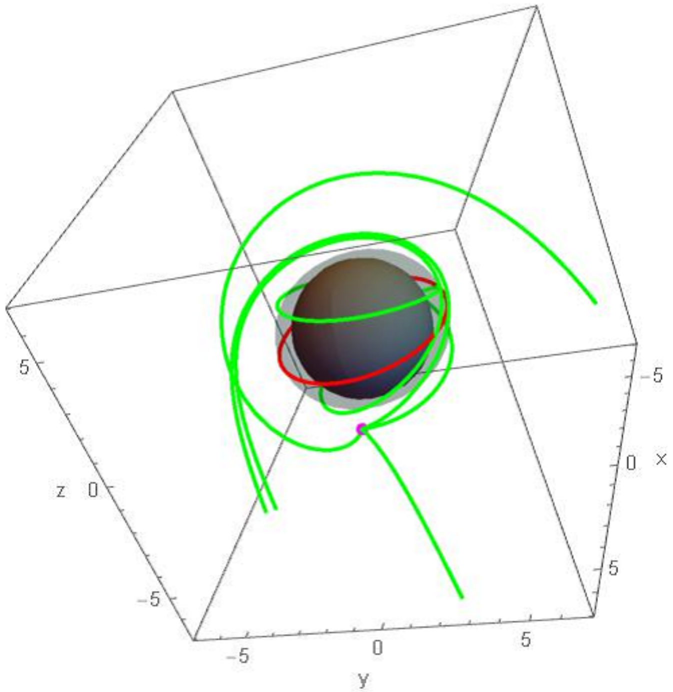
BHの強レンズ効果の観測による BHの質量と自転角運動量の測定原理



SAIDA Hiromi
(Daido Univ.,Japan)

* Under preparation for paper submission

第 9 回 BH 磁気圏研究会 (マウントレースイ，夕張)	2016.Mar.2-5 (Mar.5, talk)
第 21 回特異点研究会 (慶応大学)	2016.Jan.9-11 (Jan.10, talk)
28th Texas Sympo. (Geneva, Swiss)	2015.Dec.14-18 (Dec.17, talk)

Working Assumption

Gravity is described by the General Relativity.

$\rightarrow$ If the difference from GR would be observed,
it would be the time to consider
the modified gravity theory.

1. Introduction : Basic idea

1.1 From candidate to itself

- Best observational knowledge of BH at present

  $\rightarrow$ BH candidates by Newtonian gravity

  $\Updownarrow$ Large Gap in Physics !!

- BH is a general relativistic (GR) object

  $\rightarrow$ The method to find "BH itself" is at least a direct detection of the GR effect of BH.

What is it? How can we do it?

1.2 Meaning of BH detection in GR context

- Theoretical (mathematical) fact in GR

Uniqueness Theorem

Asymptotic flat BH spacetime is uniquely specified by 3 parameters:

$$
\begin{aligned}
M_{\rm BH} &: \text{ mass} \\
J_{\rm BH} &: \text{ spin angular momentum} \\
Q_{\rm BH} &: \text{ electric charge}
\end{aligned}
$$

◇ $Q_{\rm BH} = 0$ is expected for real situations.

$\rightarrow$ **BH is specified by $M_{\rm BH}$ and $J_{\rm BH}$.** (Kerr BH)

- Define the meaning of "direct" detection of BH

BH Detection is ...

To measure the parameters M and χ by detecting the GR effect of BH.

◇ Mass in length scale: $M = \dfrac{GM_{\rm BH}}{c^2}$ [cm]

◇ Dimensionless spin parameter: $\chi = \dfrac{a}{M}$ [no-dim]

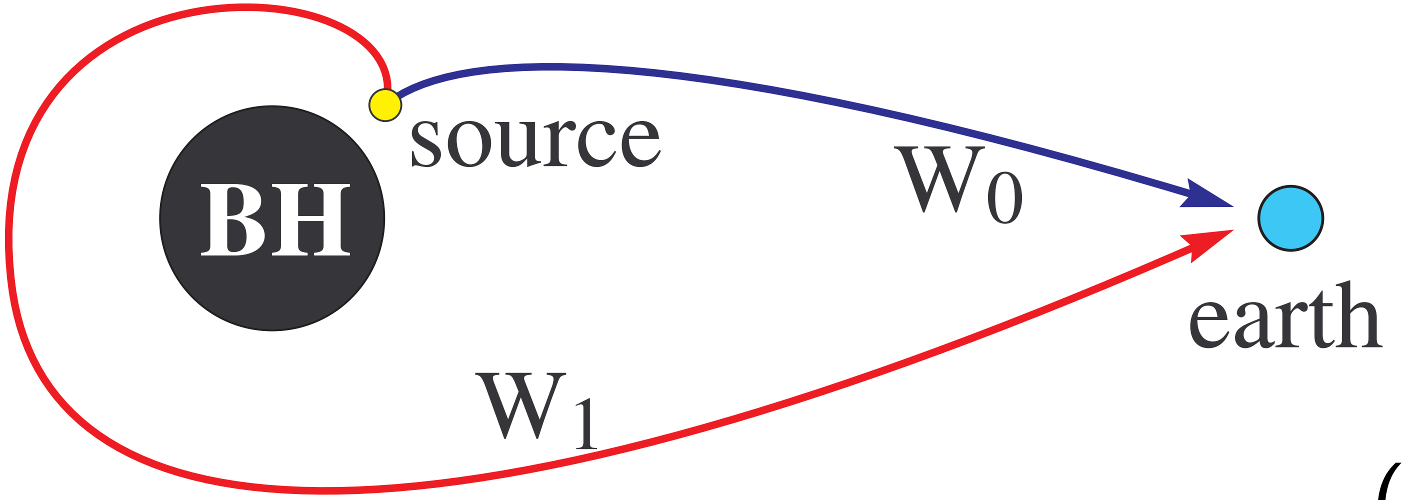
(usual spin parameter: $a = J_{\rm BH}/(M_{\rm BH}\, c)$ [cm])

◇ Kerr BH horizon radius: $r_{\rm BH} = M\left[1 + \sqrt{1-\chi^2}\,\right]$

$\Rightarrow 0 \le \chi < 1$

1.3 GR effect of BH as our target

- Target : **Spinning BH Gravitational Lens** (SGL)
- An ideal situation we want to observe:
  - ◇ Clear environment around BH except the source
  - ◇ Burst-like and spherical emission seen from the source



W_0 : "0th ray" (primary ray)

W_1 : "1st ray" (secondary ray)

$$\Downarrow$$

Basic fact in our situation

Observing two quantities of SGL

$$\begin{cases} \Delta t_{\rm obs} & : \text{ Time delay} \\ \mathcal{R}_{\rm obs} = \dfrac{F_1}{F_0} & : \text{ Flux ratio} \end{cases} \quad \text{of } W_0 \text{ and } W_1 \,,$$

gives the BH parameters $(M \,,\, \chi)$,

if the inclination angle $\theta_{\rm obs}$,

the source's motion $(\vec{x}_{\rm s} \,,\, \vec{u}_{\rm s})$,

and the source's emission spectrum $I_{\rm s}(\nu_{\rm s})$

are known.

$\rightarrow$ What should we do with observation?

• Steps for extracting (M, χ) from observation.

(a) Theory:

Prepare numerically the date set of $(\Delta t_{\rm obs}, \mathcal{R}_{\rm obs})$ with various values of $(M, \chi\, ;\, \theta_{\rm obs}, \vec{x}_{\rm s}, \vec{u}_{\rm s}, I_{\rm s})$.

(b) Observation:

Observe the target (BH candidate) and take the data $(\Delta t_{\rm obs}, \mathcal{R}_{\rm obs})$ as many as possible.

(c) Comparison:

Make the table from (a) and (b).

$\rightarrow$ See the next page $\cdots$

◇ If this table is obtained by steps (a), (b) and (c),

obs. data $(\Delta t_{\rm obs}\,,\, \mathcal{R}_{\rm obs})$	corresponding theoretical data by step (a) $(M, \chi\,;\, {\rm C})\,,\; {\rm C} = (\theta_{\rm obs}\,, \vec{x}_{\rm s}, \vec{u}_{\rm s}, I_{\rm s})$
(1.32 , 0.27)	$(9.0\,, 0.1\,; {\rm C}_0)\,,\; (\boxed{3.2\,, 0.8}\,; {\rm C}'_0)\,,\; (5.8\,, 0.8\,; {\rm C}''_0)\,,\; \cdots$
(4.05 , 0.03)	$(\boxed{3.2\,, 0.8}\,; {\rm C}_1)\,,\; (2.1\,, 0.9\,; {\rm C}'_1)\,,\; (1.9\,, 0.5\,; {\rm C}''_1)\,,\; \cdots$
(7.94 , 1.04)	$(0.8\,, 0.3\,; {\rm C}_2)\,,\; (7.4\,, 0.9\,; {\rm C}'_2)\,,\; (\boxed{3.2\,, 0.8}\,; {\rm C}''_2)\,,\; \cdots$
(9.28 , 0.44)	$(\boxed{3.2\,, 0.8}\,; {\rm C}_3)\,,\; (4.5\,, 0.5\,; {\rm C}'_3)\,,\; (1.9\,, 0.5\,; {\rm C}''_3)\,,\; \cdots$

$\rightarrow$ then we suggest $\underline{(M, \chi) = (3.2\,,\, 0.8)}$

This talk discusses the steps (a) and (b)

2. SGL's Observable Quantities

2.1 Setup for numerical calculation

- Input parameters: $M\,,\ \chi\,,\ \theta_{\rm obs}\,,\vec{x}_{\rm s}\,,\ \vec{u}_{\rm s}\,,\ I_{\rm s}(\nu_{\rm s})$
- Output parameters: $\Delta t_{\rm obs}\,,\ \mathcal{R}_{\rm obs} \leftarrow$ I calculate these quant.
- Back Ground: Kerr spacetime

$$ds^2 = g_{tt}\,dt^2 + 2g_{t\varphi}\,dt\,d\varphi + g_{rr}\,dr^2 + g_{\theta\theta}\,d\theta^2 + g_{\varphi\varphi}\,d\varphi^2$$

$$\begin{cases} g_{\mu\nu} = g_{\mu\nu}(r,\theta\,;\,M,\chi) & \text{determined by } M\,,\ \chi \\ x^\mu = (\,t\,,\ r\,,\ \theta\,,\ \varphi\,) & \text{Boyer-Lindquist coord.} \end{cases}$$

2.2 Steps to calculate $(\Delta t_{\rm obs}\,,\,\mathcal{R}_{\rm obs})$

Step1. Solve Null Geodesic Eq. which connects
the source and observer (semi-shooting)

$\rightarrow$ Time delay Δt is obtained.

Step2. Solve Geodesic Deviation Eq.

$\rightarrow$ Visible solid-angle $\Delta\Omega$ is obtained.

Step3. Specify the source's velocity $\vec{u}_{\rm s}$ and
specific intensity $I_{\rm s}(\nu_{\rm s})$ $[{\rm erg/s\,cm^2\,Hz\,\Omega}]$.

$\rightarrow$ Flux ratio $\mathcal{R}_{\rm obs}$ is obtained.

2.3 Step1: Null geodesics, $\Delta t_{\rm obs}$ and Doppler

- Some notes on Kerr BH :

◇ BH horizon at t =const. is the sphere of radius $r_{\rm BH}$

$$r_{\rm BH} = M\left[1+\sqrt{1-\chi^2}\right] \ [{\rm cm}]$$

◇ Ergo-surface : $r_{\rm erg} = M\left[1+\sqrt{1-\chi^2\cos^2\theta}\right]$

$\rightarrow$ Radial motion $(\theta, \varphi =$ const.$)$ is

impossible in the ergo-region $r < r_{\rm erg}$.

$\rightarrow$ Any object rotates with BH spin in "$r \leq r_{\rm erg}$".

◇ Geodesic motion is "three-dimensional" in general, except for on the equatorial plane $\theta = \pi/2$.

- (Null) Geodesic eq. in Hamilton's formalism

◇ Dynamical Variables : $\begin{cases} x^\mu(\eta) & : \text{null geodesic} \\ k_\mu(\eta) & : \text{tangent 1-from} \end{cases}$

→ In Kerr spacetime, (η : affine para.)

$$k_\mu = (-\varepsilon \, , \, k_r(\eta) \, , \, k_\theta(\eta) \, , \, l \,)$$

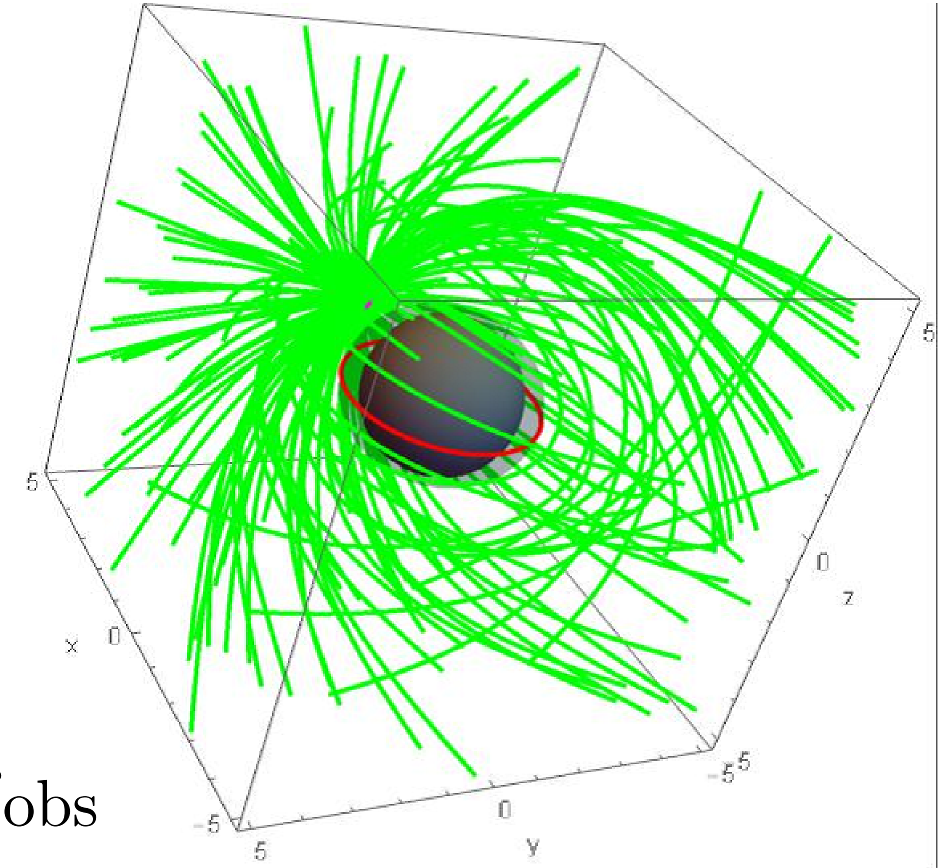
◇ Hamiltonian : $\mathcal{H} = \frac{1}{2} k_\mu k_\nu g^{\mu\nu}(x) \quad \left(= \frac{1}{2} k^2 \right)$

◇ Hamilton's eq.: $\frac{\mathrm{d}x^\mu}{\mathrm{d}\eta} = \frac{\partial \mathcal{H}}{\partial k_\mu} \, , \; \frac{\mathrm{d}k_\mu}{\mathrm{d}\eta} = - \frac{\partial \mathcal{H}}{\partial x^\mu}$

→ Solve these six ODEs numerically.

∗ note: $\mathrm{d}\mathcal{H}/\mathrm{d}\eta \equiv 0 \;\Rightarrow\; k^2 = 0$ holds automatically

- Solving null geodesic eq. by semi-shooting:
  - ∘ Fix $r_{\rm obs}$, $\vec{x}_{\rm s}$
  - ∘ Shoot light rays in many directions at $\vec{x}_{\rm s}$
  - ∘ Store necessary data at every point on the sphere of radius $r_{\rm obs}$



then $\cdots$

(ex. $r_{\rm obs} = 100\, r_{\rm BH}$)

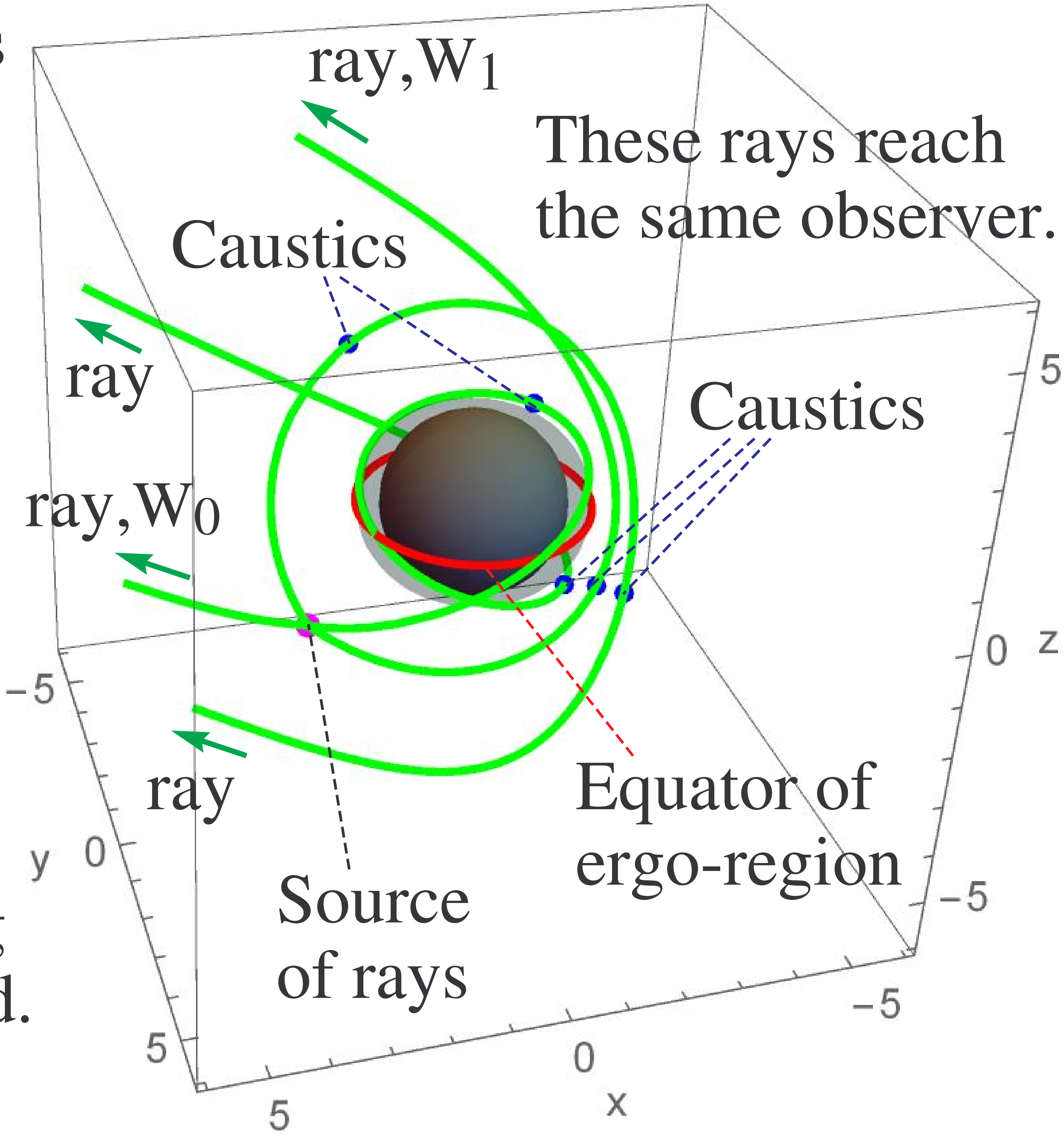
Some examples of light rays:

$M = 1.0$
$\chi = 0.8$

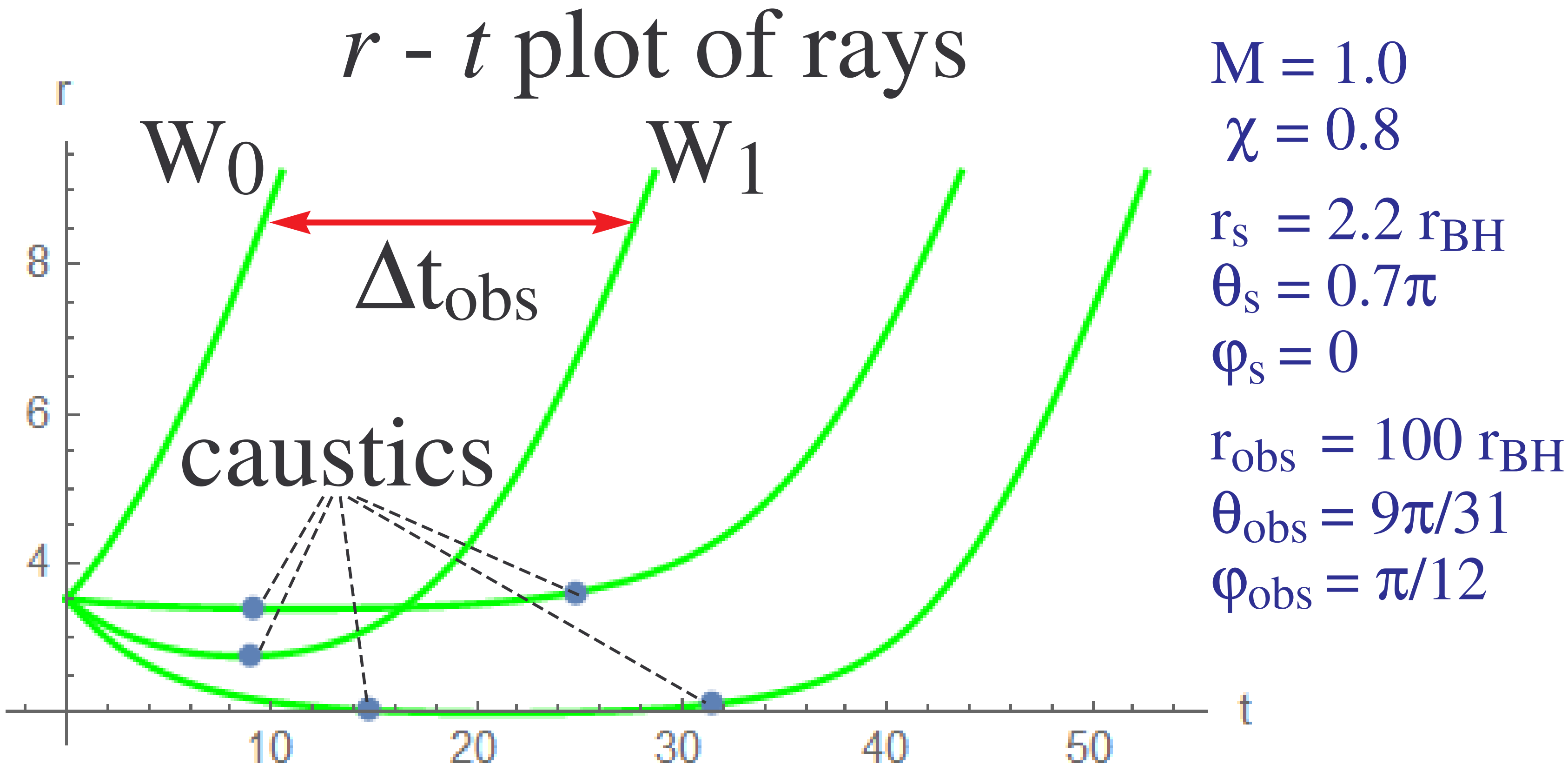
$r_s = 2.2\, r_{BH}$
$\theta_s = 0.7\pi$
$\varphi_s = 0$

$r_{obs} = 100\, r_{BH}$
$\theta_{obs} = 9\pi/31$
$\varphi_{obs} = \pi/12$

Higher winding rays are omitted.



- Time delay $\Delta t_{\rm obs}$ is read from the "r-t plot" of the primary ray W_0 and secondary ray W_1.



- Total Doppler effect: $\dfrac{\nu_{\rm s}}{\nu_{\rm obs}} = \dfrac{k_\mu u^\mu_{\rm s}}{k_\mu u^\mu_{\rm obs}}$ (k^μ : null vector)

2.4 Def. of Specific Flux: preparation for next

- Observed Specific Flux $[\mathrm{erg/s\ cm^2\,Hz}]$

$$F_{\rm obs}(\nu_{\rm obs}) = I_{\rm obs}(\nu_{\rm obs})\,\Delta\Omega_{\rm obs} = \left(\frac{\nu_{\rm obs}}{\nu_{\rm s}}\right)^3 I_{\rm s}(\nu_{\rm s})\,\Delta\Omega_{\rm obs}$$

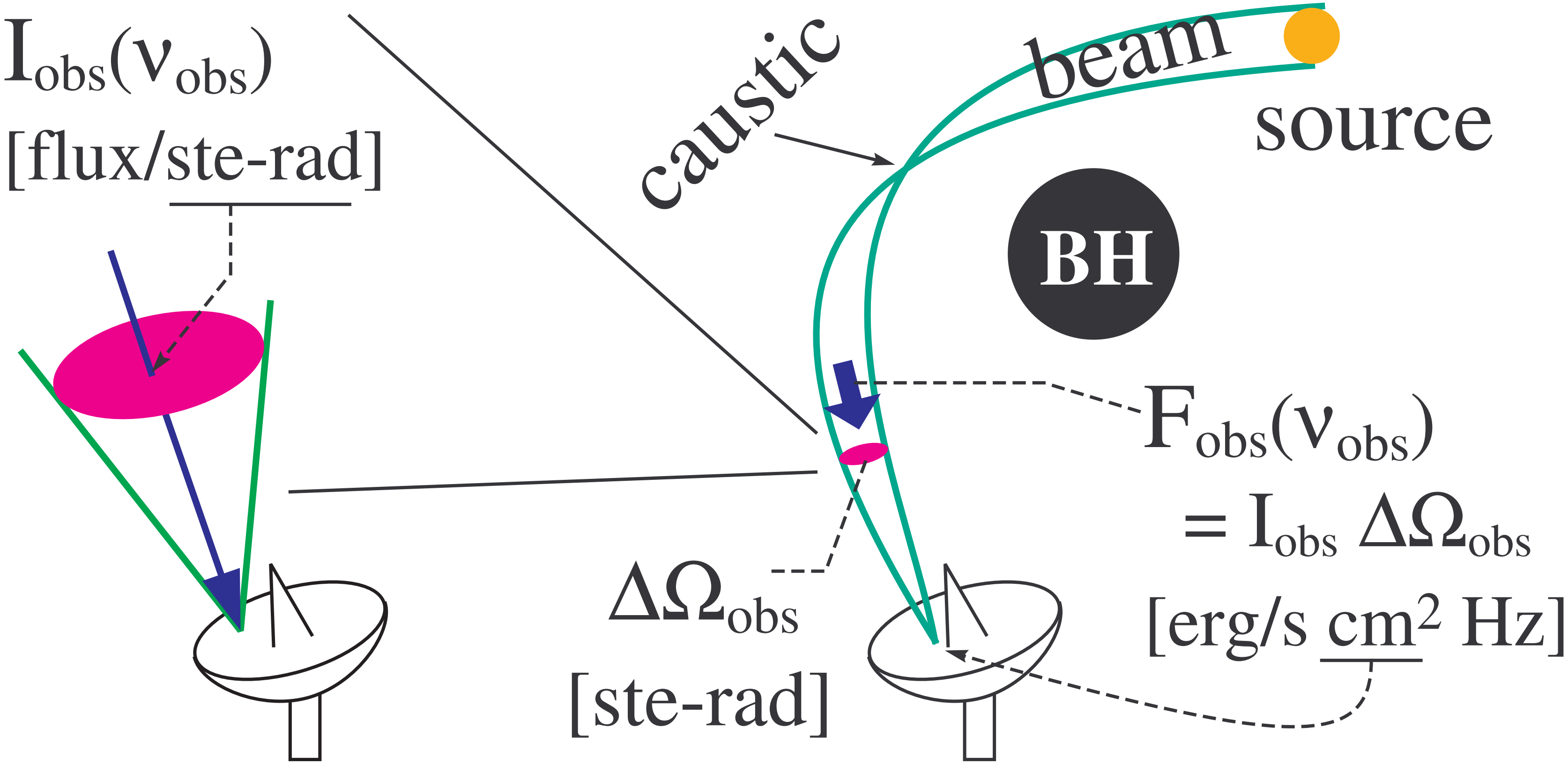
where
$$\begin{cases} \text{freq. at observer} : \nu_{\rm obs} = -u^\mu_{\rm obs}k_\mu\big|_{\rm obs} \\ \text{freq. at emission} : \nu_{\rm s} = \nu_{\rm s}(\nu_{\rm obs}) = -u^\mu_{\rm s}k_\mu\big|_{\rm s} \\ \text{Visible Solid-angle} : \Delta\Omega_{\rm obs} \\ \text{Specific Intensity} : I_{\rm s}(\nu_{\rm s}) \end{cases}$$

$[\mathrm{erg/s\,cm^2\,Hz\,ste\text{-}rad}]$

→ see next fig.

Liouville's Thm:

$$\frac{I_{\rm obs}}{\nu_{\rm obs}^3} = \frac{I_{\rm s}}{\nu_{\rm s}^3}$$



∘ "specific" = per unit frequency

∘ Intensity $I(\nu)$ is the flux per unit solid-angle.

∗ [ste-rad] is the unit of solid-angle.

(Full sky = 4π [ste-rad])

• Why the specific intensity $I(\nu)$?

$\rightarrow I(\nu)$ $[\mathrm{erg/s\,cm^2\,Hz\,ste\text{-}rad}]$ gives a useful scalar quantity along each null geodesic!

The scalar quantity along a flow line

The collisionless Boltzmann equation (applied to photons) reveals the following qunatity is a scalar along each flow line (null geodesic):

$$\frac{I(\nu)}{\nu^3} = \text{const. along each flow line}$$

$*$ ref: Misner, Thorne & Wheeler, *Gravitation*, p587

2.5 Visible solid-angle – Step2

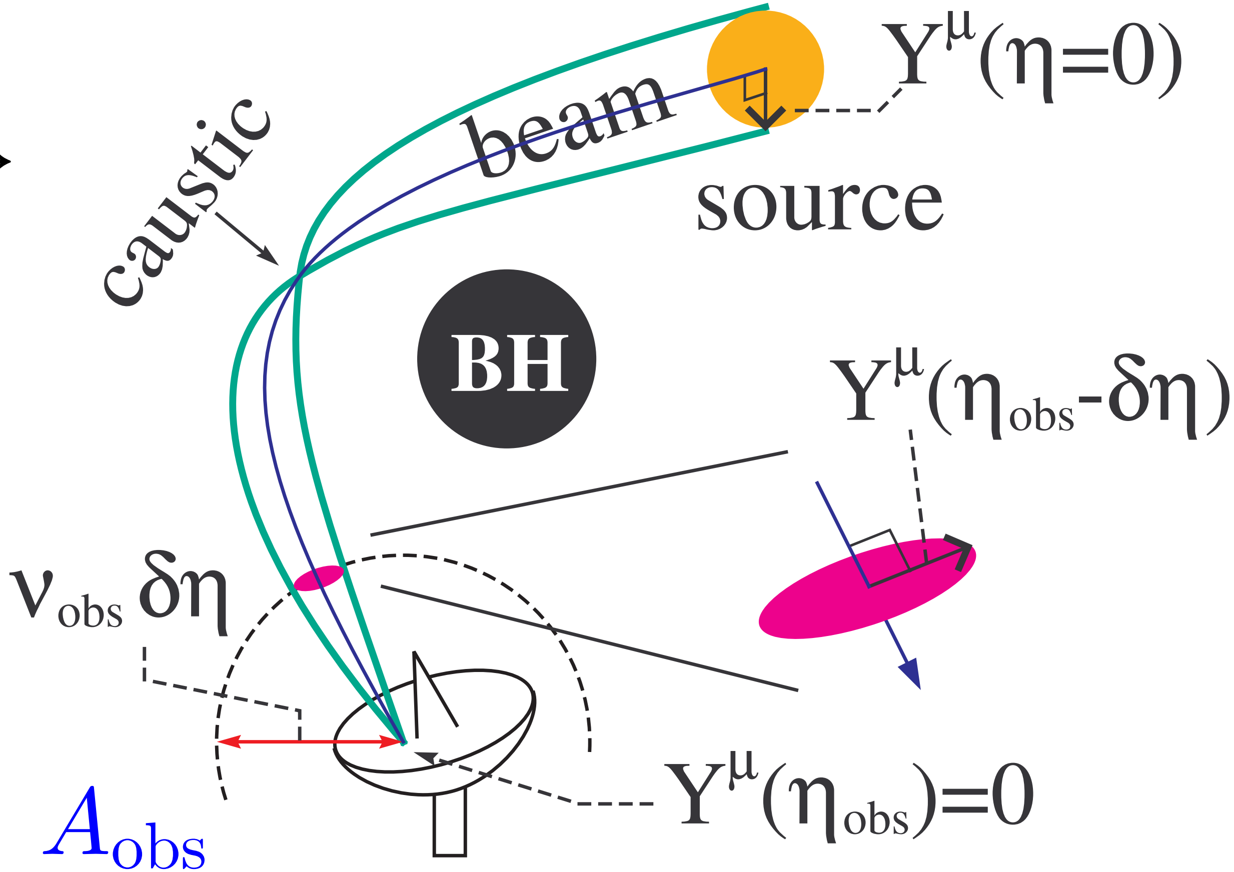
- Geodesic deviation
  $Y^\mu(\eta)$ with cond.$\rightarrow$

$$\Downarrow$$

$$Y^\mu(\eta - \delta\eta) = -\delta\eta \left.\frac{\mathrm{D}Y^\mu}{\mathrm{d}\eta}\right|_{\eta=0}$$

$$\Downarrow$$

$$\Delta\Omega_{\rm obs} = \frac{\delta\eta^2 A_{\rm obs}}{(\nu_{\rm obs}\delta\eta)^2} = \frac{A_{\rm obs}}{\nu_{\rm obs}^2}$$



where $A_{\rm obs}$ = "area swept by $\mathrm{D}Y^\mu/\mathrm{d}\eta(0)$".

- Null geodesic deviation eq. (Jacobi eq.)

◇ A null tetrad basis: $e^{\mu}_{(\mathrm{a})} = \{\, k^{\mu}\,,\ l^{\mu}\,,\ e^{\mu}_{(1)}\,,\ e^{\mu}_{(2)}\,\}$

$$g_{\mu\nu} e^{\mu}_{(\mathrm{a})} e^{\nu}_{(\mathrm{b})} = \left[\begin{array}{cc|cc} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \quad (\mathrm{a}, \mathrm{b} = k, l, 1, 2)$$

→ Parallel transport along the null geodesic $k^{\mu}(\eta)$

$$k^{\nu}(\eta)\nabla_{\nu} e^{\mu}_{(\mathrm{a})}(\eta) = 0 \quad \cdots \quad \text{solve numerically}$$

→ Tetrad basis "field" $e^{\mu}_{(\mathrm{a})}(\eta)$ on the null geodesic.

◇ Tetrad component:

$$Y_{(\mathrm{a})} = Y_\mu e^\mu_{(\mathrm{a})} \quad \leftarrow \text{scalar}$$

◇ Jacobi eq.: $$\frac{\mathrm{d}^2 Y_{(\mathrm{a})}}{\mathrm{d}\eta^2} = -\underline{\mathrm{Rmn}^{(\mathrm{b})}{}_{(\mathrm{k})(\mathrm{a})(\mathrm{k})}}\, Y_{(\mathrm{b})}$$

↑

(Tetrad component of Riemann curvature tensor)

→ (a) $= (1), (2) \cdots$ by symmetry of $\mathrm{Rmn}^\mu{}_{\nu\alpha\beta}$

→ Solve these two ODEs numerically

to obtain $Y^\mu(\eta)$

→ Area element A_{obs} is calculated.

2.6 Step3: Flux ratio

- Observer 4-velocity : $u^{\mu}_{\rm obs} = \dfrac{\partial}{\partial t}$

  $\rightarrow \nu_{\rm obs} = -u^{\mu}_{\rm obs} k_{\mu}\Big|_{\rm obs} = -k_{ct}$ (const. of motion)

- Input source's velocity $u^{\mu}_{\rm s}$

  $\rightarrow \nu_{\rm s} = -u^{\mu}_{\rm s} k_{\mu}\Big|_{\eta=0}$, $k^{\mu}(\eta)$: tangent of null geodesic

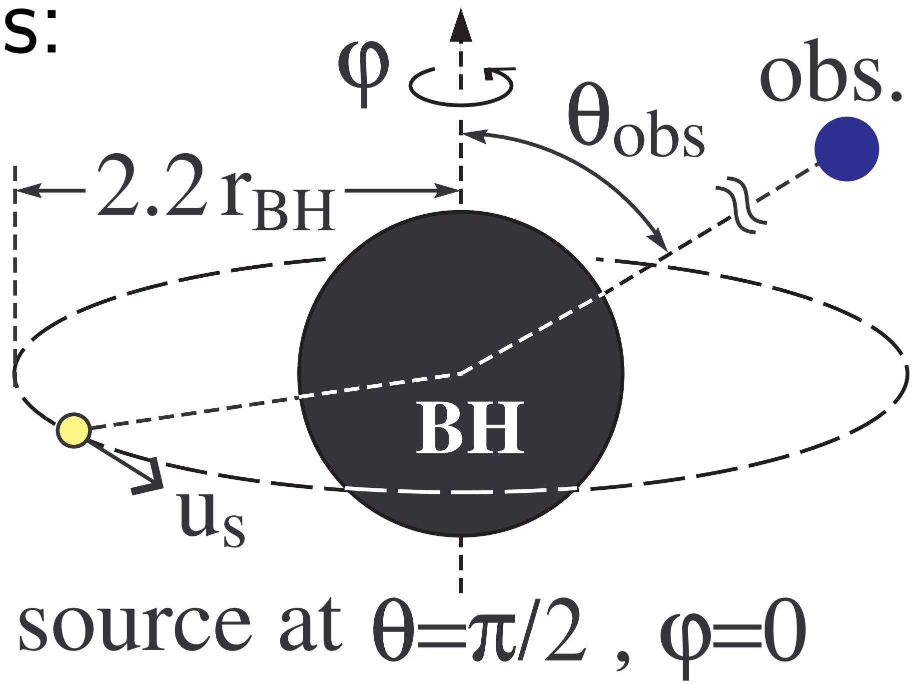
  $\rightarrow \nu_{\rm s} = \nu_{\rm s}(\nu_{\rm obs})$ Total Dopper effect is obtained

- Input specific intensity at source $I_{\rm s}(\nu_{\rm s})$

  $\rightarrow$ Flux $\mathcal{F}_{\rm obs}(\nu_{\rm obs})$ is obtained !

2.7 Ex. of numerical results: preliminary

- Parameters of next figures:
  - Configuration: $\longrightarrow$



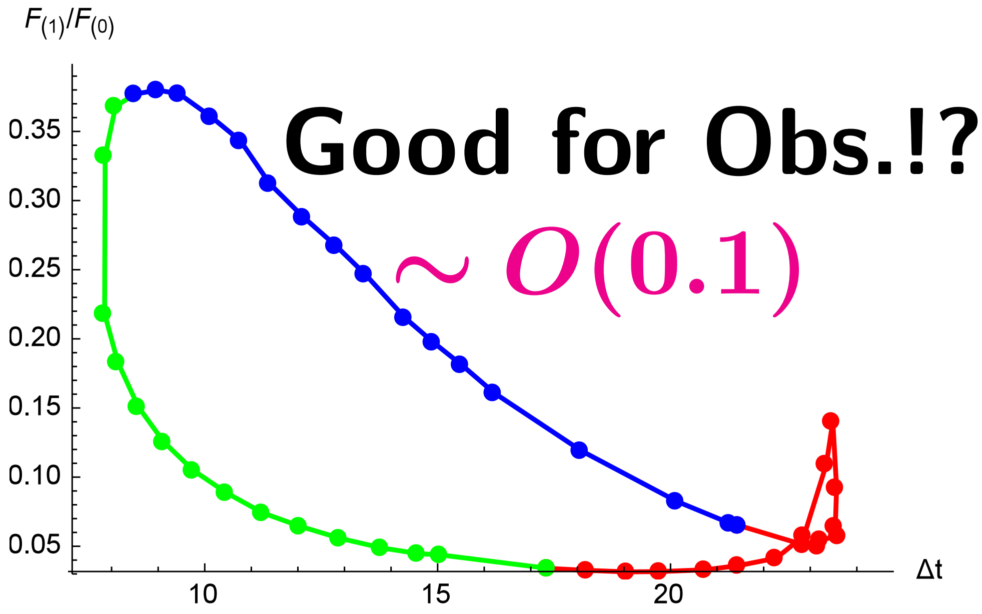
  - BH: $(M, \chi) = (1.0\,,\, 0.8)$
  - Source velocity:
    $u_{\rm s}^{\mu} = (1.49\,,\, 0\,,\, 0\,,\, 0.05)$
  - Inclination: $\theta_{\rm obs} = \dfrac{12}{31}\pi$
  - Line emission: $I_{\rm s}(\nu_{\rm s}) = \delta(\nu_{\rm s} - \nu_{\rm c})$, $\nu_{\rm c}$ is const.
  - Emission at $\nu_{\rm c}$ $\to$ Obs. with $\nu_{\rm obs(0)}$ and $\nu_{\rm obs(1)}$

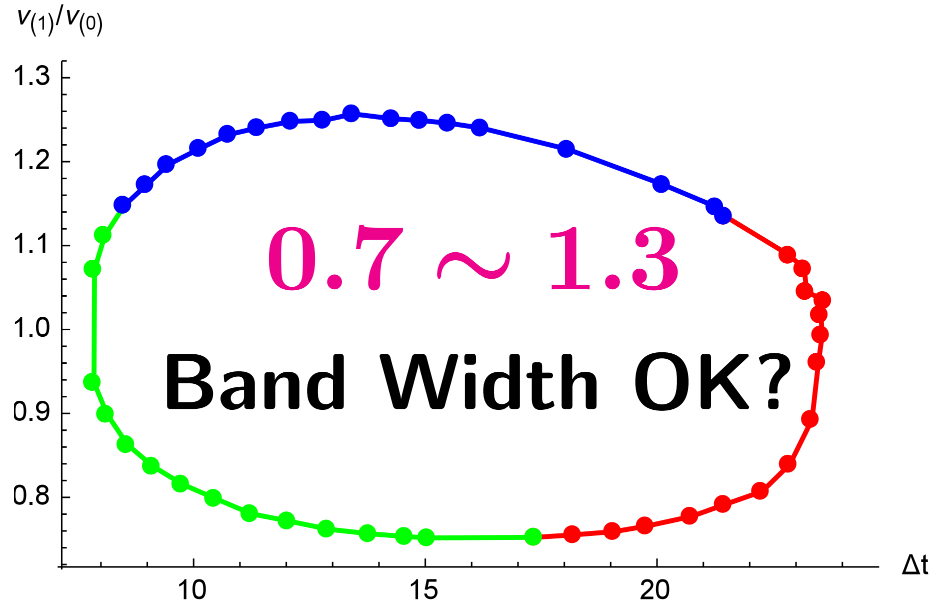
- Calculate $\Delta t_{\rm obs}$, $F_{\rm obs(1)}/F_{\rm obs(0)}$, $\nu_{\rm obs(1)}/\nu_{\rm obs(0)}$ at every azimuthal angle of obs. $\varphi_{\rm obs}$

◇ coloring of $\varphi_{\rm obs}$:

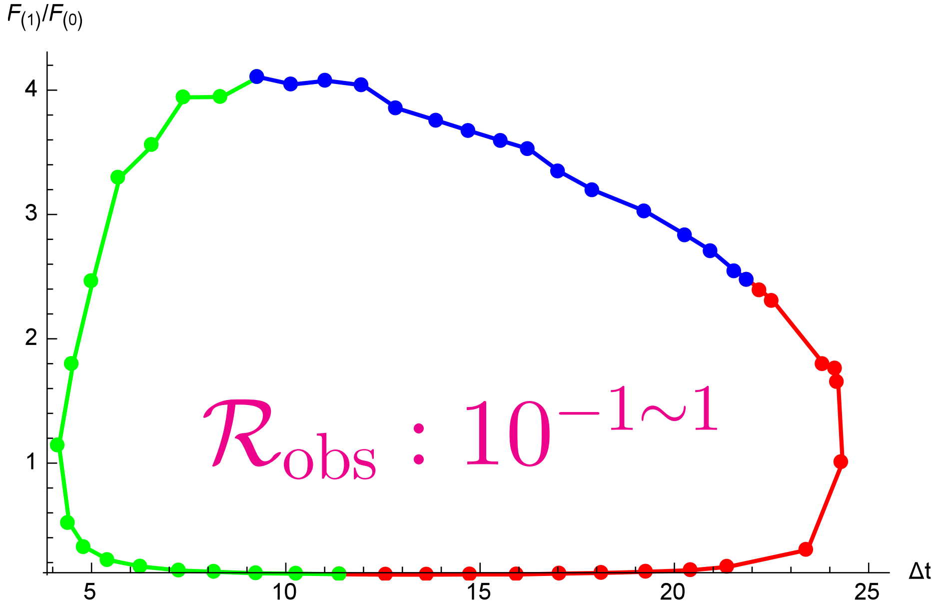
$\varphi_{\rm obs}$	**0** $\longrightarrow$ **2π**
color	**R** $\to$ **G** $\to$ **B**

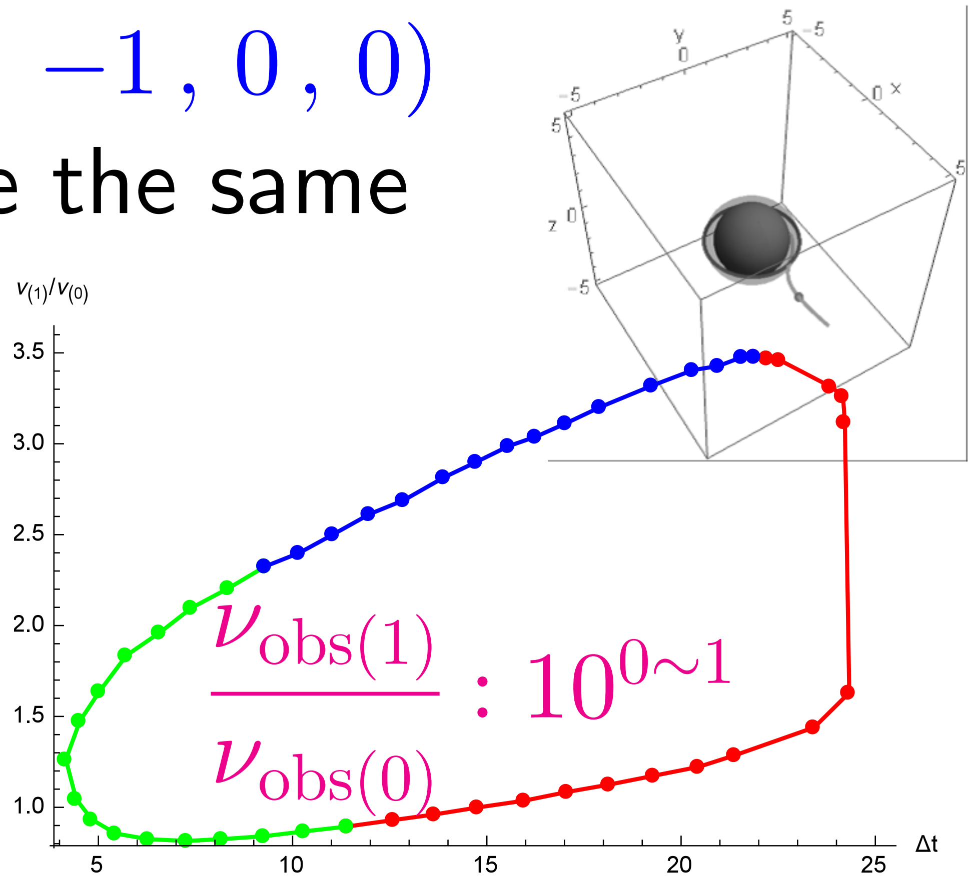
◇ hor. : $\Delta t_{\rm obs}/M$ — ver. : F_1/F_0(left) , ν_1/ν_0(right)





- Replace with $u_{\rm s}^{\mu} = (2.7\,,\,-1\,,\,0\,,\,0)$
  the other parameters are the same





◇ Various values of $\mathcal{R}_{\rm obs}$ is possible!

◇ Typically $O(\Delta t_{\rm obs}) \sim O(\pi r_{\rm s})$ ($= 10$ for this case)

⇒ There should be the case which is
detectable by the present telescope capability!

2.8 現在の望遠鏡での検出可能性

- 例：NICT鹿島34m電波望遠鏡

情報通信研究機構 (NICT)
次世代時空計測グループHPより

S/N：$R_{\rm sn} = \dfrac{F_{\rm obs}}{F_{\rm sefd}} \sqrt{2\,\delta\nu\,\delta t}$

◇ システム雑音：$F_{\rm sefd} = 300{\rm Jy}$

◇ 観測バンド幅：$\delta\nu = 1024{\rm MHz}$

◇ 観測継続時間：$\delta t = 60{\rm sec}$

◇ 信号検出基準：$R_{\rm sn} > 6$

$(1{\rm Jy} = 10^{-26}{\rm W/m^2Hz})$

→ 検出可能な光線の強度条件：$F_{\rm obs} > 0.005{\rm Jy}$

0.005Jy以上の光線が来れば検出できる。

- 観測対象の例：SgrA*

→ 典型的な放射強度： 1Jy

→ W$_0$ (primary ray) の強度の仮定： $F_0 = 0.1$Jy

→ W$_1$ (secondary ray) が観測可能であるためには：

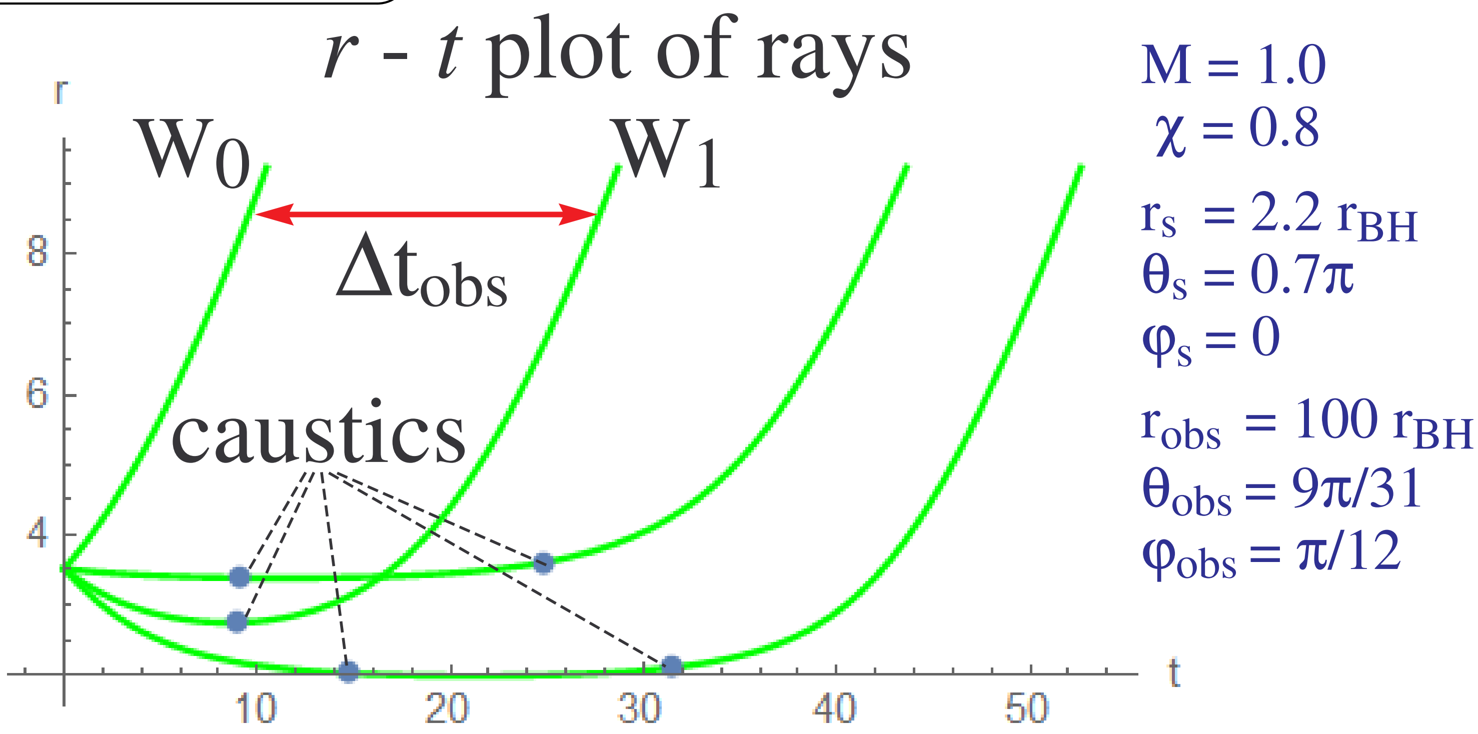
$$F_1 > 0.005\text{Jy} \Rightarrow \boxed{\mathcal{R}_{\rm obs} := \frac{F_1}{F_0} > 0.05}$$

現在の望遠鏡性能で $(\Delta t_{\rm obs}, \mathcal{R}_{\rm obs})$ の
観測可能性が期待できる！

3. SGL in the Light Curve

3.1 Indication by sec.2

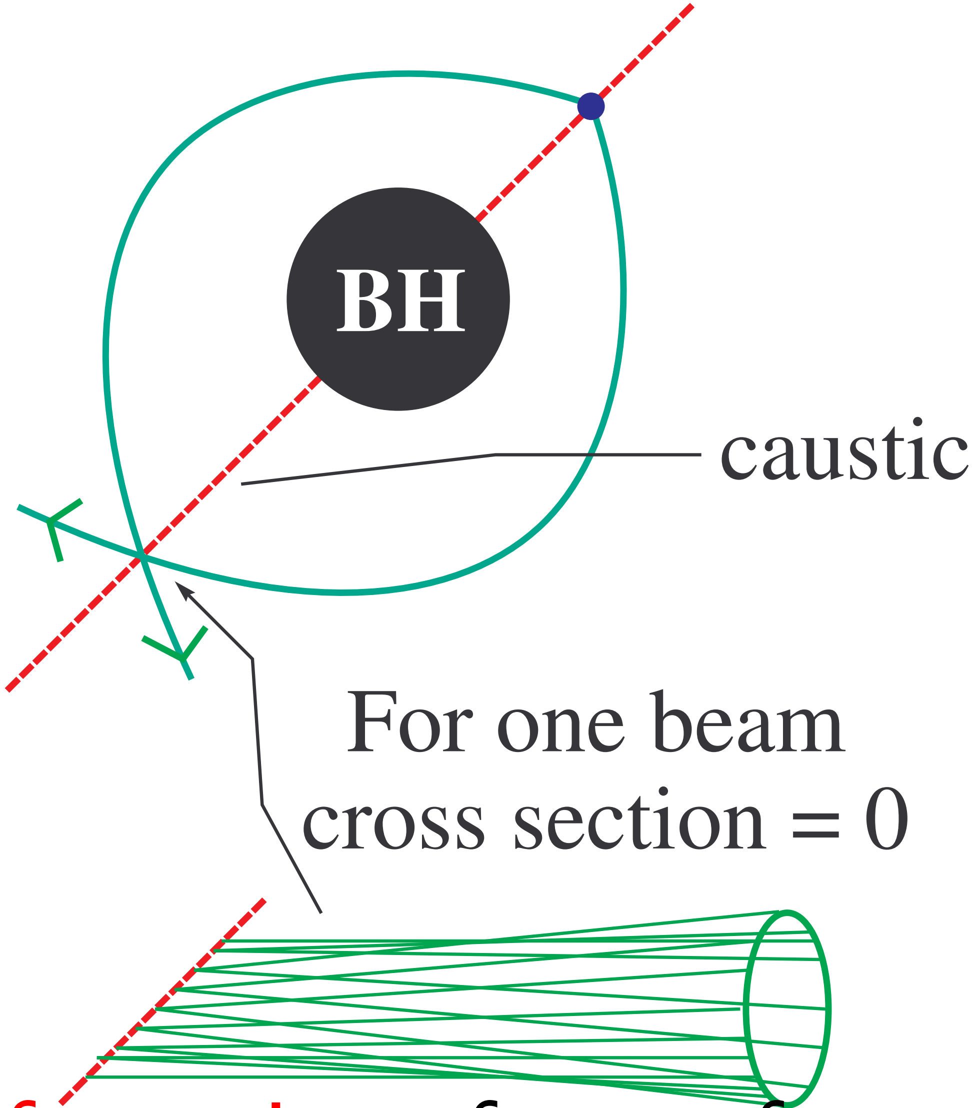
- The ray W_0 passes no causitic.
- The ray W_1 passes one caustic.



$\rightarrow$ The effect of caustic on the light curve may be important for the observation.

3.2 Gouy phase shift: wave optics issue (not GR)

Phase shift of waves when passing the caustic (an interference effect)



◇ positive freq. mode

$\rightarrow$ phase shift by $-\pi/2$

◇ negative freq. mode

$\rightarrow$ phase shift by $+\pi/2$

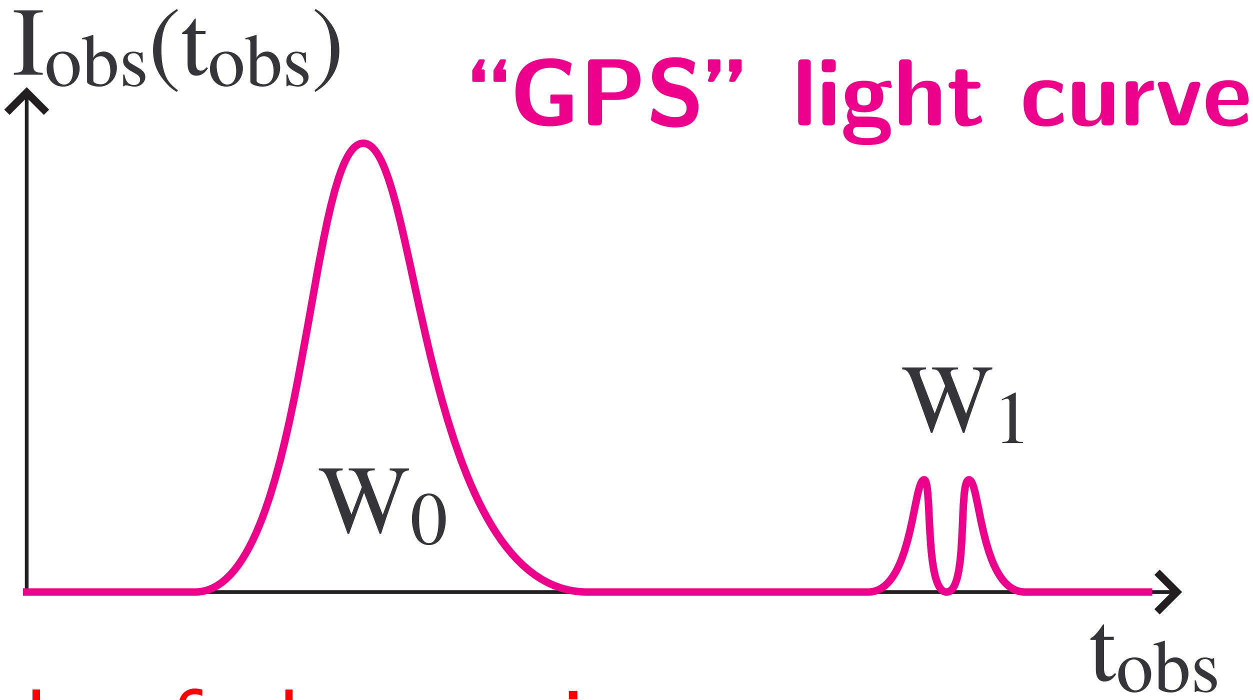
$\Rightarrow$ ex. $\cos(\omega t) \leftrightarrow \sin(\omega t)$

$\Rightarrow$ This is the Hilbert transformation of wave form.

3.3 Expected feature of light curve

- Gouy Phase Shift
  $\Leftrightarrow$ Hilbert trans. of wave form

- Observed Flux
  $F_{\rm obs} \propto |E_{\rm obs}|^2$
  ($E_{\rm obs}$: amplitude)



Principle of observation

Find the GPS (Gous Phase Shifted) light curve from the time series data taken by a telescope. Then, the delay $\Delta t_{\rm obs}$ and ratio $\mathcal{R}_{\rm obs}$ are obtained.

4. Summary

- "Direct" BH detection is
  to measure M , χ through GR effects.
- Focus on the Spinning BH's Gravitational Lens
- Obs. quantities $(\Delta t_{\rm obs}\,,\, F_1/F_0)$ seem to be
  detectable by the present telescope capability !?
  $\rightarrow$ Already estimated for a radio telescope in Japan.
  How about X-ray telescope ?
- Light curve $\rightarrow$ the Gouy effect may appear.
- If $\nu_{(1)}/\nu_{(0)}$ is also an observable, it is useful.