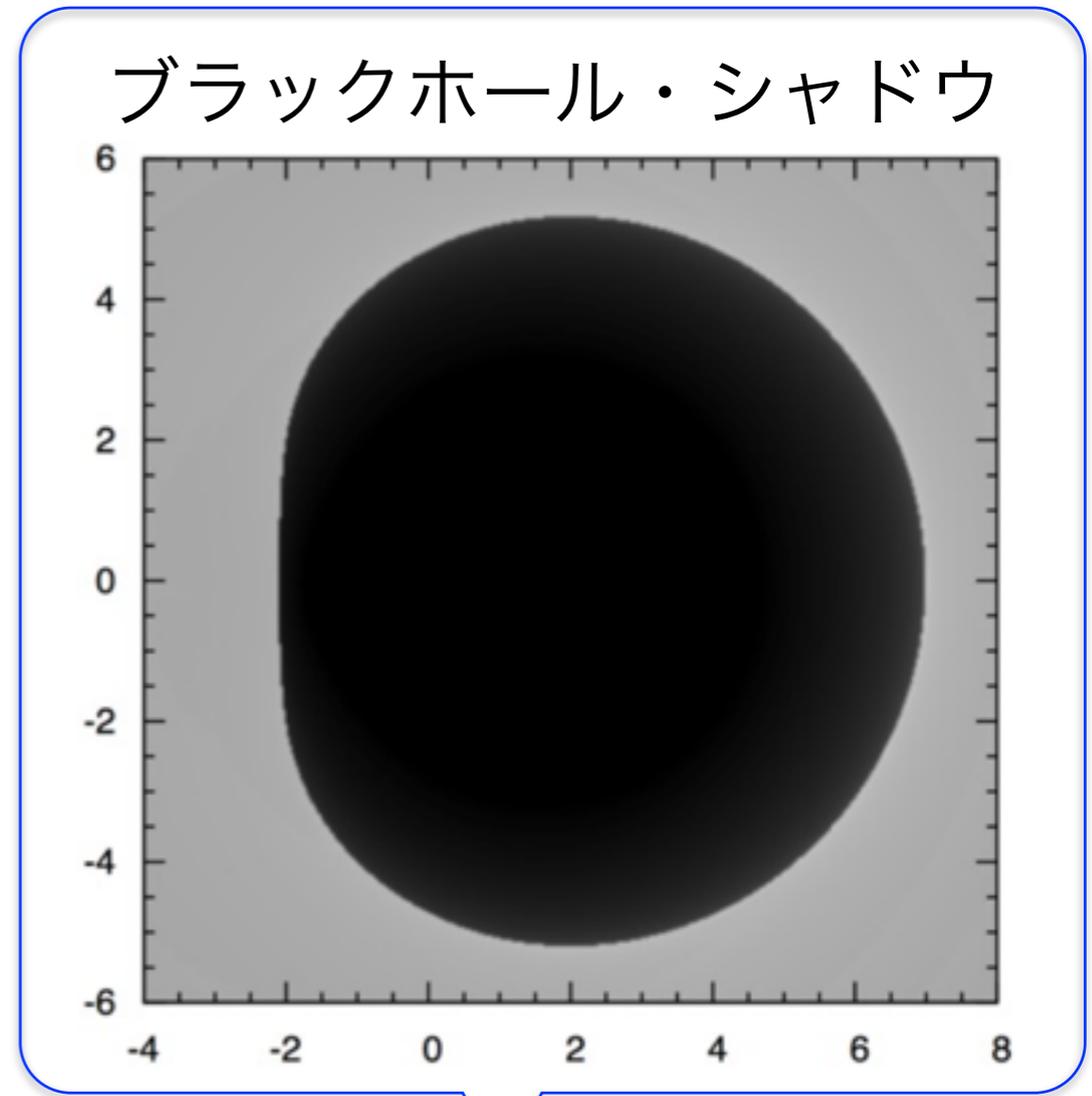
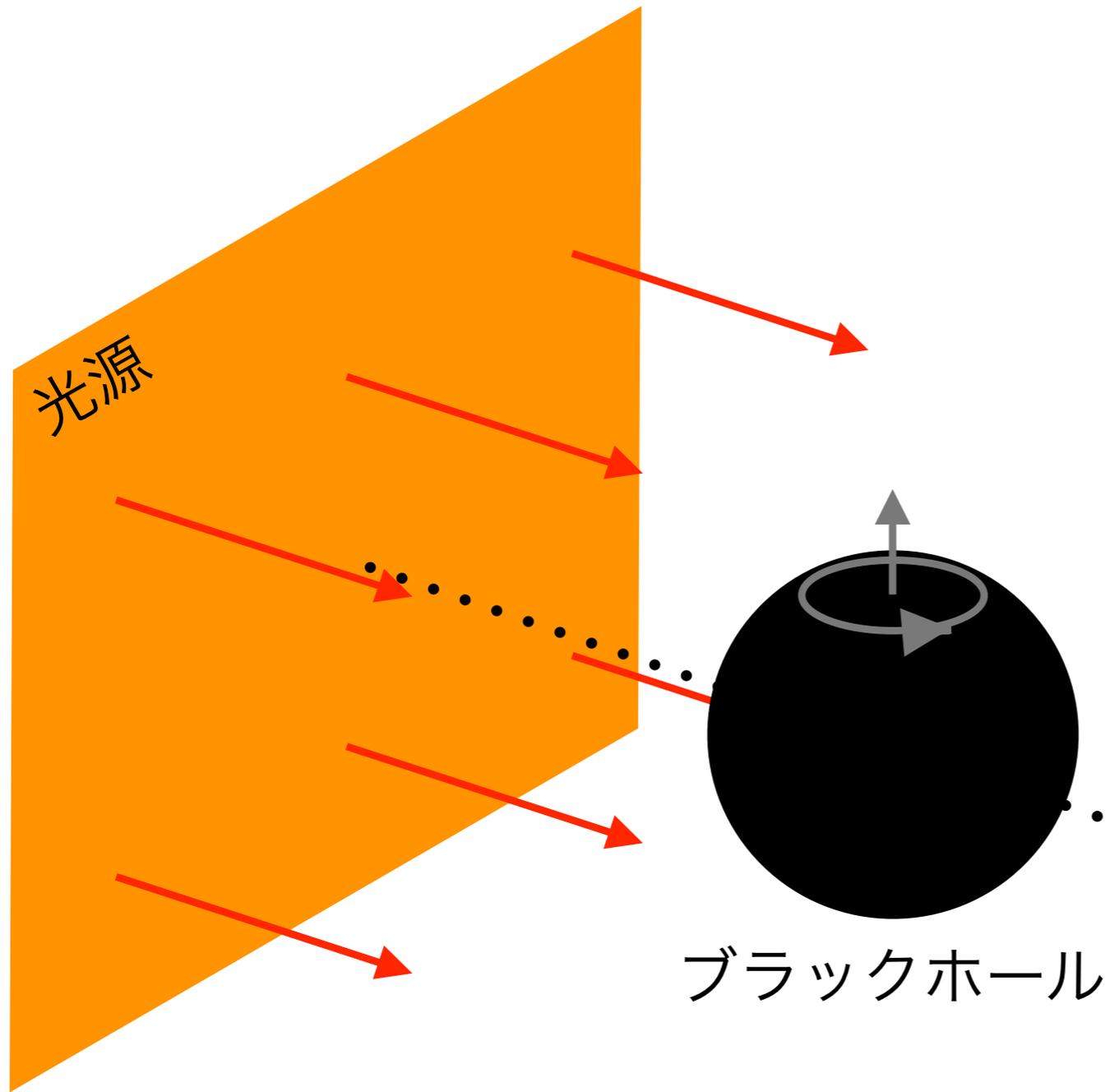
# 観測者の全天にブラックホールが占める割合

シュバルツシルト・ブラックホールの場合

カー・ブラックホールの場合



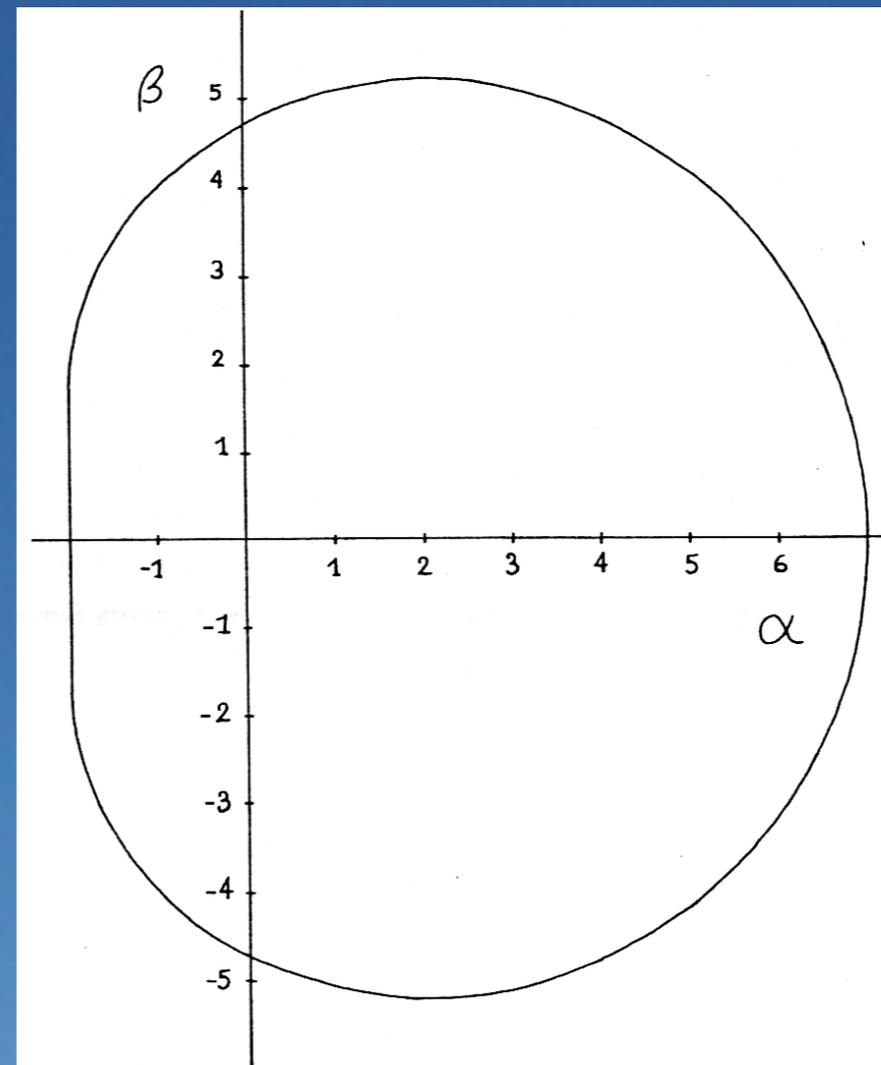
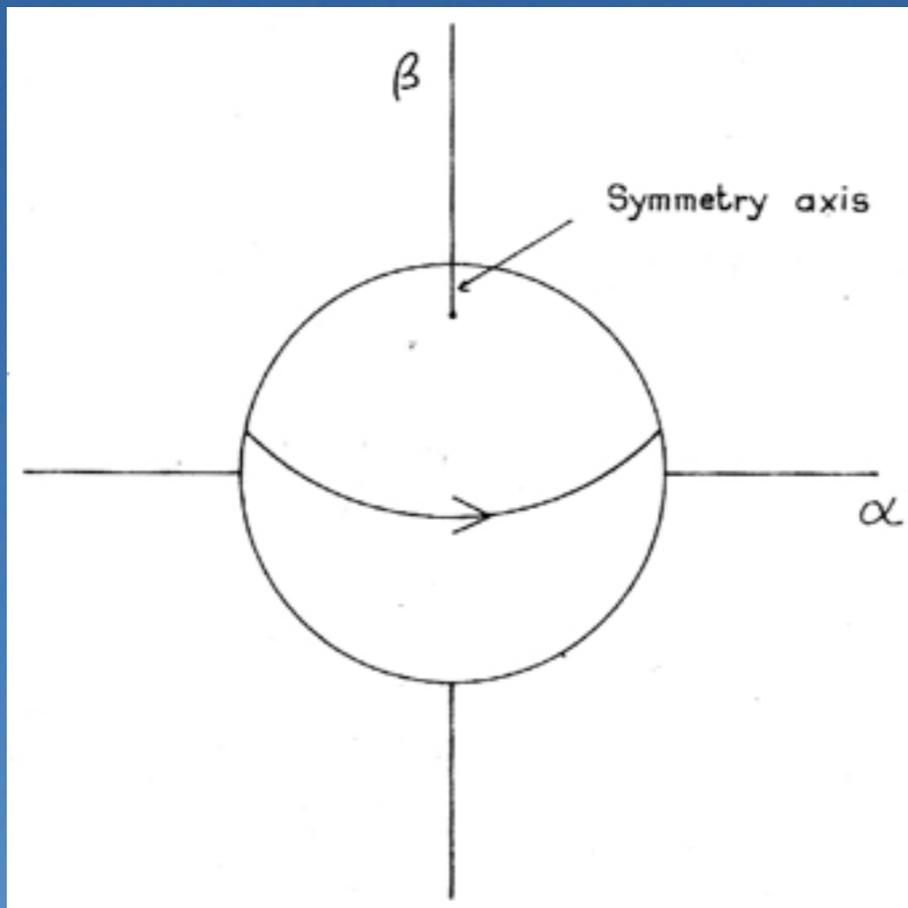
# ブラックホール・シャドウ



# Bardeen 1973

J. M. Bardeen, in Black Holes ed. C. DeWitt & B. S. DeWitt (1973)

“Timelike and Null Geodesics in the Kerr Metric”



$$\alpha = \frac{\lambda}{\sin \theta_0}$$

$$\beta = \pm [\eta + \cos^2 \theta_0 - \lambda^2 \cot^2 \theta_0]^{1/2}$$

Photon Sphere  $\rightarrow$

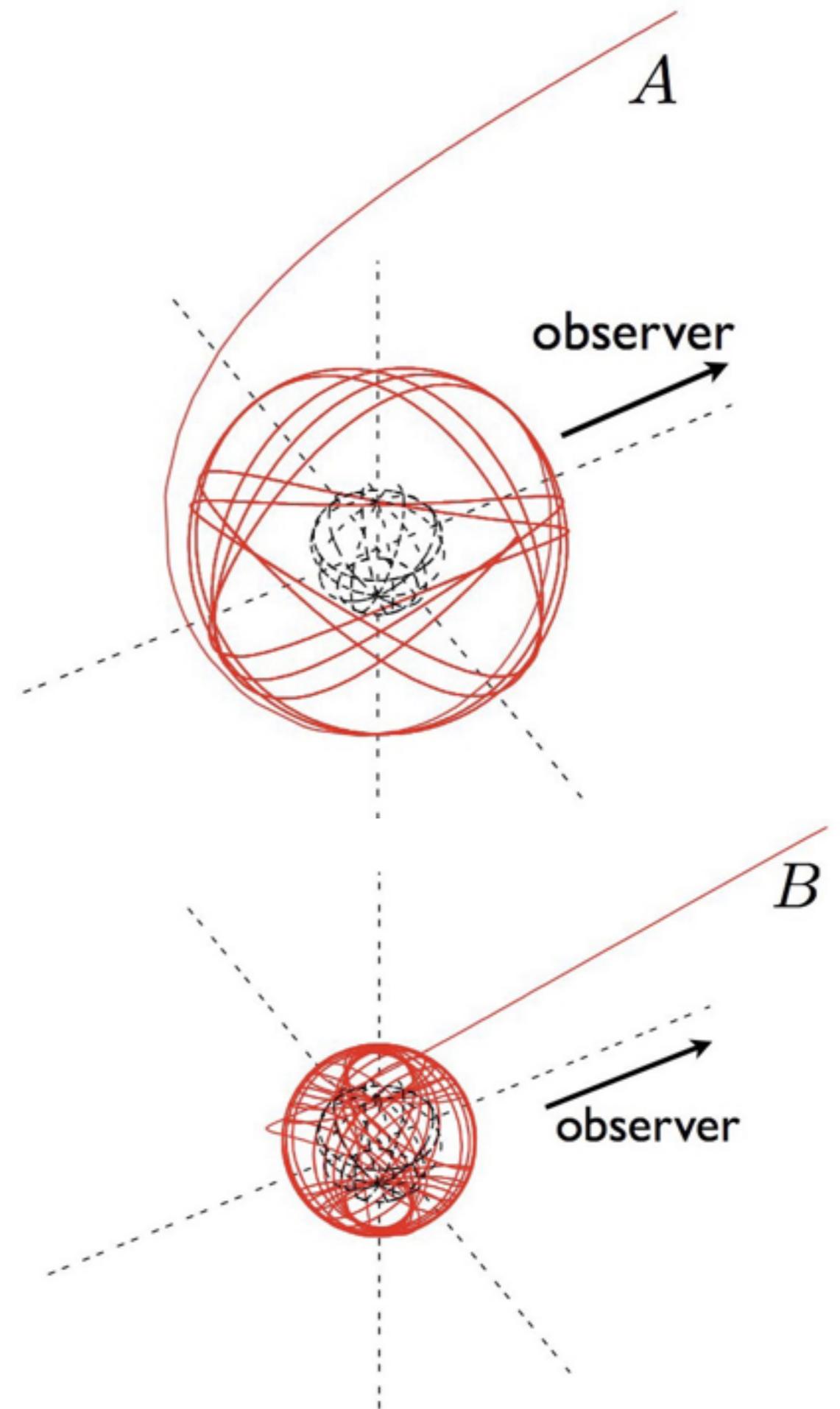
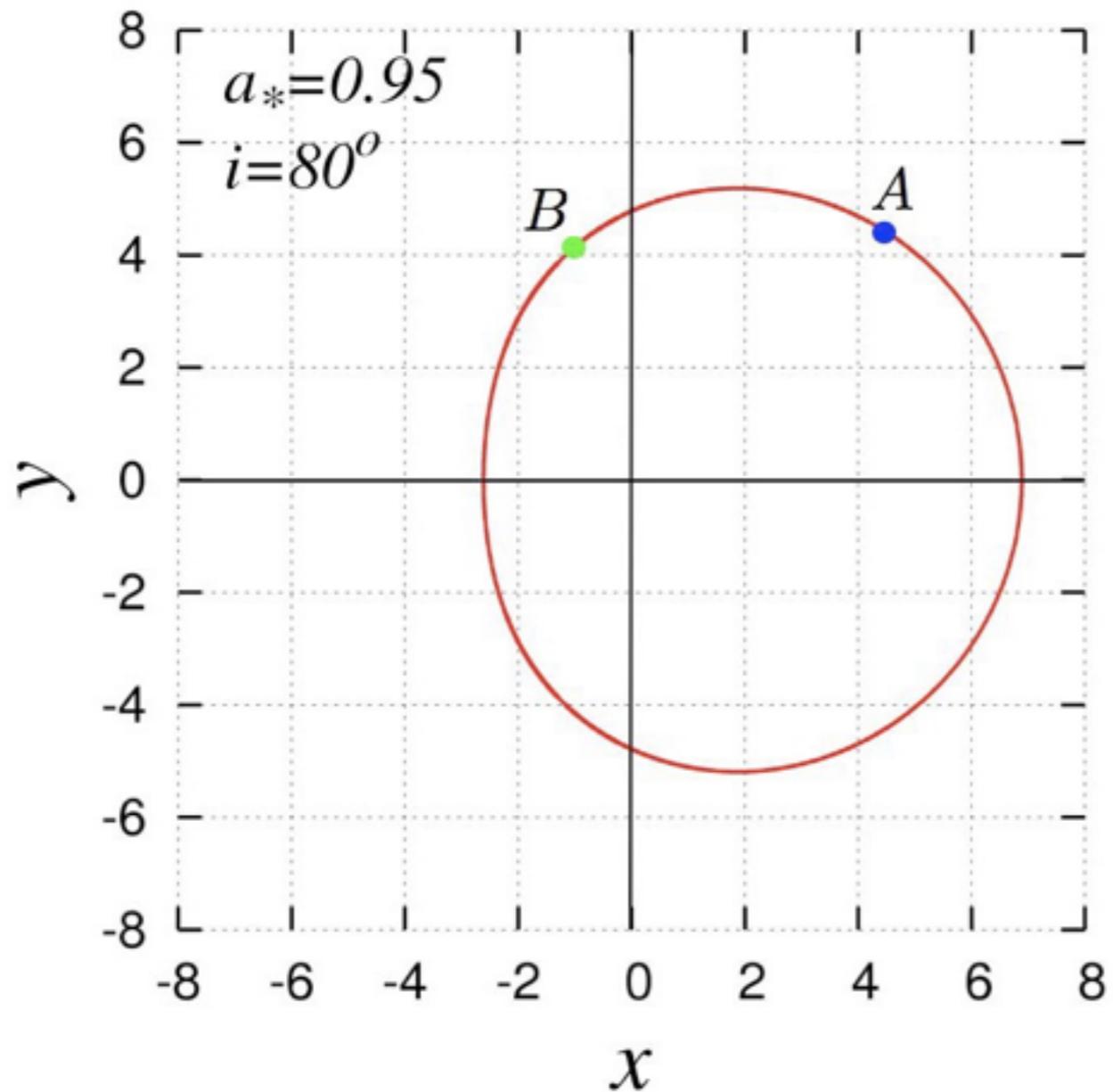
$$\lambda = \frac{-r^3 + 3mr^2 - a^2(r + m)}{a(r - m)}$$

$$\eta = \frac{r^3 [4a^2m - r(r - 3m)^2]}{a^2(r - m)^2}$$

# Photon Sphere

$$\dot{r}^2 \propto R(r)$$

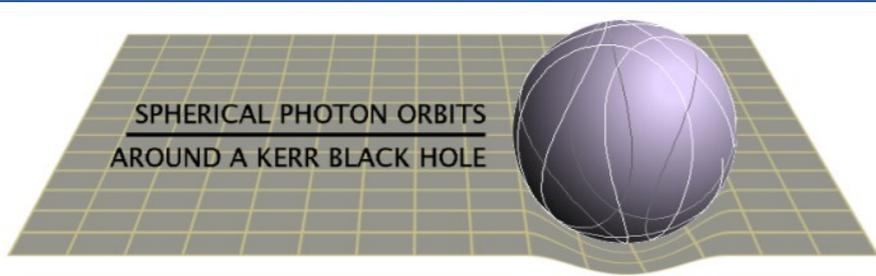
$$R = \frac{dR}{dr} = 0, \quad \frac{d^2R}{dr^2} > 0 \text{ (unstable)}$$



# Photon Sphere around a Kerr BH

E. Teo, General Relativity and Gravitation 35,1909 (2003)

“Spherical Photon Orbits around a Kerr Black Hole”



## Complete Table of Example Orbits

Here we focus on two values of  $a$ , namely the extreme case  $a = M$  and a non-extreme case  $a = 0.5M$ .  $\Phi$  takes the range  $-7M < \Phi < 2M$  in the former case, while it takes the range  $-6.1382M < \Phi < 4.0963M$  in the latter case. Example orbits are chosen at more or less equal intervals along these ranges.

In most of the examples, the orbits have been plotted for an integer number of latitudinal oscillations until they return to near the starting point. Again, clicking on each figure gives a three-dimensional wire-frame model of the orbit.

	$a = M$	$a = 0.5M$
$\Phi = -6.9M$		
$\Phi = -6M$		
$\Phi = -5M$		

$\Phi = -4M$		
$\Phi = -3M$		
$\Phi = -2M$		
$\Phi = -M$		
$\Phi = 0$		

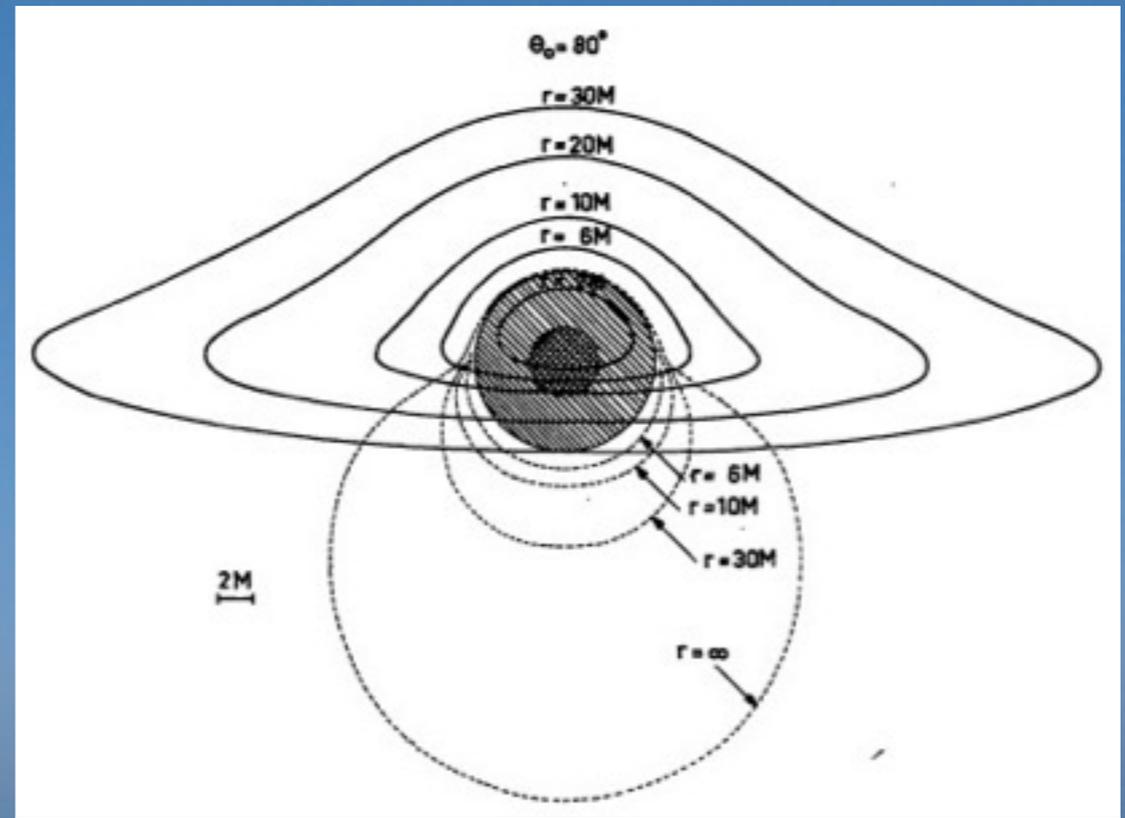
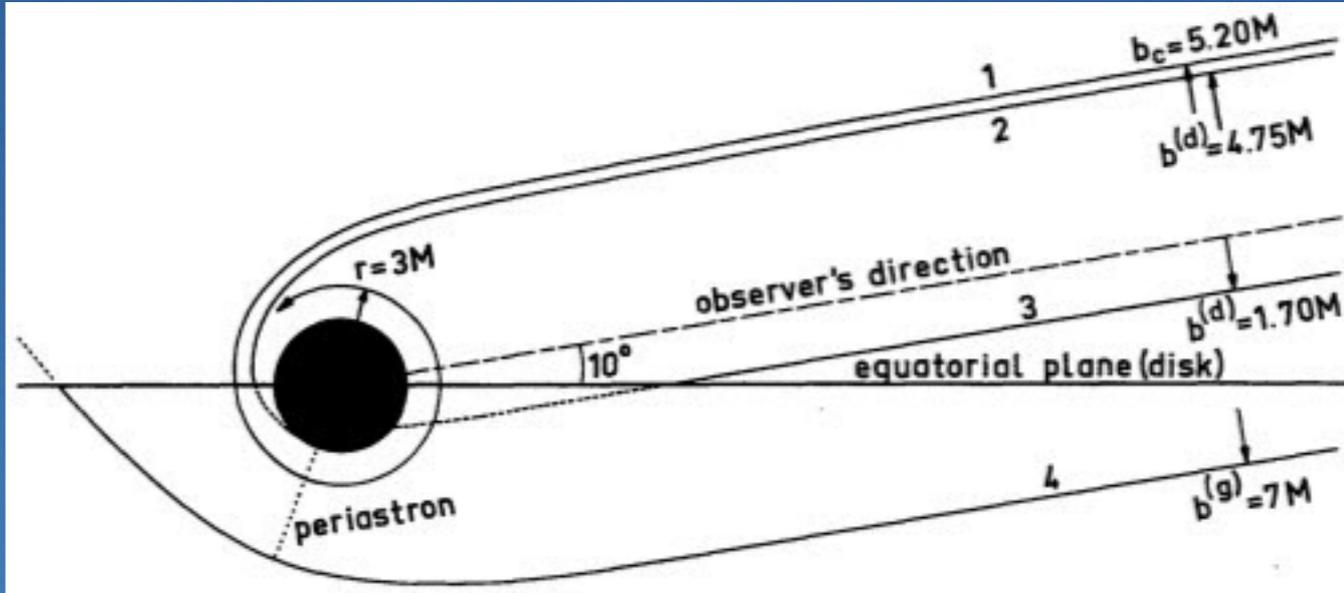
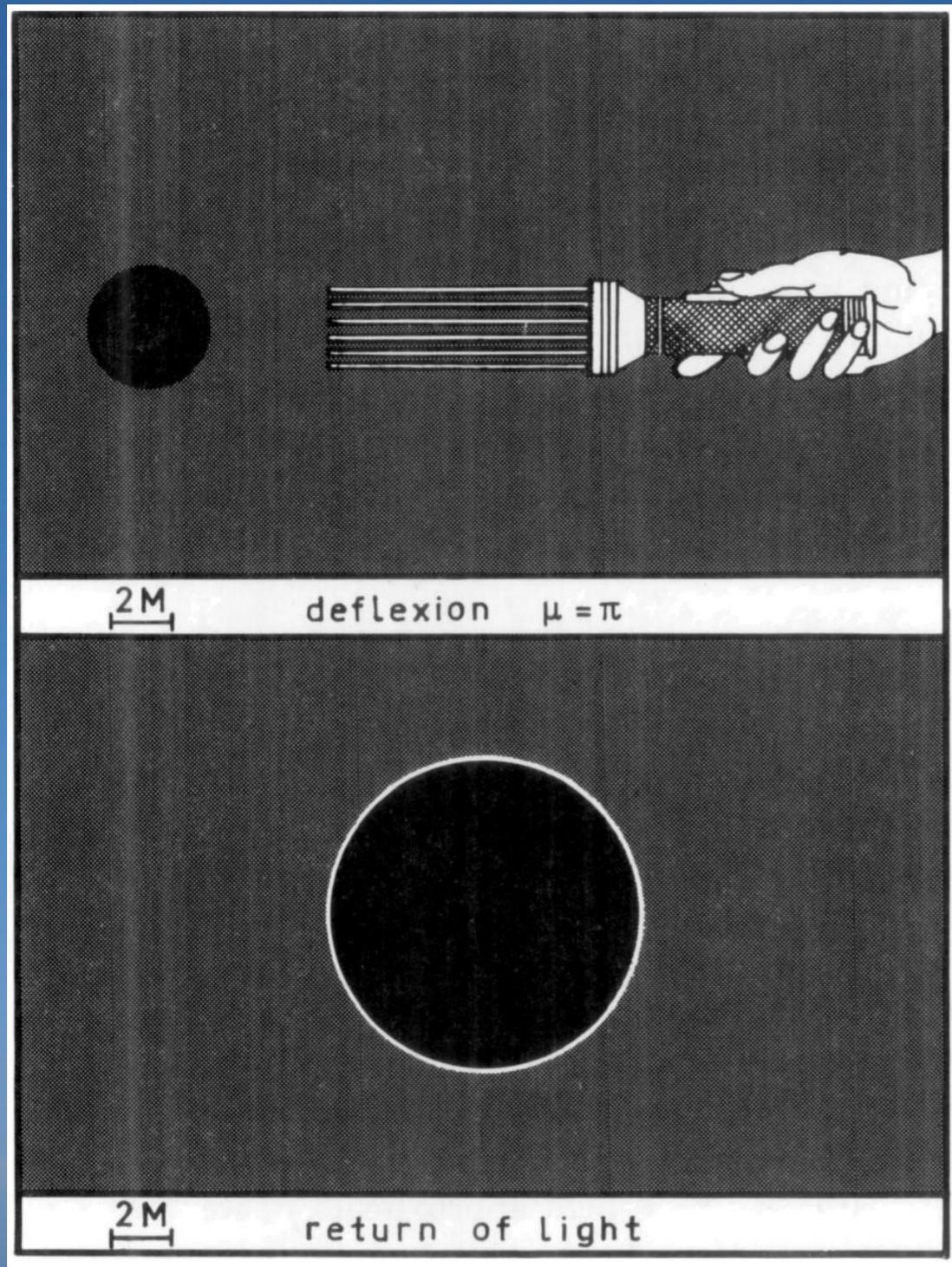
$\Phi = M$		
$\Phi = 1.9M$ (for $a = M$ ) $\Phi = 2M$ (for $a = 0.5M$ )		
$\Phi = 3M$		
$\Phi = 4M$		

# 降着円盤中のブラックホール・シャドウ

# Luminet 1979

J.-P. Luminet, Astron. Astrophys. 75, 228-235 (1979)

“Image of a Spherical Black Hole with Thin Accretion Disk”



# Luminet 1979

J.-P. Luminet, *Astron. Astrophys.* 75, 228-235 (1979)

“Image of a Spherical Black Hole with Thin Accretion Disk”



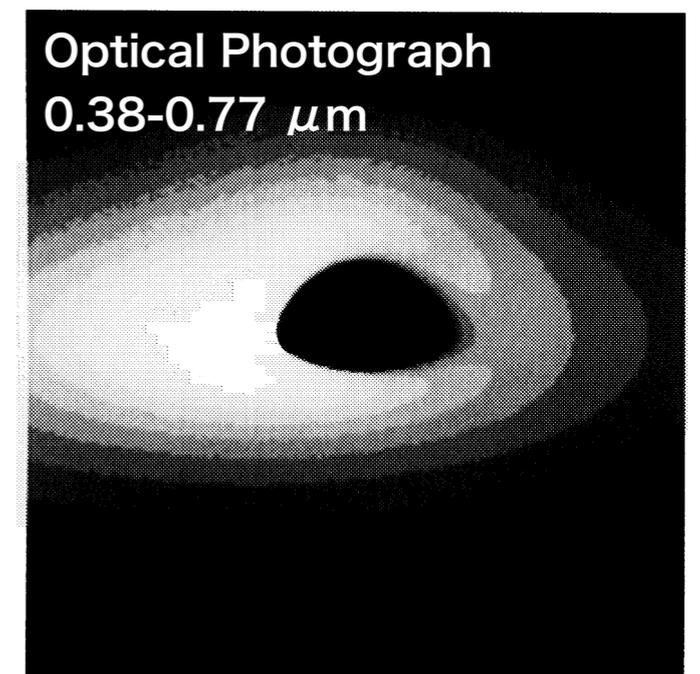
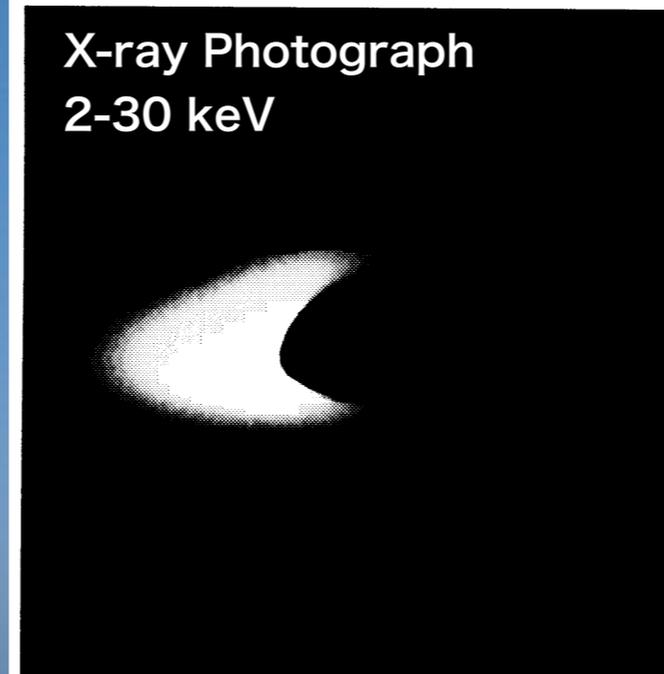
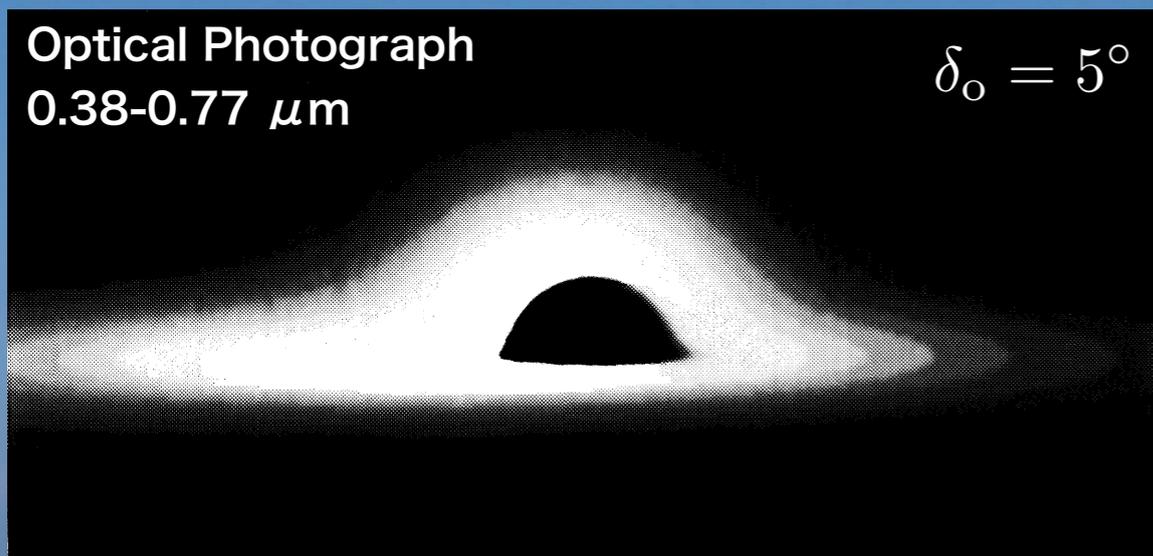
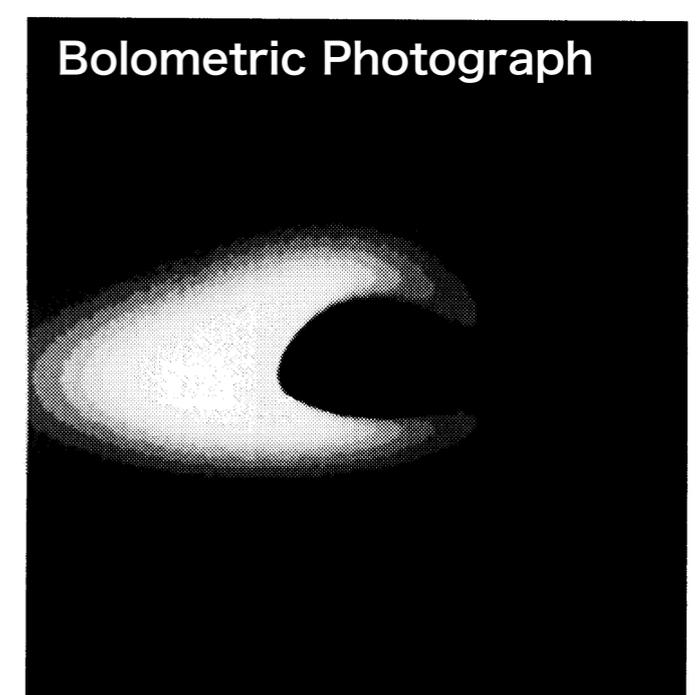
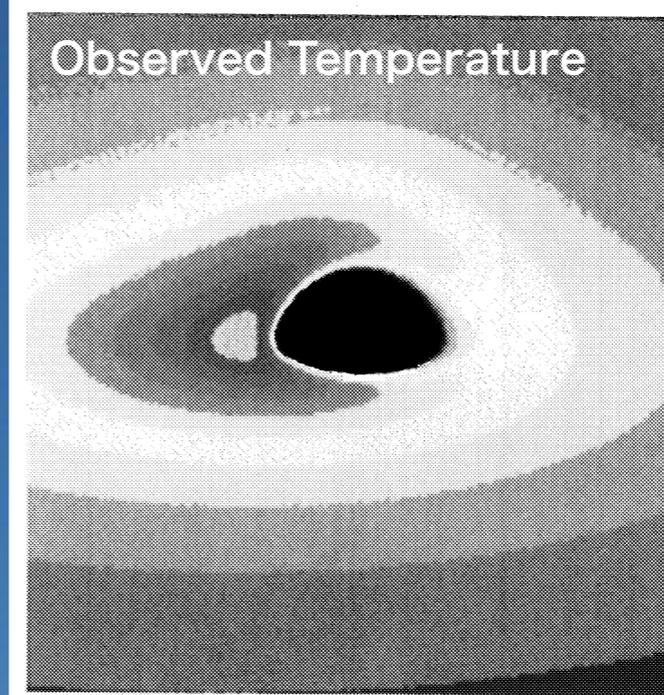
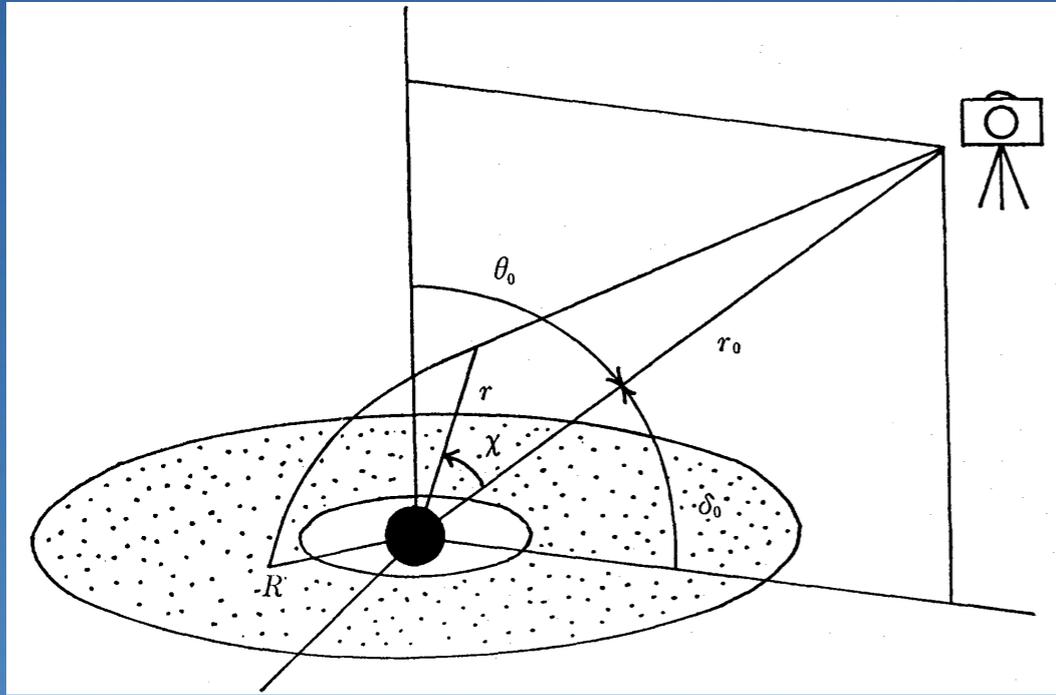
Bolometric Photograph

# Fukue & Yokoyama 1979

J. Fukue & T. Yokoyama, Publ. Astron. Soc. Japan 40, 15-24 (1988)

“Color Photographs of an Accretion Disk around a Black Hole”

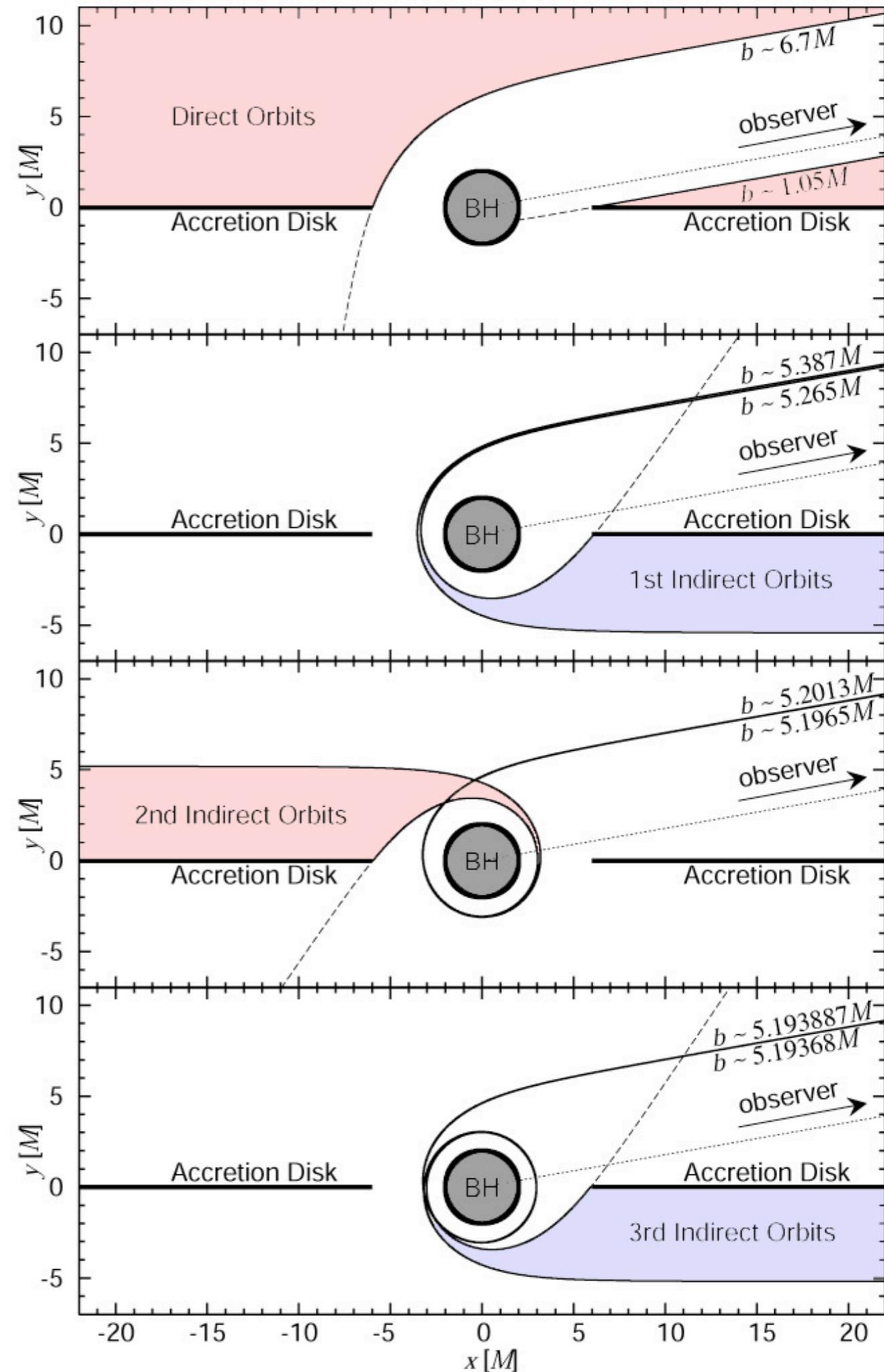
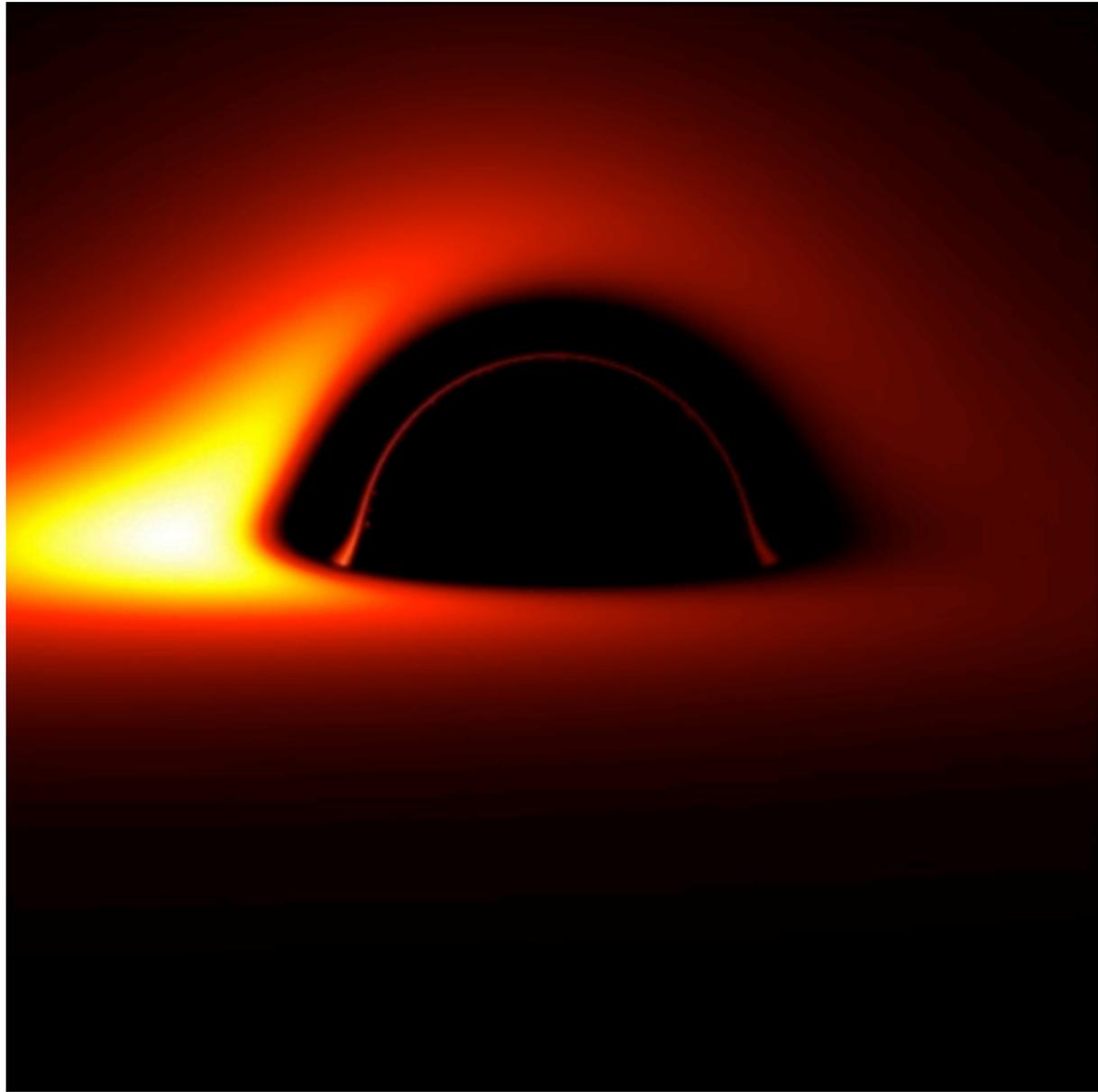
$$\delta_o = 20^\circ$$



(c)

(d)

# イメージと軌道の対応



# BHスピン

$a=0.5$

仰角 10度

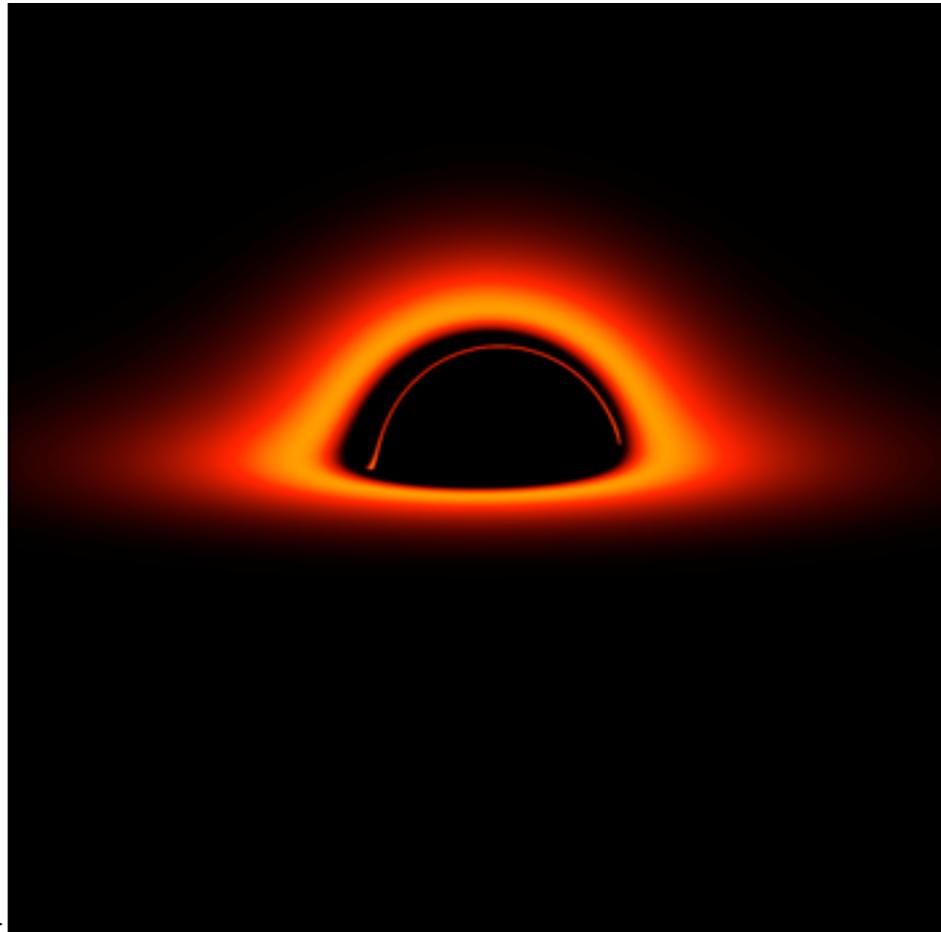
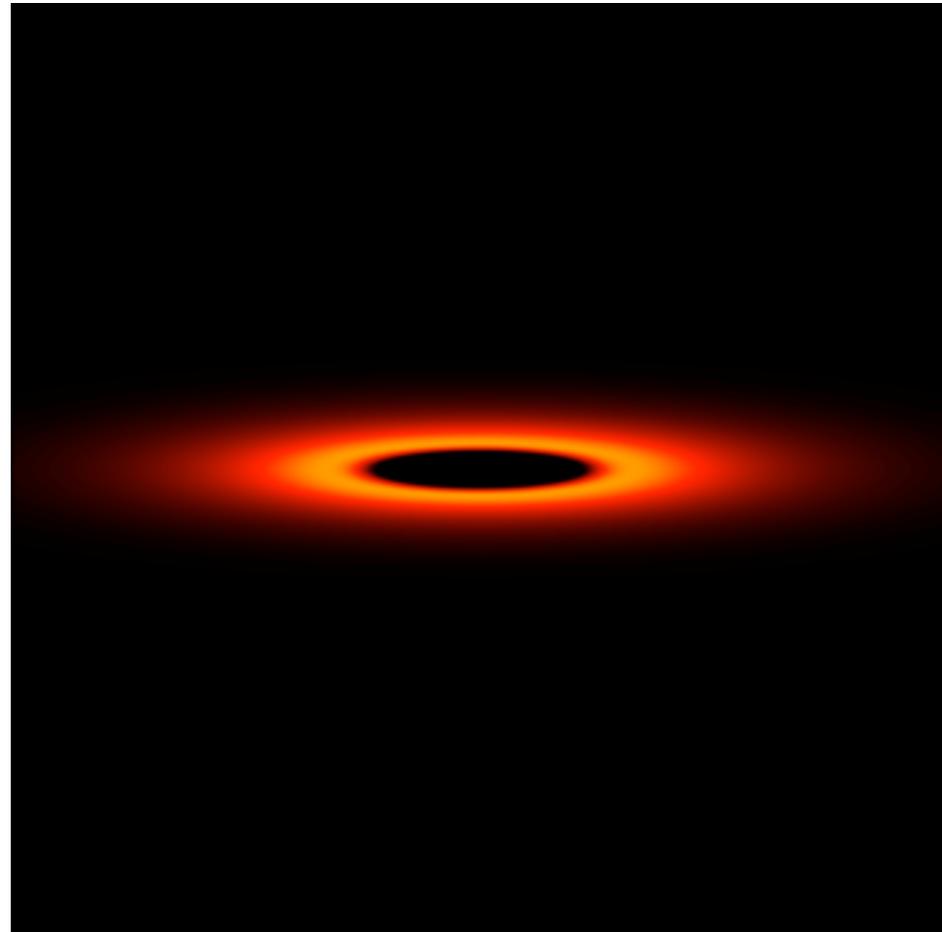
観測波長 1keV (X線)

BH質量 10太陽質量

質量降着率

0.1 Eddington降着率

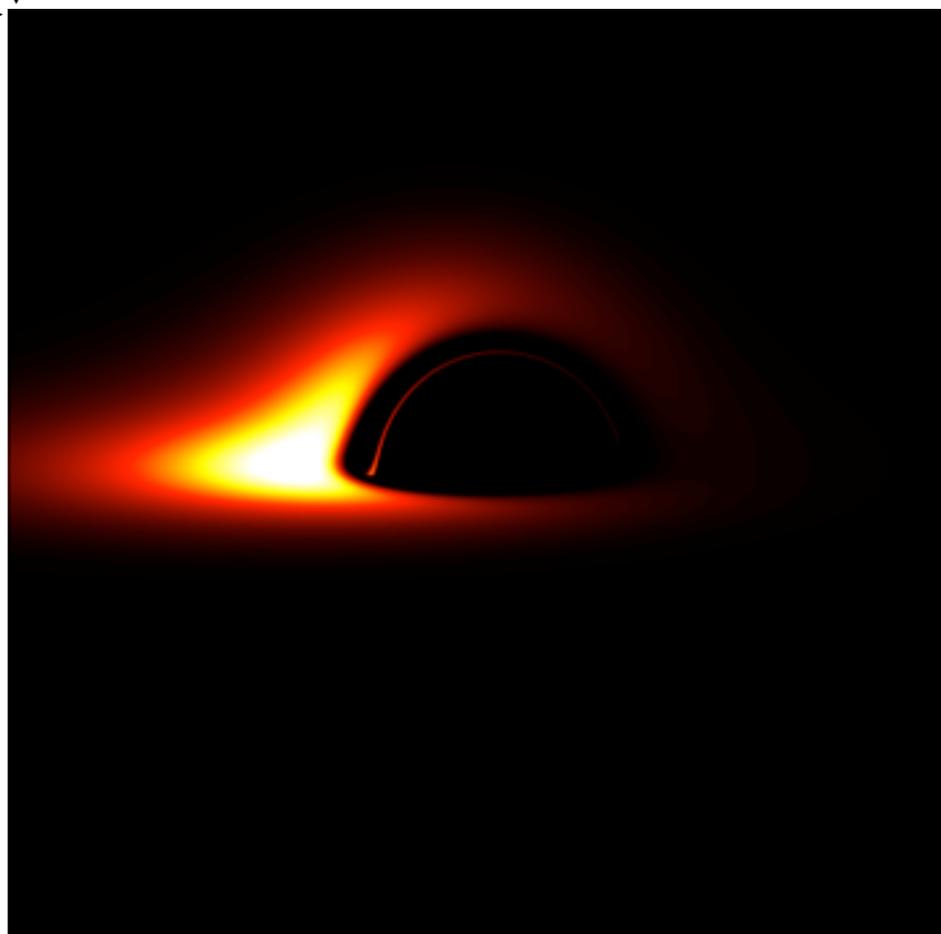
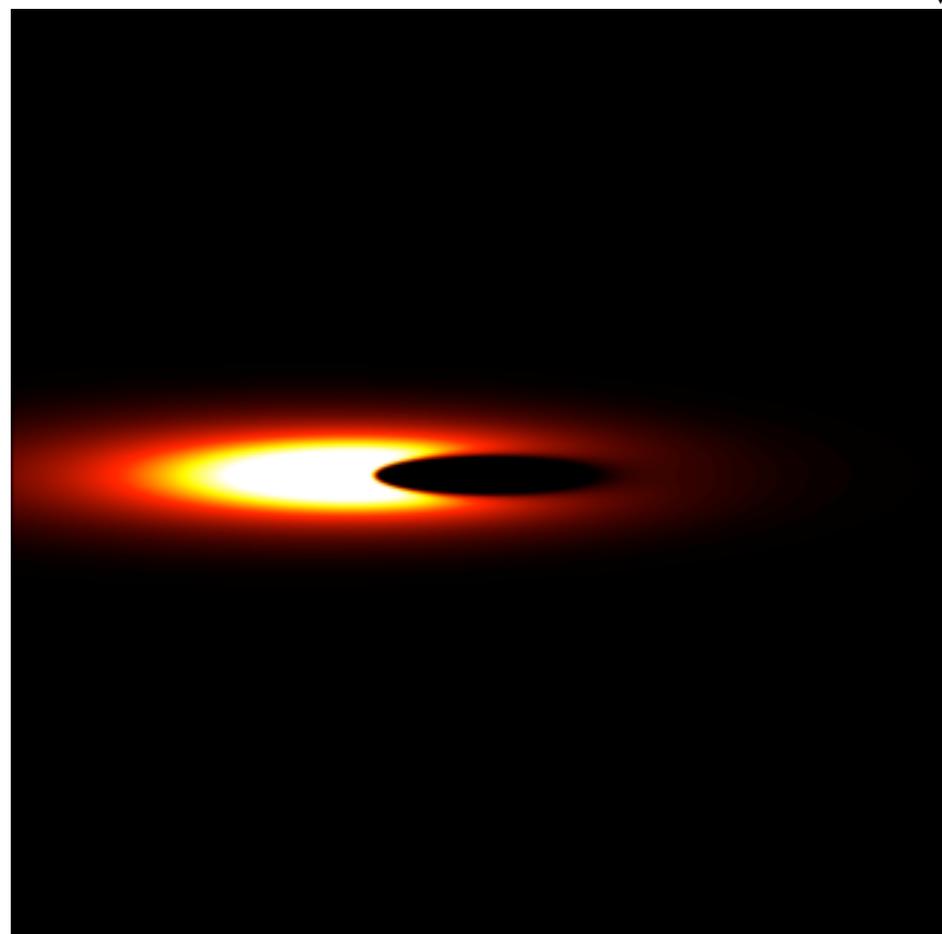
輝度範囲 0~4500



光の軌道の湾曲

円盤回転によるドップラー効果

円盤回転によるドップラー効果  
重力赤方偏移 時空の引きずり

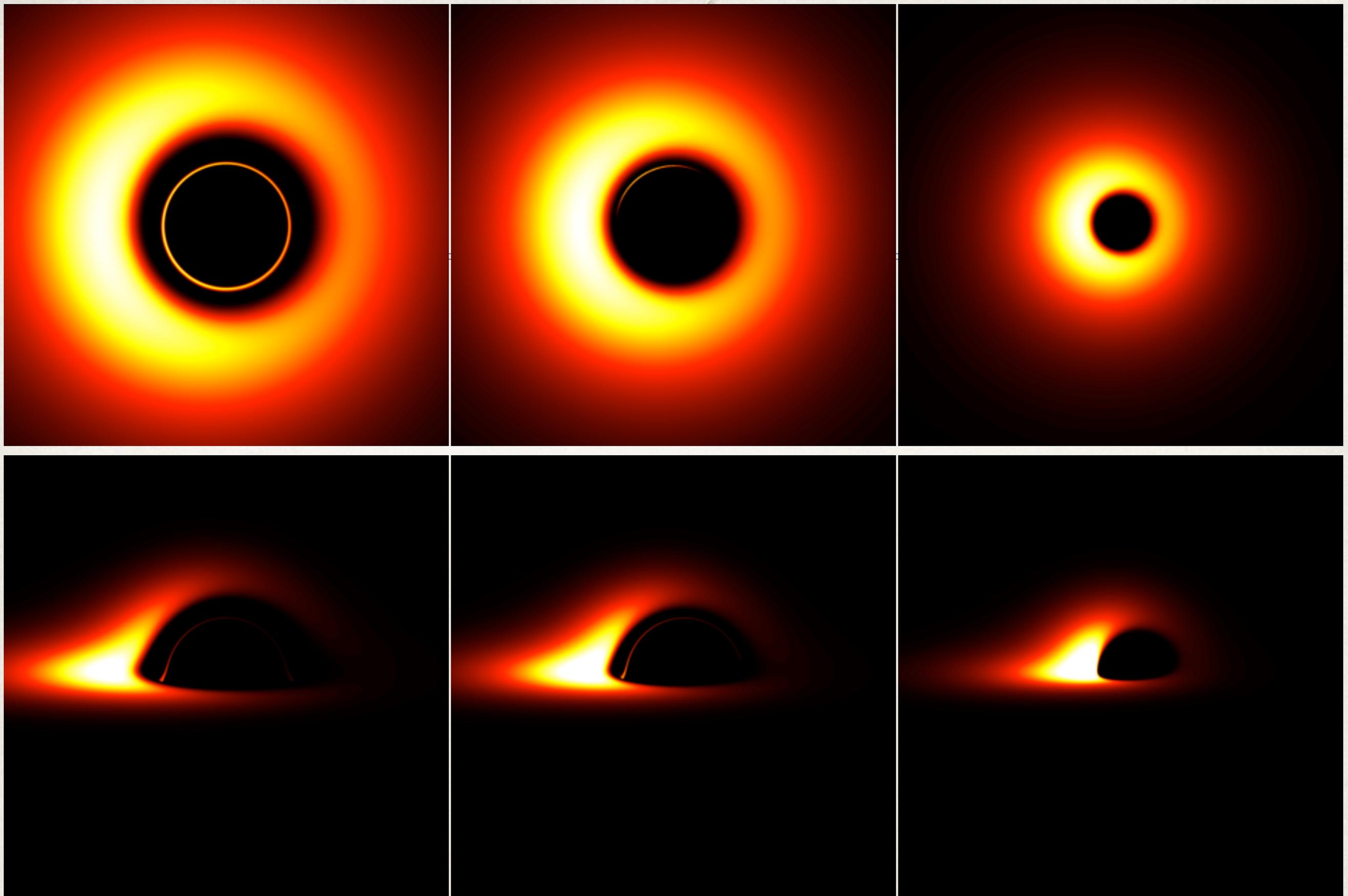


光の軌道の湾曲  
重力赤方偏移  
時空の引きずり

青字：  
特殊相対論効果

赤字：  
一般相対論効果

表示範囲  $20R_s \times 20R_s$   
( $R_s$  : Schwarzschild半径)  
(図の中心がBHの中心)

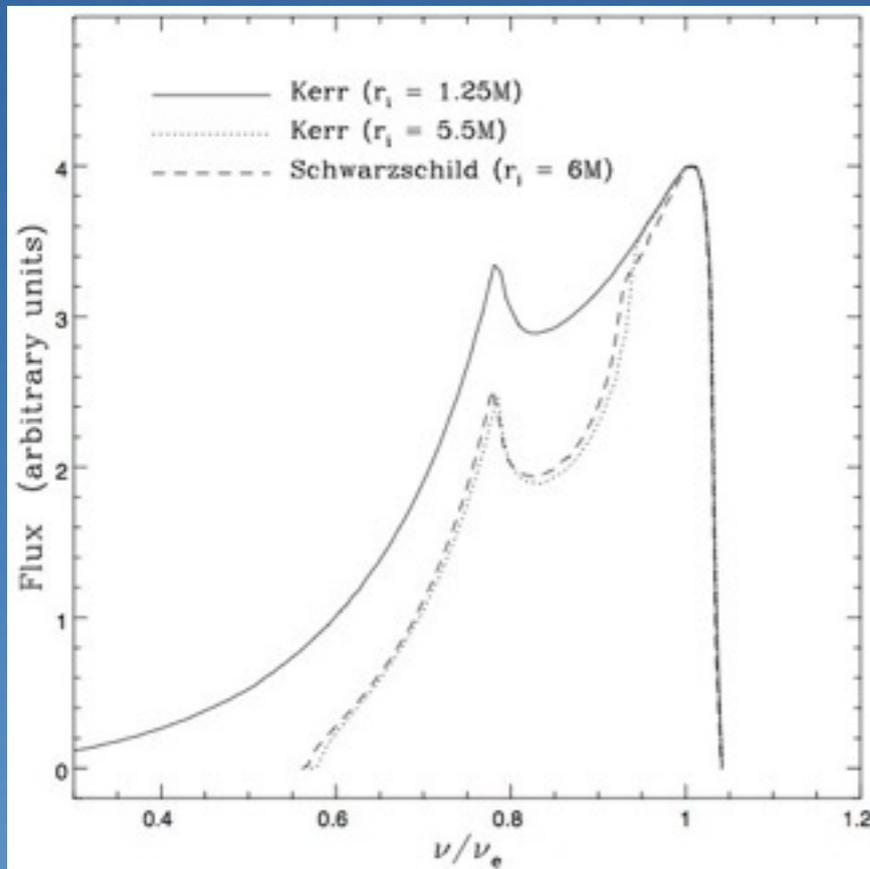


**光学的に厚く幾何学的に薄いブラックホール降着流のイメージ** Kerrブラックホールの スピン(回転の度合いを表す):  $a^*=0, 0.5, 0.999$  (左図から右図), 観測者が見る位置:  $i=10$ 度(上の図),  $85$ 度(下の図), これらは一般相対論的輻射輸送計算により計算した。

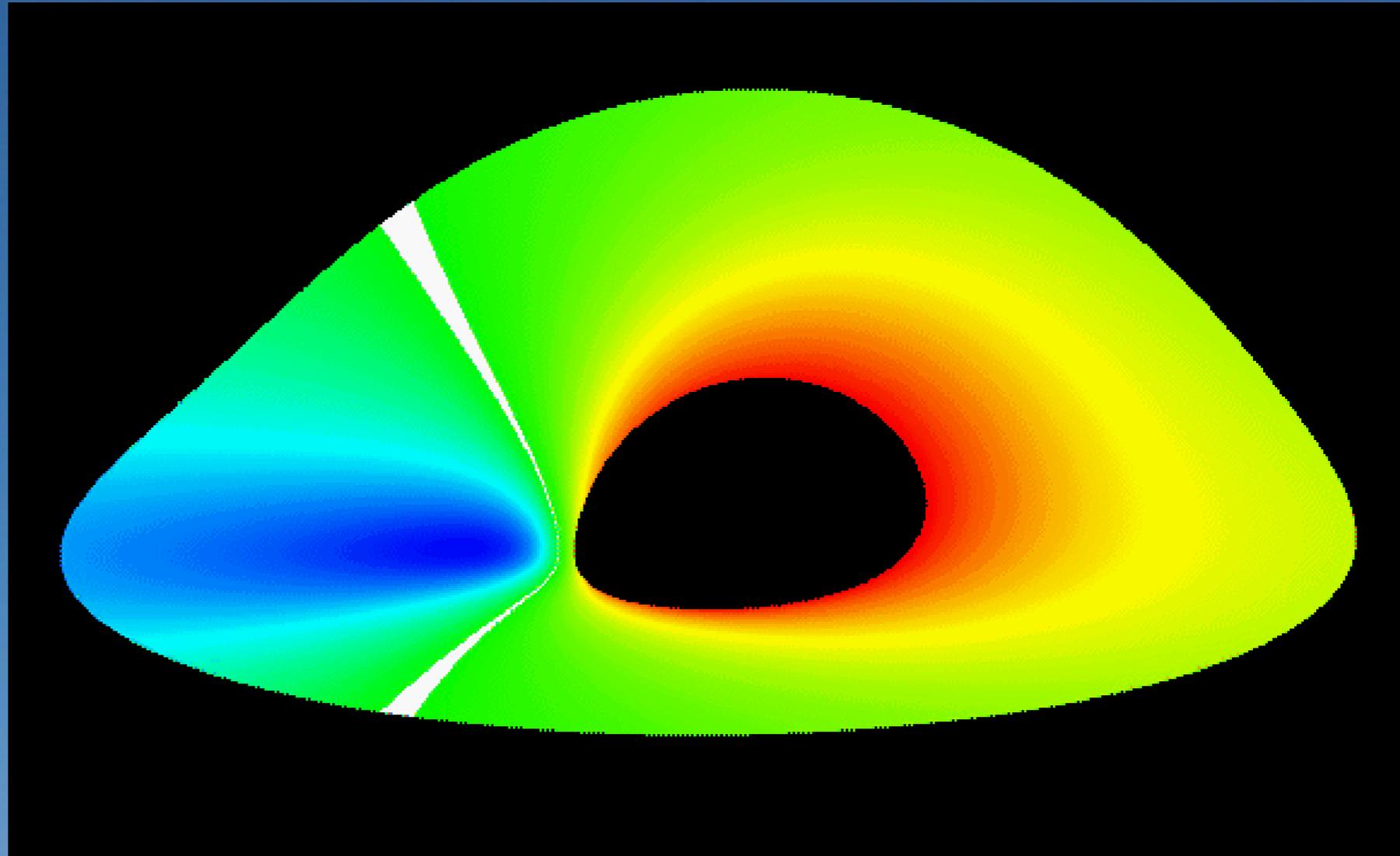
# Bromely, Chen & Miller 1997

B. C. Bromley, K. Chen & W. Miller, The Astrophysical Journal 475, 57-64 (1997)

“Line Emission From an Accretion Disk around a Rotating Black Hole: Toward a Measurement of Frame Dragging”

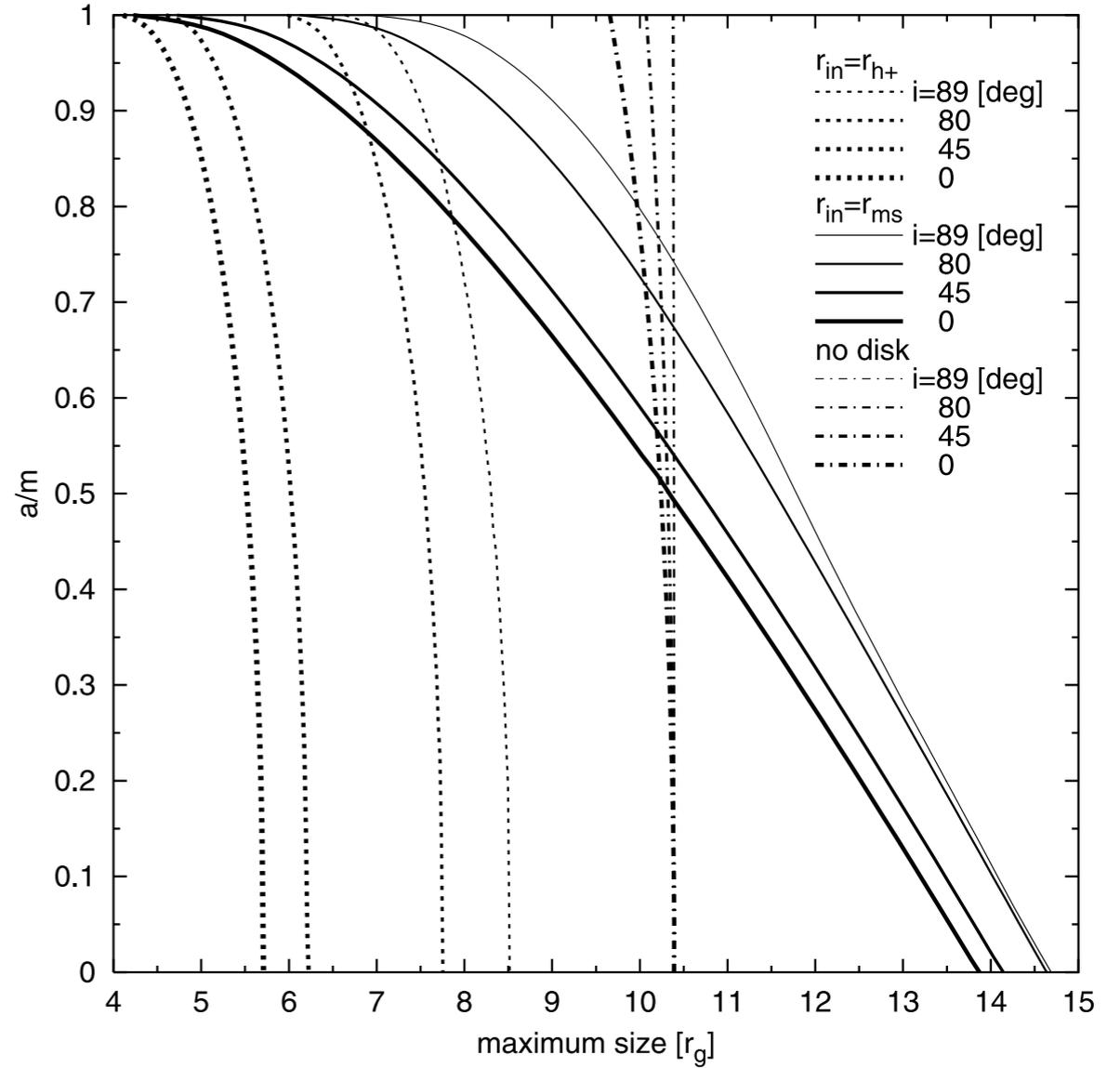
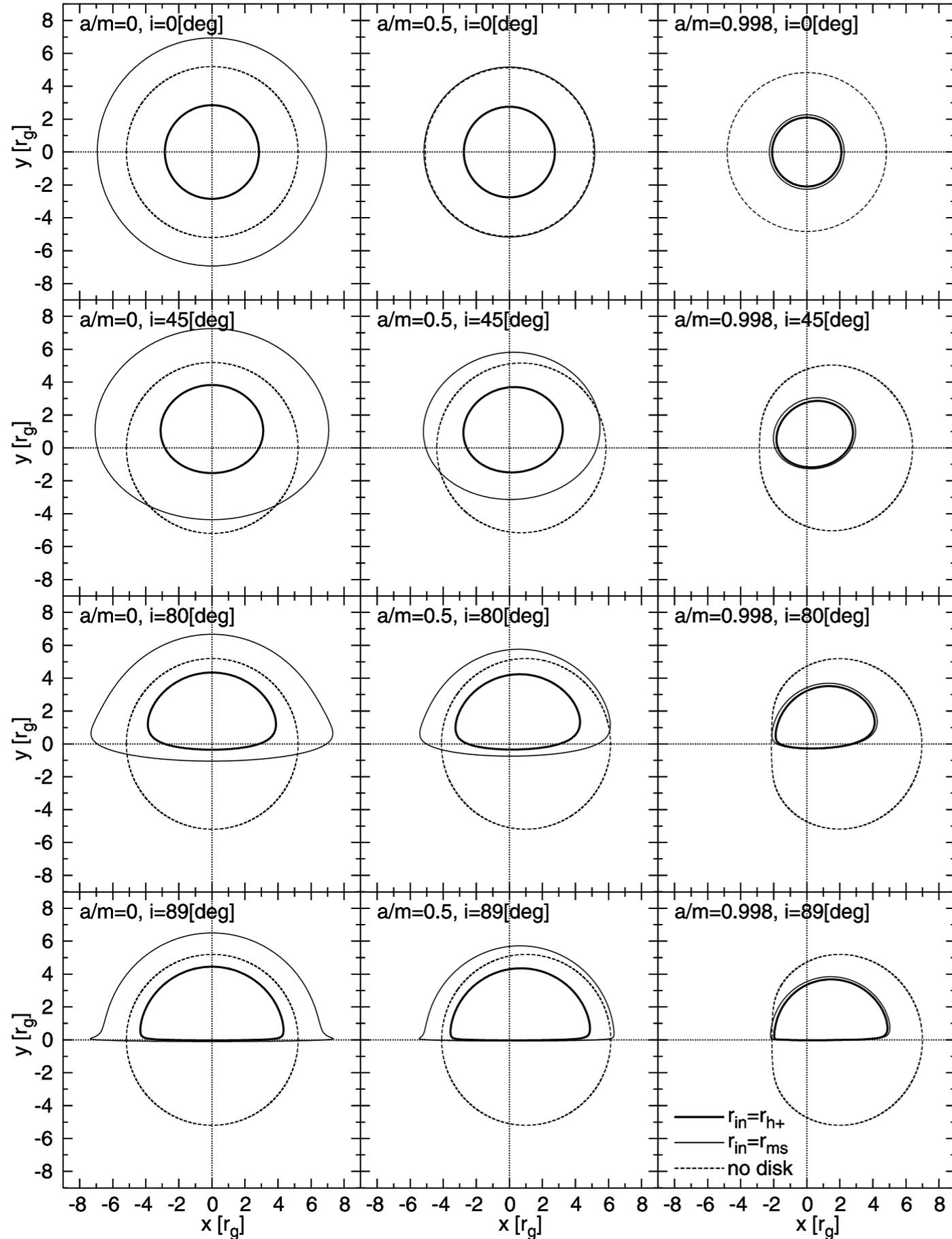


Line Spectrum

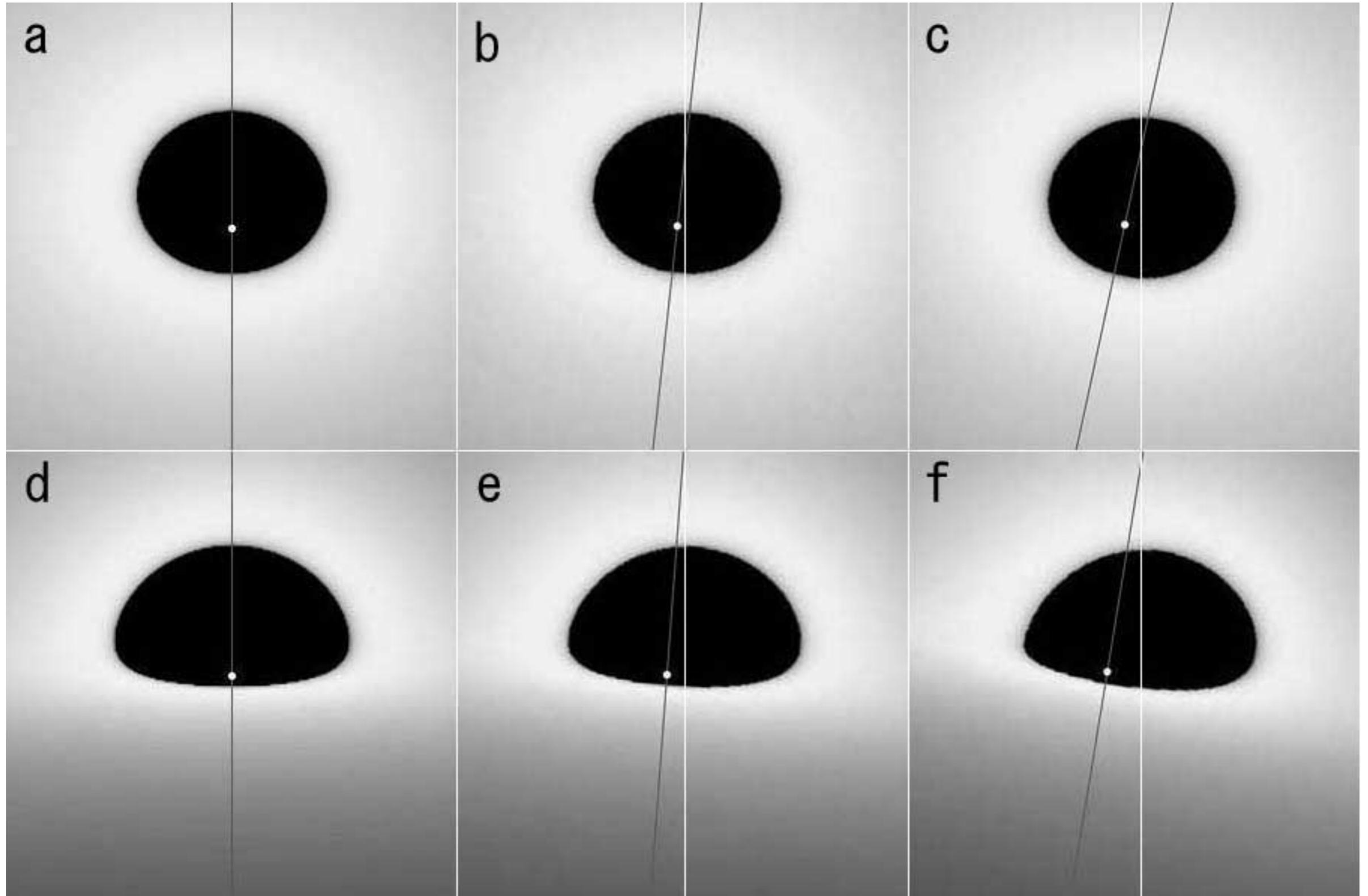


- \* maximum Kerr ( $a=M$ ), inclination angle=75[deg]
- \* image of redshift factor
- \* 1200 x 1200 pixels
- \* 15 minutes using 128 nodes on a Cray T3D Supercomputer

# 降着円盤中のBHシャドウの輪郭



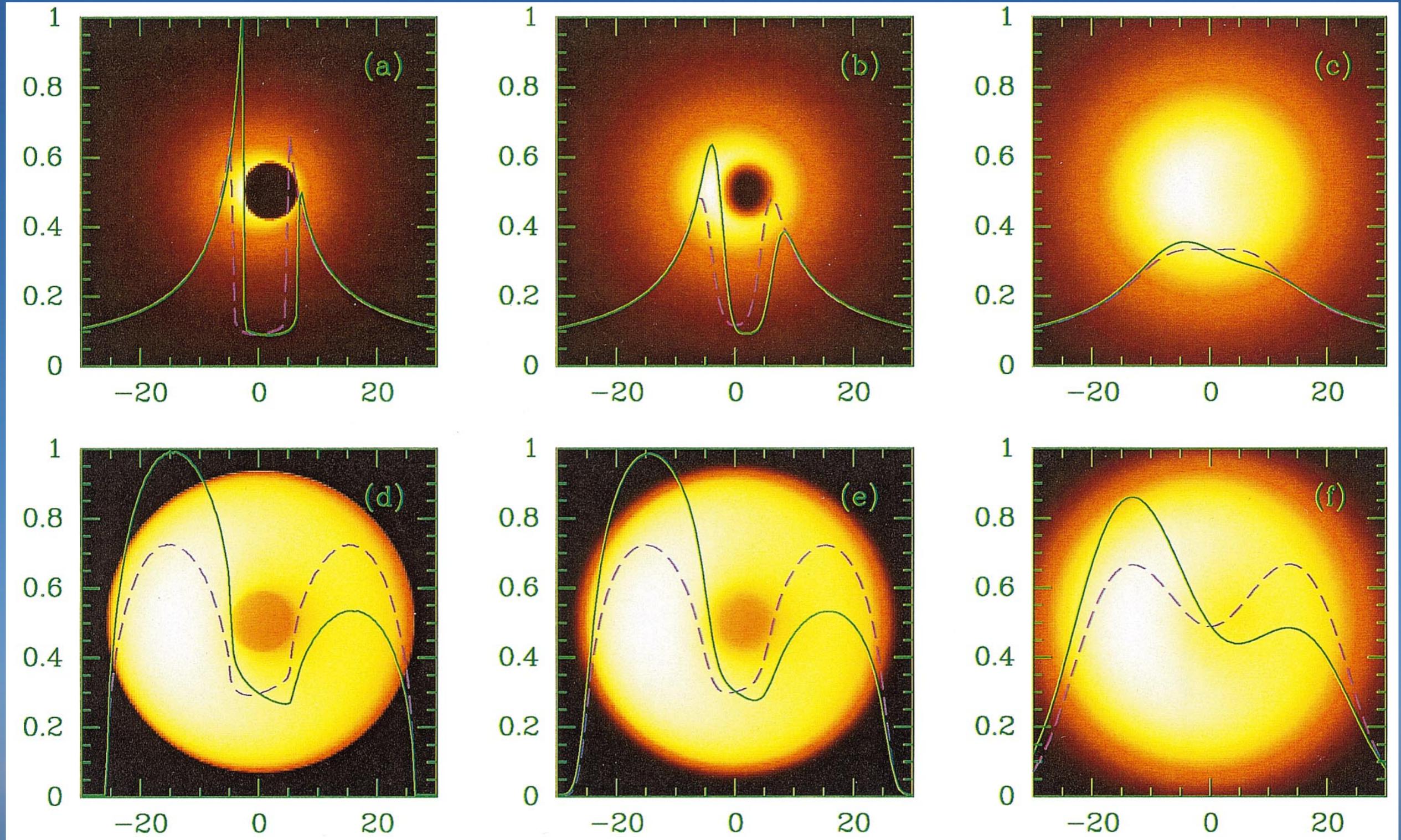
# 降着円盤中のBHシャドウの輪郭

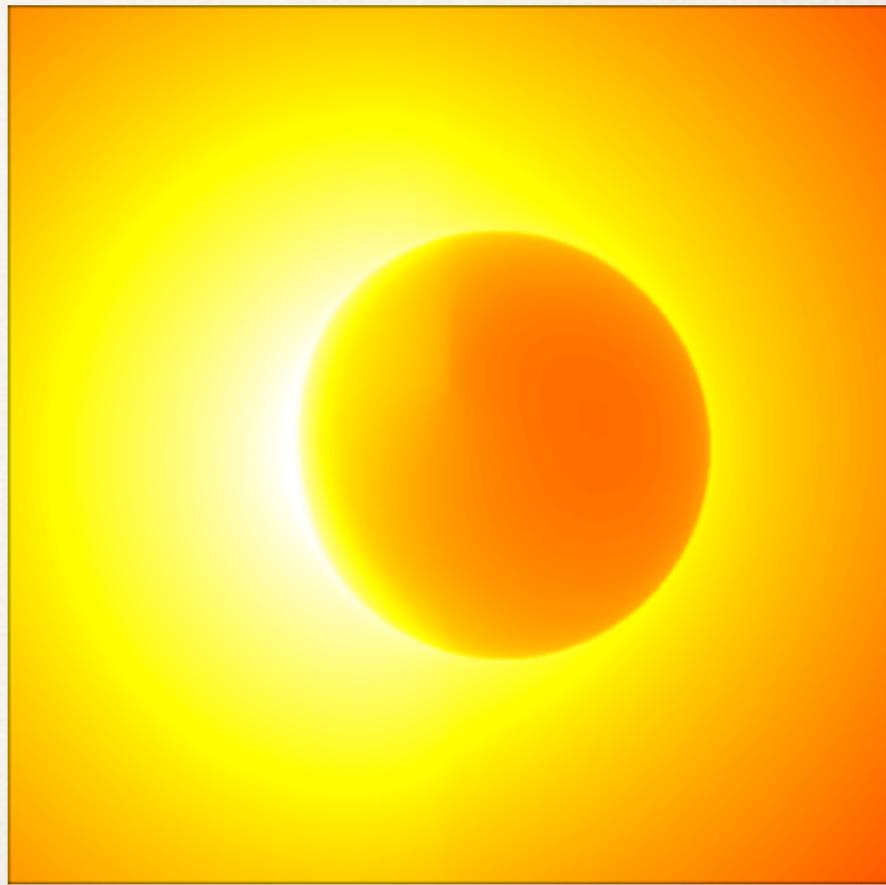
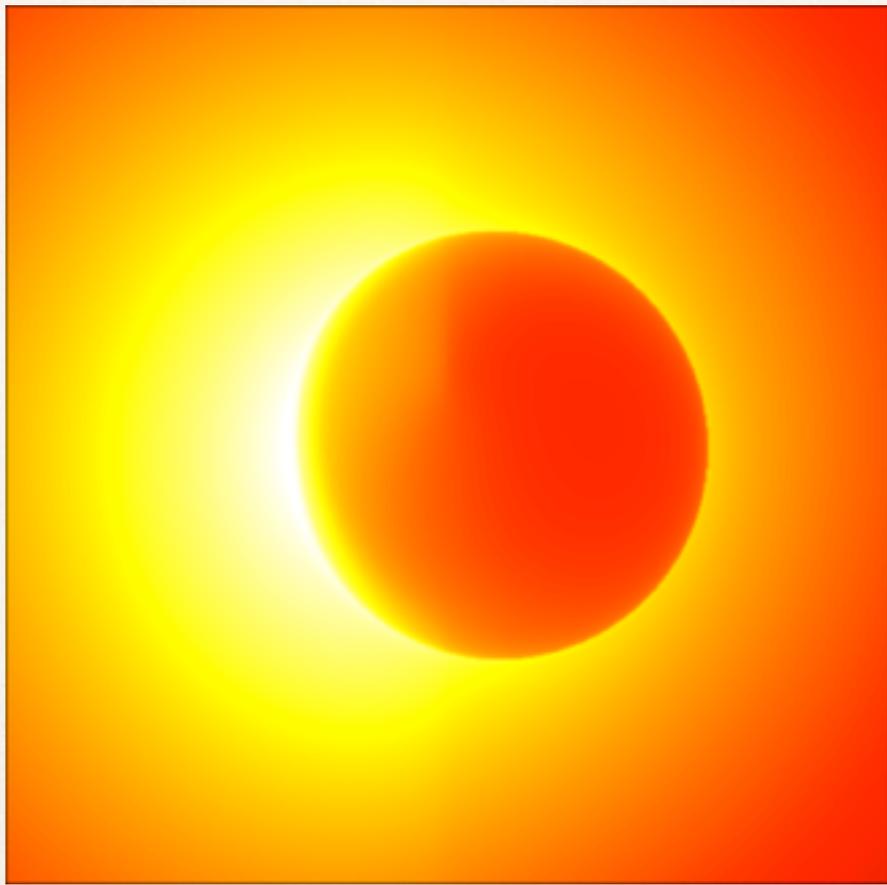
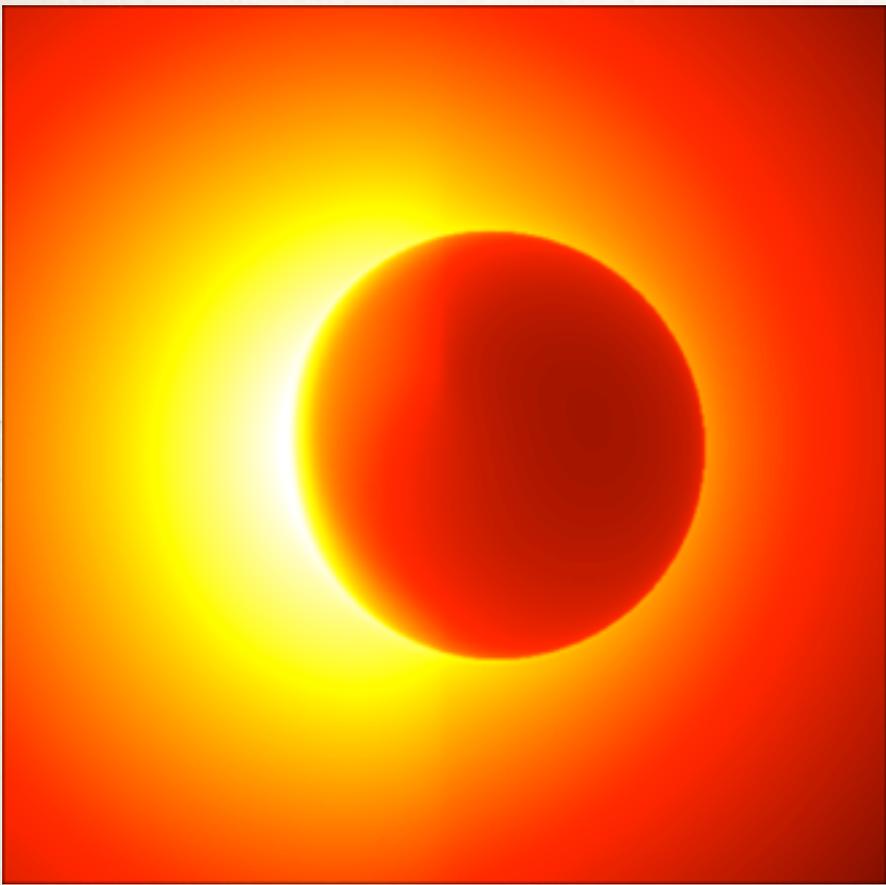
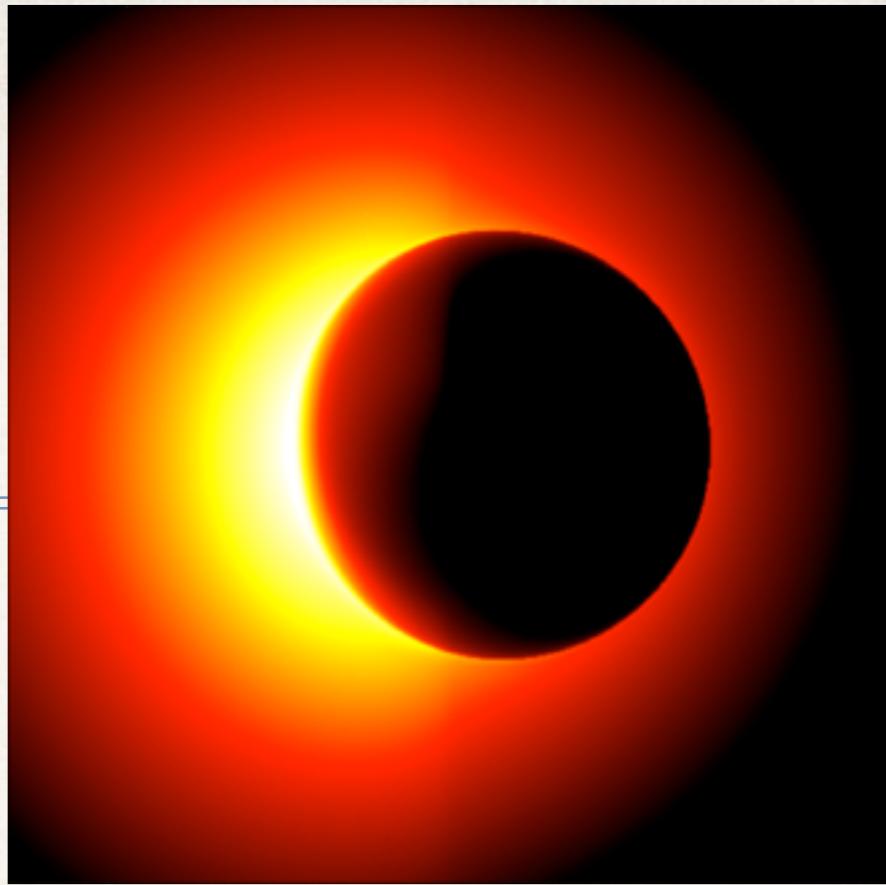
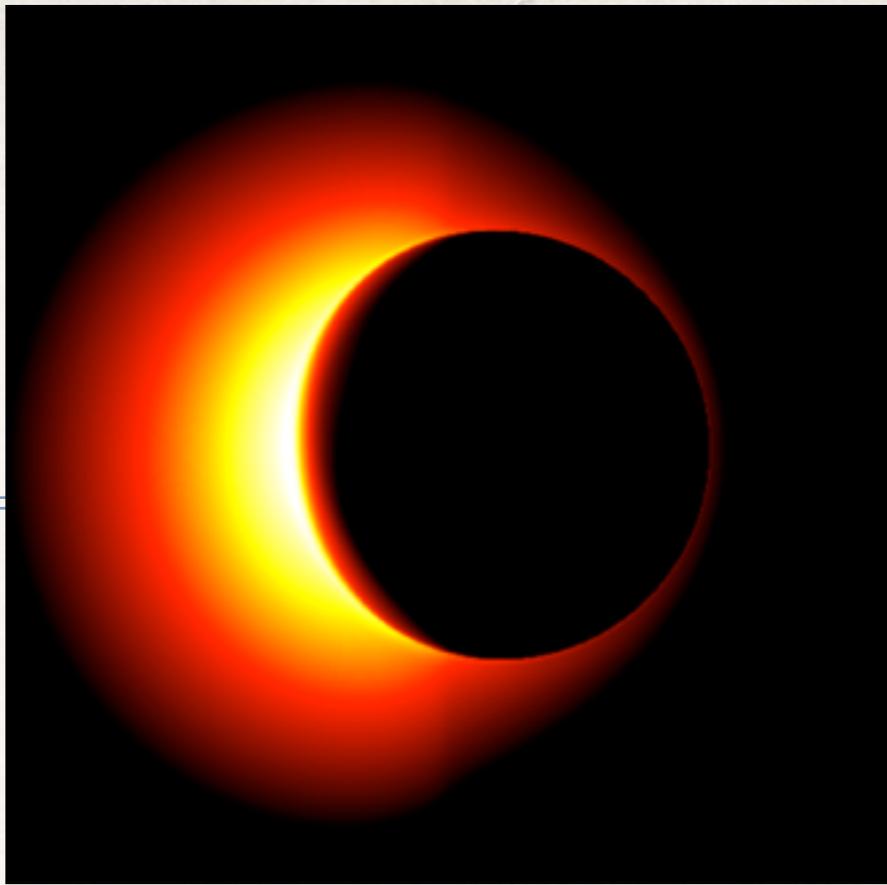
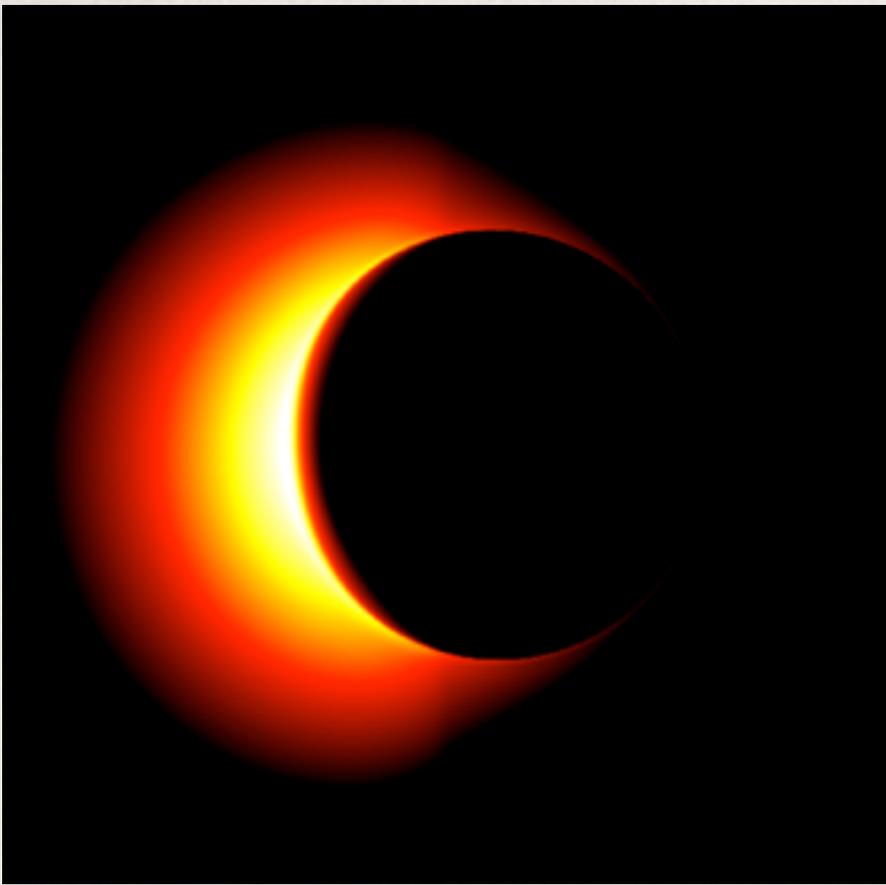


# Falcke, Melia & Agol 2000

H. Falcke, F. Melia & E. Agol, The Astrophysical Journal 528, L13-L16 (2000)

“Viewing the Shadow of the Black Hole at the Galactic Center”

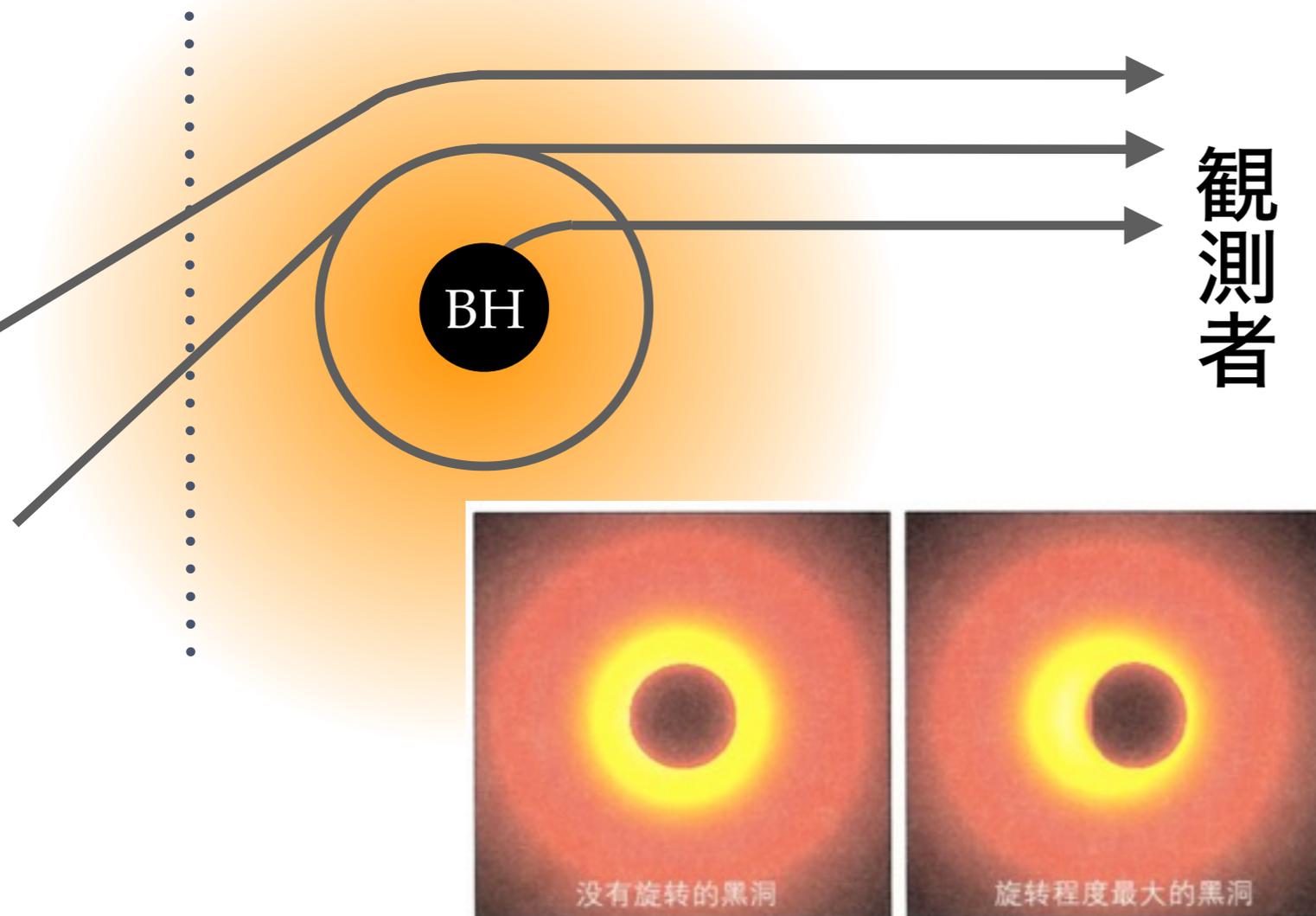




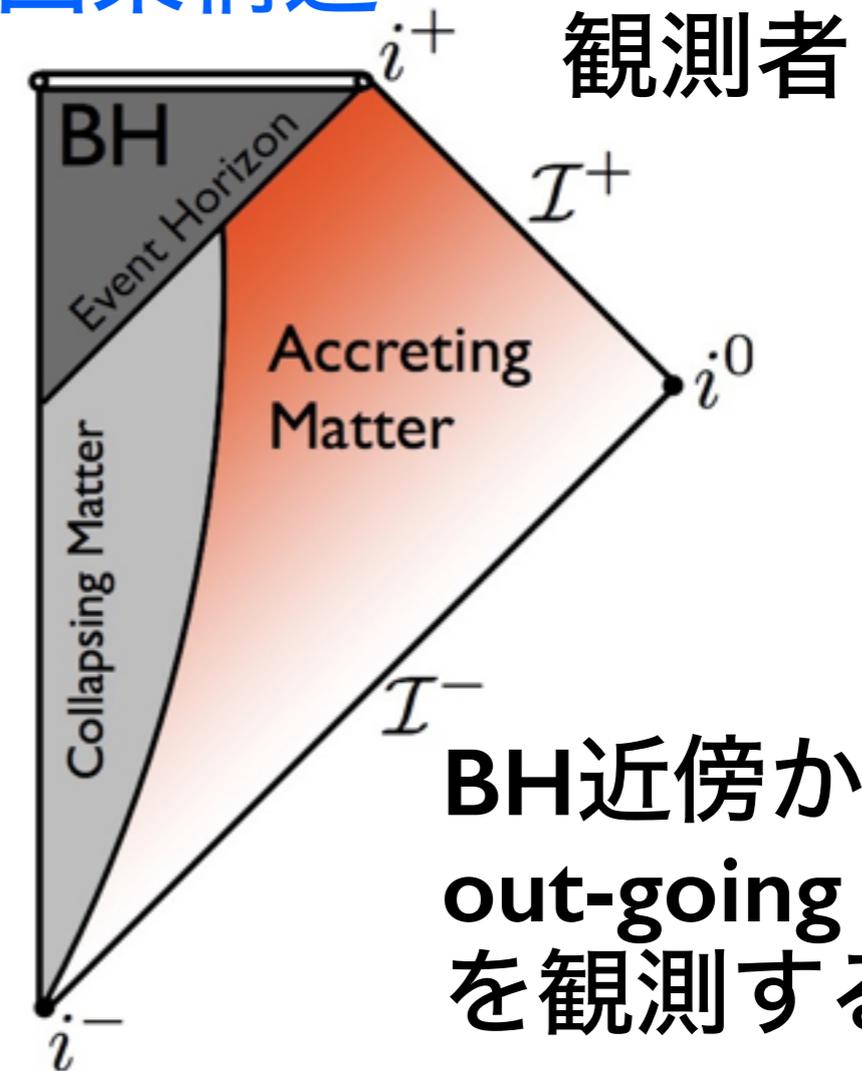
**光学的に薄く幾何学的に厚いブラックホール降着流のイメージ** 最大回転するブラックホールを仮定し、観測者が見る位置は85度とした。同じデータに対し、カラーの付け方を変えることにより異なるイメージを作成した。最も明るく輝いている部分は、ブラックホール回転が引き起こす時空の引きずりの効果により青方偏移している部分に対応する。

# 電磁波では直接BHを見ることはできない

光学的に薄い降着流



因果構造



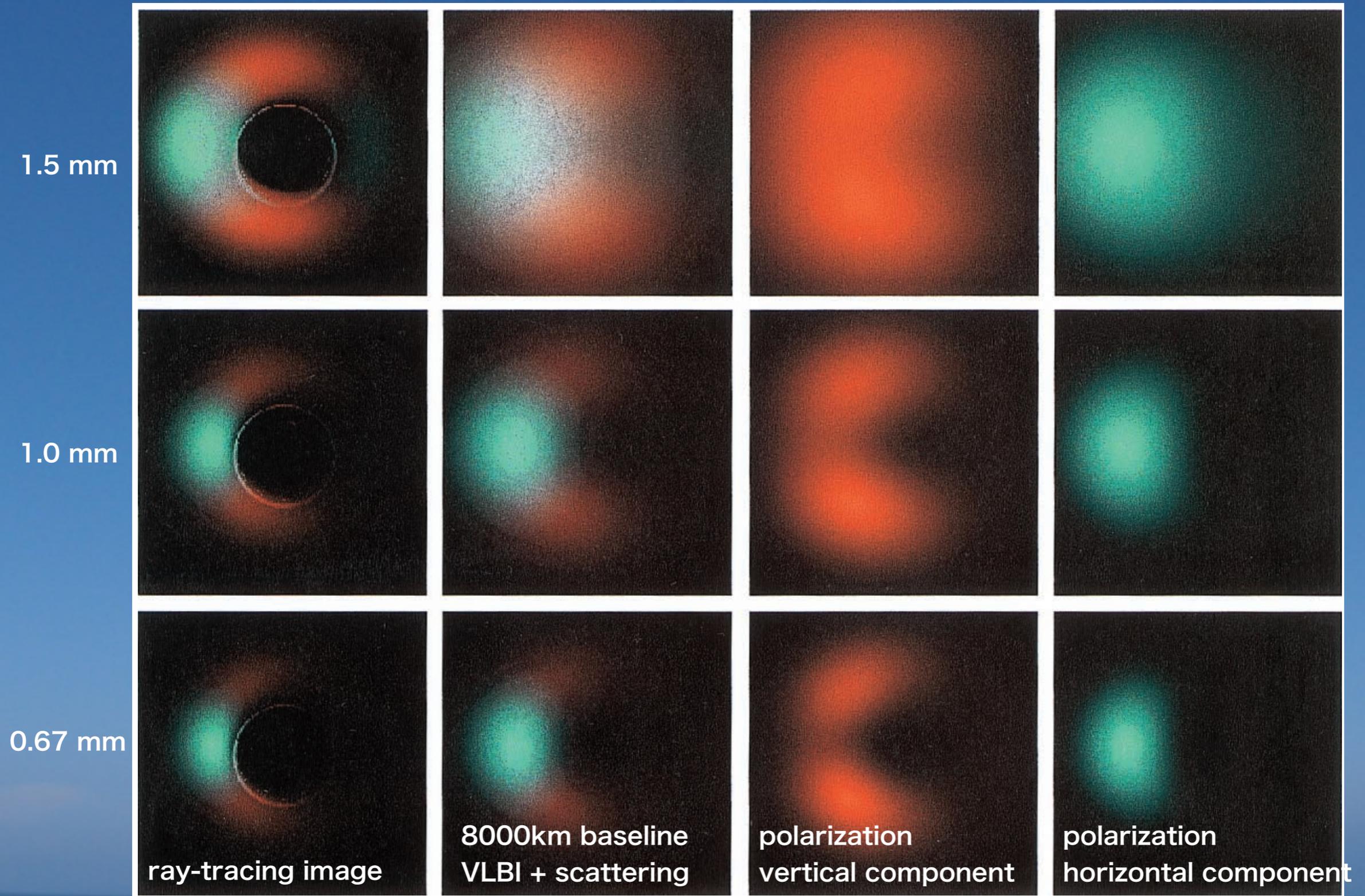
BH近傍から来る  
out-going null  
を観測する。

- 背景光の手前のBHのシルエットが見える。間接的に見る。
- 中心の低輝度領域をBH shadowと呼ぶ。
- 光学的に薄い波長での観測が必要（一般に暗いイメージ）。

# Bromley, Melia & Liu 2001

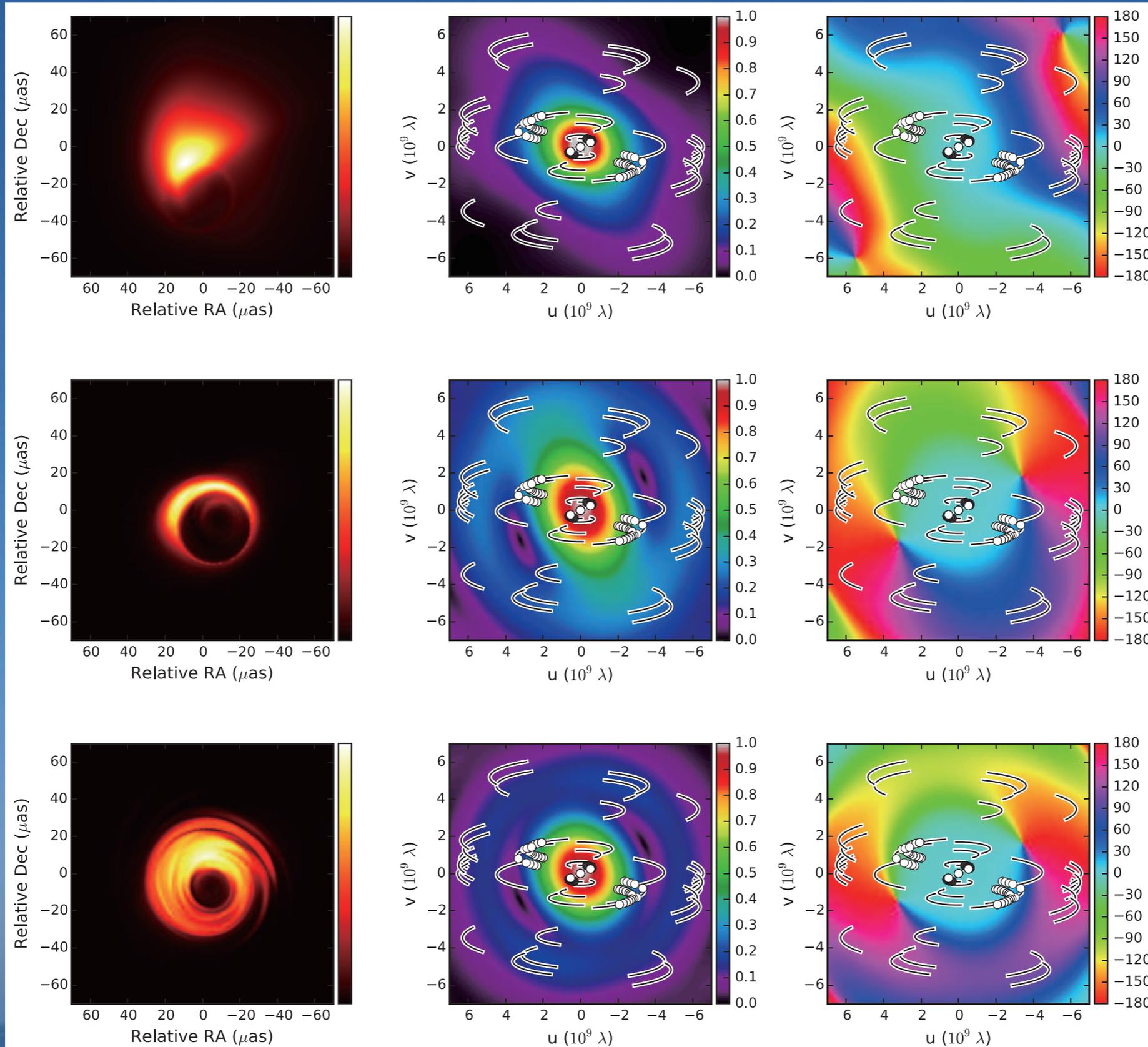
B. C. Bromley, F. Melia & S. Liu, The Astrophysical Journal 555, L83-L86 (2001)

“Polarimetric Imaging of the Massive Black Hole at the Galactic Center”



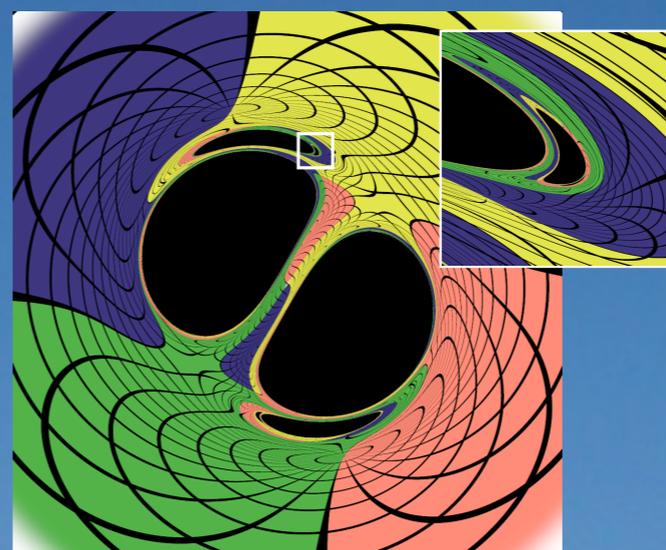
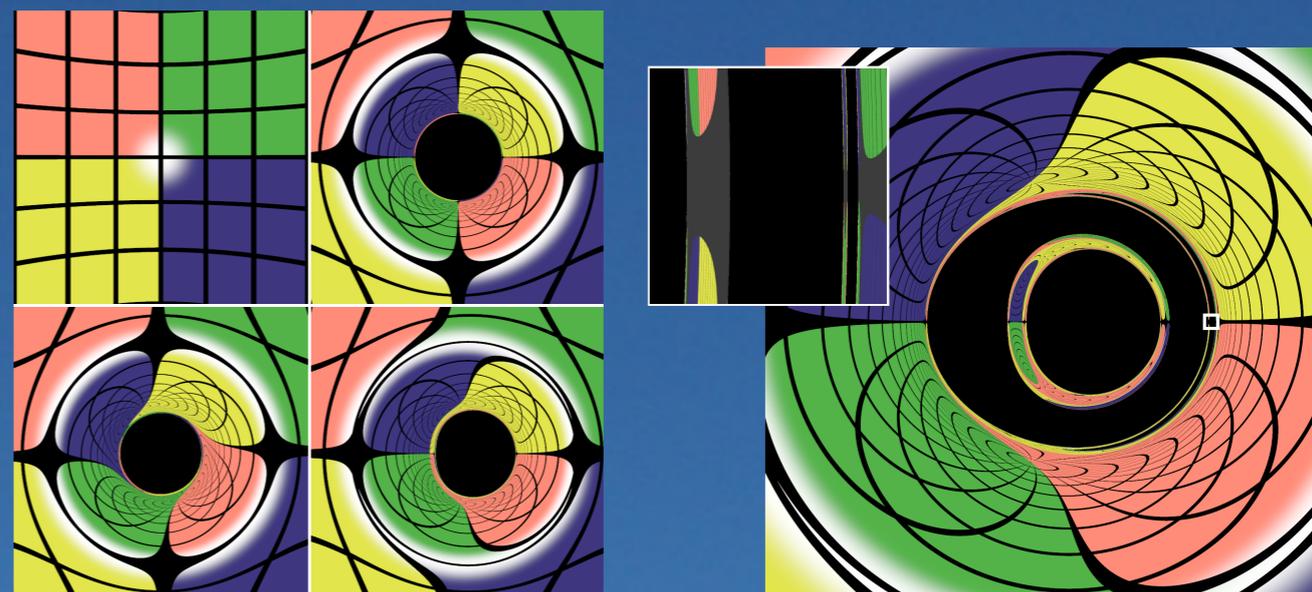
K. Akiyama et al., astro-ph/1505.0354 (2015)

# “230 GHz VLBI Observations of M87: Event-Horizon-Scale Structure at the Enhanced Very-High-Energy $\gamma$ -ray State in 2012”



A. Bohn et al., Classical and Quantum Gravity, 32, 065002 (2015)

“What does a binary black hole merger look like?”



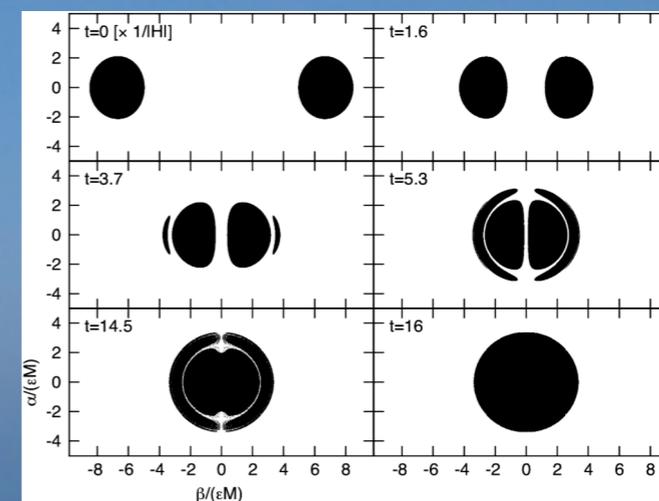
see, also,

D. Nitta, T. Chiba, N. Sugiyama, Physical Review D, 84, 063008 (2011)

“Shadows of colliding black holes”

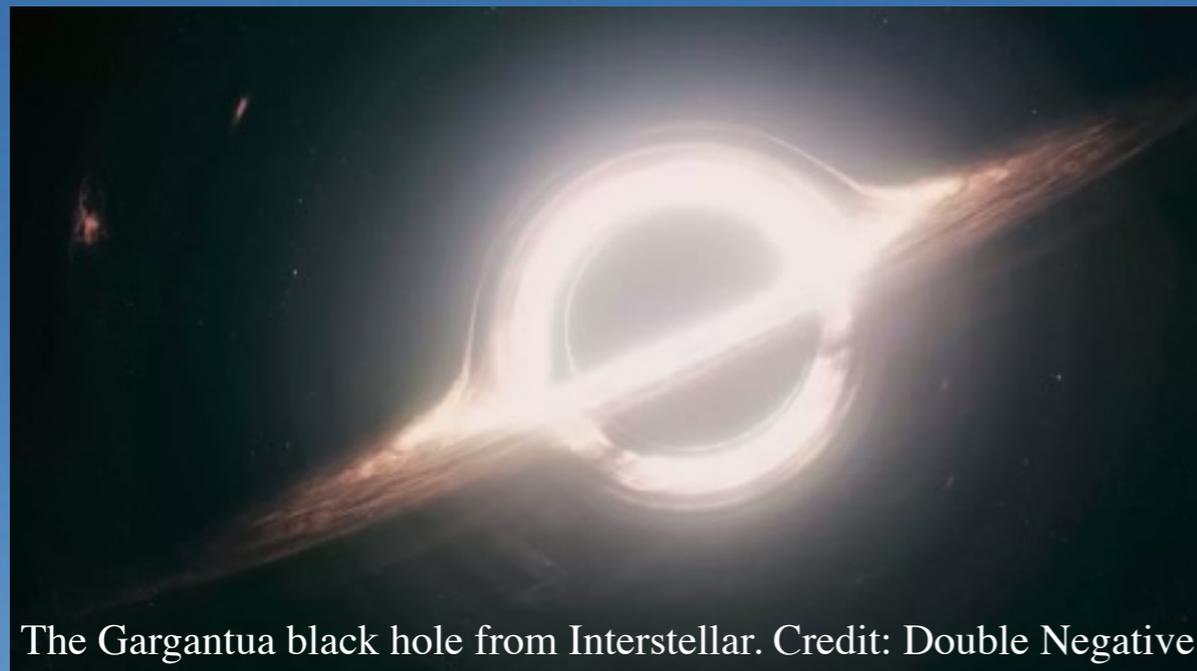
A. Yumoto, D. Nitta, T. Chiba, N. Sugiyama, Physical Review D, 86, 103001 (2012)

“Shadows of multi-black holes: Analytic exploration”

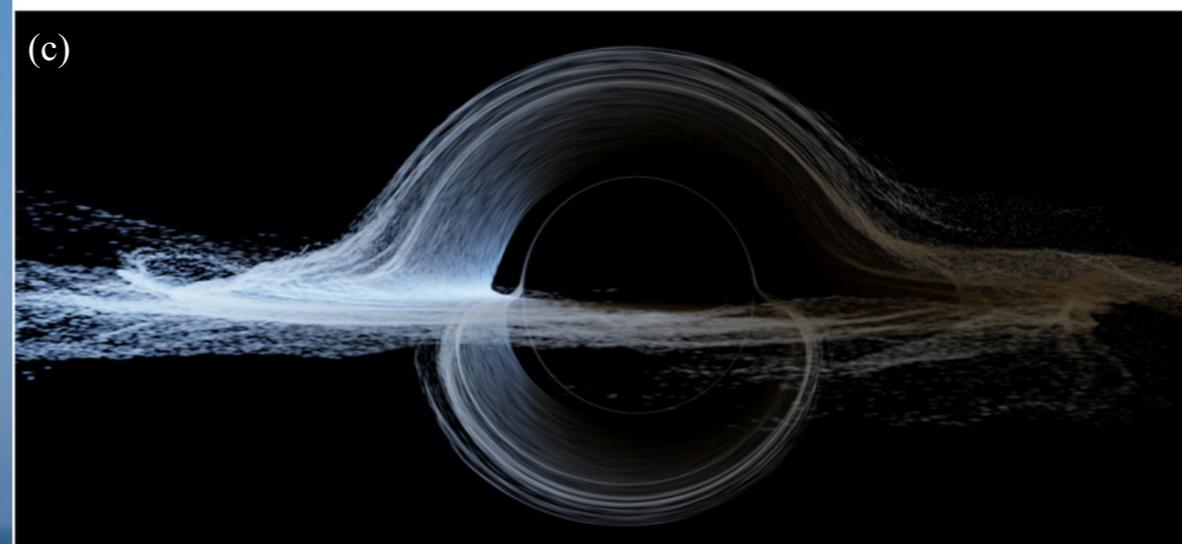
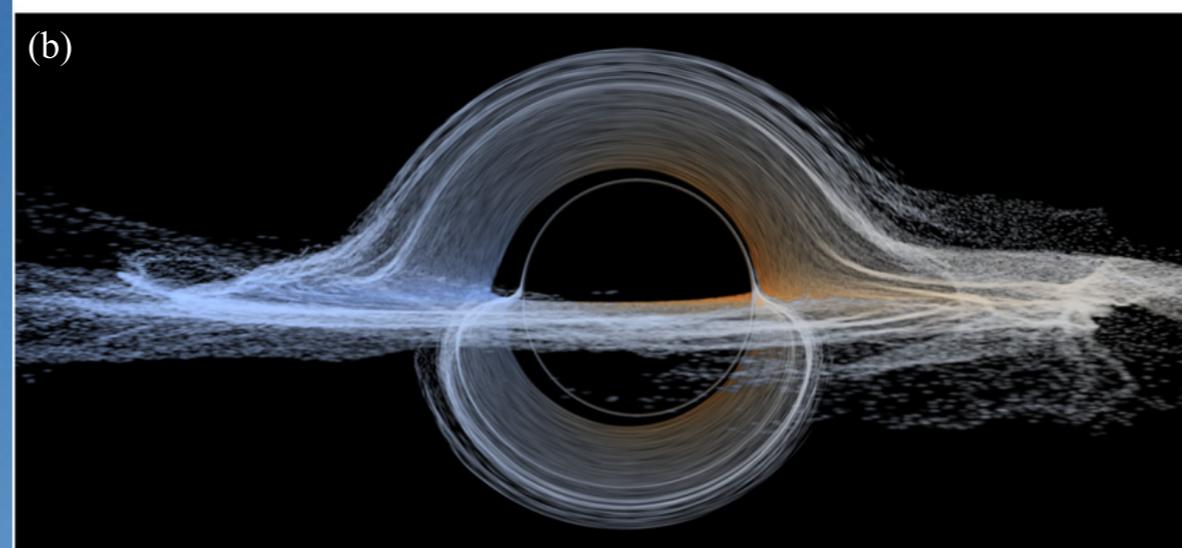


O. James et al., *Classical and Quantum Gravity*, 32, 065001 (2015)

“Gravitational lensing by spinning black holes in astrophysics, and in the movie *Interstellar*”

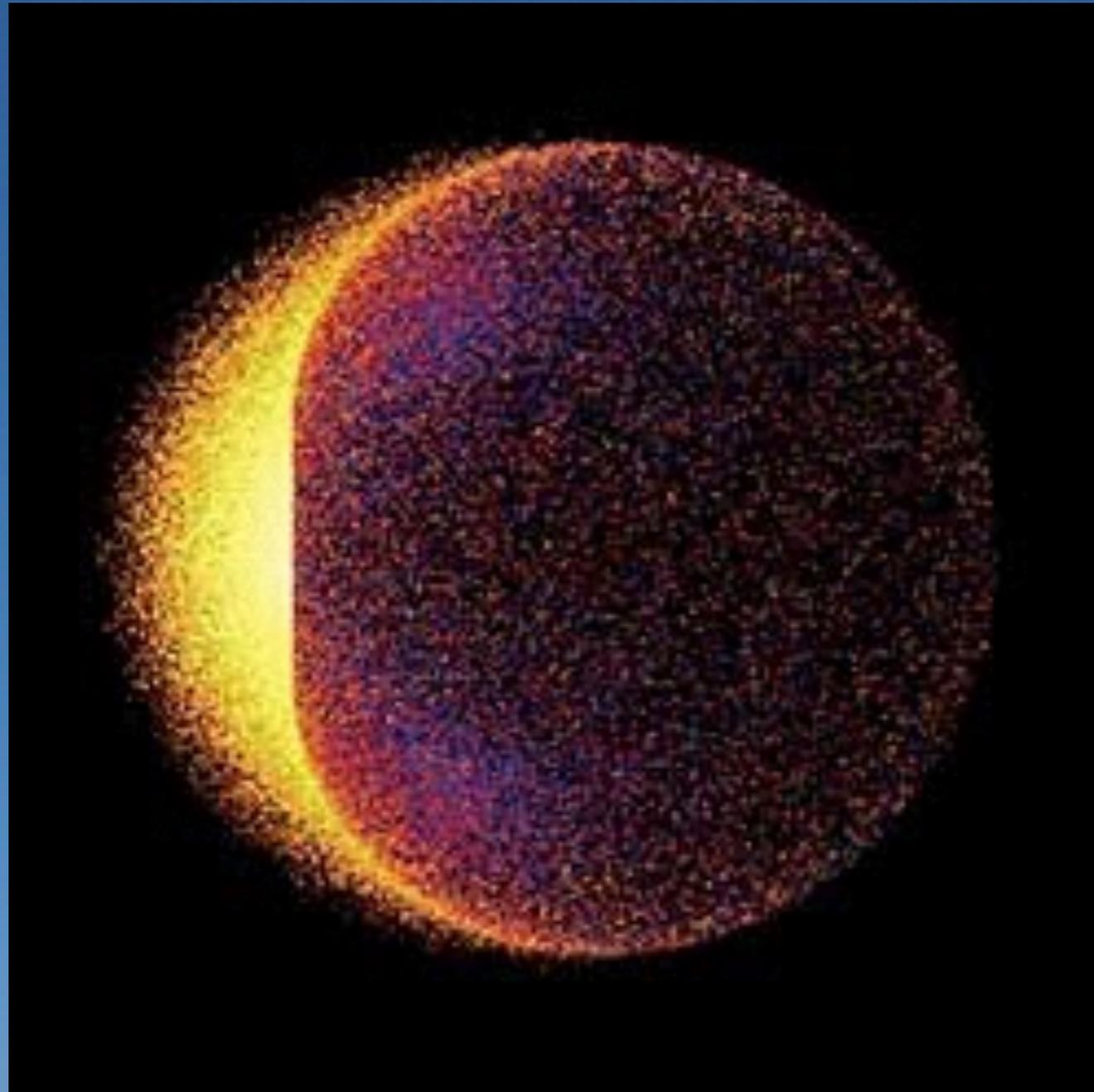


The Gargantua black hole from *Interstellar*. Credit: Double Negative



J. D. Schnittman, APS April Meeting 2015, Baltimore

“The Distribution and Annihilation of Dark Matter around Black Holes”



see, also,

J. D. Schnittman, Physical Review Letters, 113, 261102 (2014)

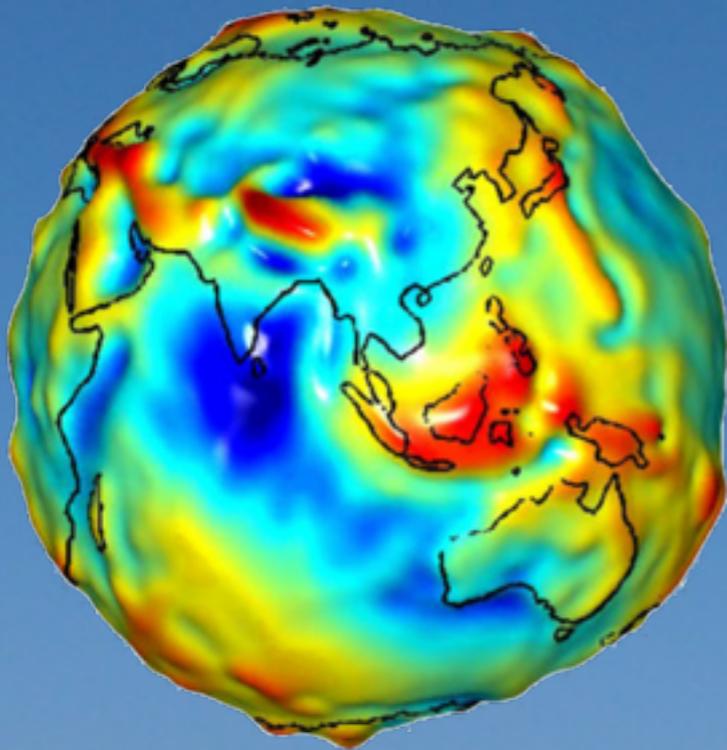
“Revised Upper Limit to Energy Extraction from a Kerr Black Hole”

# Beyond Kerr Spacetime

## \* 重力場の分布

- 地球や月で測定されている
- 重力場分布の記述  
→ 球面調和関数展開

$$f(r, \theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^l a_{lm} r^l Y_l^m(\theta, \phi)$$



GRACE Gravity Model

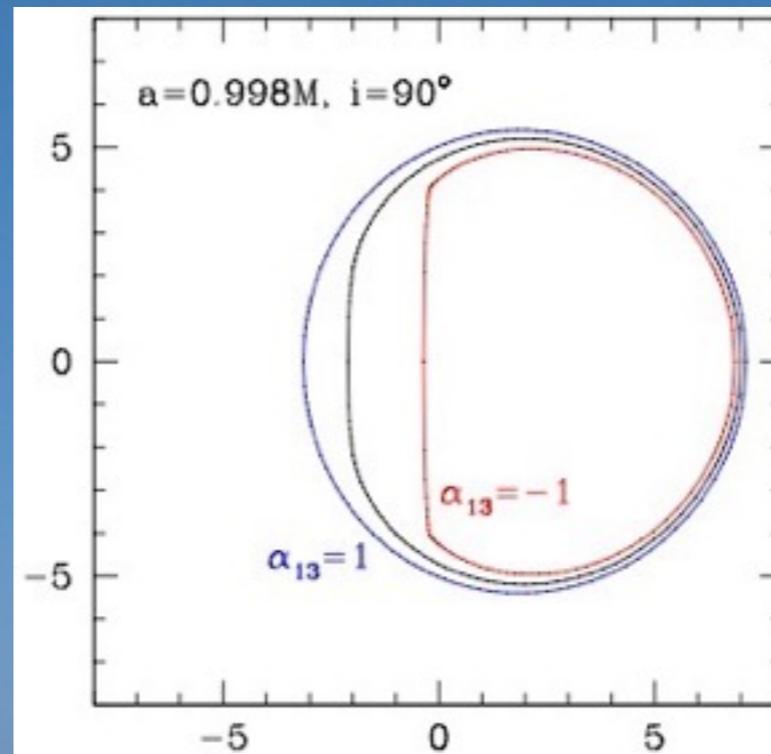
- \* No Hair Theorem  $M_l + iS_l = M(ia_*)^l$   
( $M_l$  : mass multipole,  $S_l$  : current multipole)  
Geroch(1970), Hansen (1974)

## \* multipole structure の測定

→ Kerr時空の検証, 重力理論の検証

$$M_l + iS_l = M(ia_*)^l + \delta M_l + i\delta S_l$$

方法 A : Kerr時空からの摂動をできるだけ一般的に記述  
(発展途上)



e.g.

Johannsen & Psaltis (2011)

Johannsen (2013)

Cardoso, Pani & Rico (2014)

摂動Kerr時空でのBHシャドウの輪郭  
T. Johannsen, ApJ, 777, 170 (2013)

方法 B : Case-by-case analysis

→ Black Hole Shadow Zoo

# Black Hole Shadow Zoo

Schwarzschild BH  
(Synge 1966)

Kerr BH  
(Bardeen 1973)



## \* Contour of a Shadow

Kerr-Newman BH (de Vries 2000), Schwarzschild-de Sitter BH (Bakala et al. 2007), Sen BH (Hioki & Miyamoto 2008), braneworld Kerr BH (Schee & Stuchlik 2009), Tomimatsu-Sato spacetime (Bambi & Yoshida 2010), dynamical Chern-Simons BH (Amarilla, Eiroa & Giribet 2010), Kastor-Traschen cosmological multi-BH solution (Nitta, Chiba & Sugiyama 2011), rotating braneworld BH (Amarilla & Eiroa 2012), Majumder-Papapetrou solution (Yumoto et al. 2012), Kalza-Klein rotating dilaton BH (Amarilla & Eiroa 2013), Kerr-Taub-NUT BH (Abdujabbarov et al. 2013), rotating Horava-Lifshitz BH (Atamurotov, Abdujabbarov & Ahmedov 2013), rotating non-Kerr BH (Atamurotov, Abdujabbarov & Ahmedov 2013), Einstein-Maxwell-dilaton-axion BH (Wei & Liu 2013), Kerr-Newman-NUT(Newman-Unti-Tamburino) BH with a cosmological constant (Grenzebach, Perlick & Lammerzahl 2014), a five-dimensional rotating Myers-Perry BH (Papnoi et al. 2014), Schwarzschild-MOG(modified gravity) & Kerr-MOG (Moffat 2015), binary BH (Bohn et al. 2015), Plebanski-Demianski spacetime (Grenzebach, Perlick & Lammerzahl 2015), a Schwarzschild BH in an external gravitational field (Abdolrahimi, Mann & Tzounis 2015)

他に、

- \* Black Hole Shadow in an Accretion Disk
- \* Shadow of a naked singularity (e.g. super-spinar, overspinning Kerr)
- \* Shadow of other objects (e.g. boson star)

他にもあるだろう・・・

## まとめ

- # 過去のブラックホール・シャドウの研究をいくつか紹介した。  
(全てを網羅したわけではない)
- # 電波干渉計での観測により、もうすぐシャドウがとらえられるかもしれない。現在は、フーリエ空間中の位相情報が観測的に得られ始めた。
- # 将来的には、Kerr時空の検証、重力理論の検証などが重要。

おしまい