高速回転ブラックホール付近での天体 (粒子) の高速衝突

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第 4 **回 「**BH **磁気圏勉強会」研究会**, 2011 **年** 2 **月** 28 **日**-3 **月** 2 **日**
T. Harada and M. Kimura, PRD83:024002,2011
T. Harada and M. Kimura, arXiv:1102.3316

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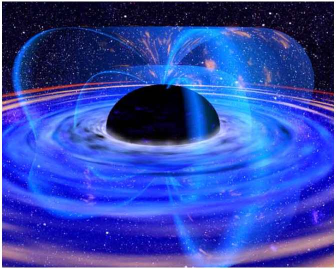
Astrophysical black holes



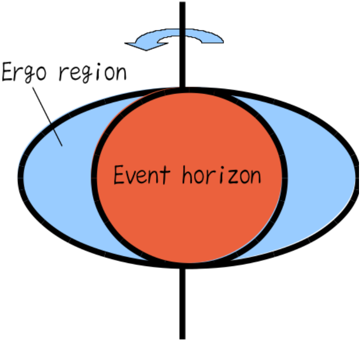
- Black hole candidates
  - X-ray binary: several-10 $M_\odot$, accretion disk
  - Galactic centre: $10^6 - 10^9 M_\odot$
- Towards "direct" observation
  - High-resolution observation with electromagnetic waves, Black hole "shadow"
  - Direct observation of spacetime geometries with gravitational waves

注釈

- ところどころで一般相対論に馴染みのない方のための注釈を入れておきます。
- 時間も長めなのでいつでも質問してください。

Rotating black holes

- Black holes are usually rotating.
- Rotating black holes are uniquely described by a Kerr metric, if ....
- The Kerr spacetime is parametrized by the mass M and the spin a.
- $0 \leq |a| \leq M$: black hole, $|a| > M$: naked singularity
- $a_* = a/M$: nondimensional Kerr parameter



注釈

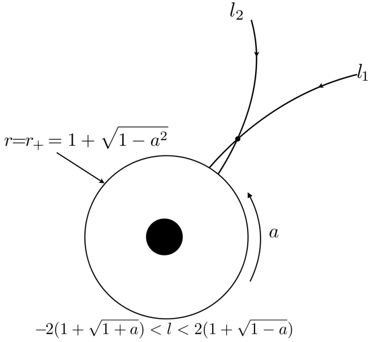
- 幾何単位系: $G = c = 1$
  この単位系では、次元としては時間=長さ=質量になる。例えば $1M_\odot = 1.5\text{km} = 5\mu\text{s}$。
- $a = 0$ の場合
  球対称静的なブラックホール。すなわち Schwarzschild ブラックホールになる。
- a の次元
  a は長さの次元をもつ。スピン角運動量を J として $a = J/M$。
- Event horizon
  無限遠に信号を送ることができる領域の境界。光的測地線によって生成される光的超曲面。
- Ergo region
  ブラックホールが定常に見える観測者が存在しない領域。

Bañados, Silk and West (2009)

- Let particles 1 and 2 of rest mass m_0 which begin at rest at infinity fall into a maximally rotating black hole of mass M on the equatorial plane.
- The specific angular momentum $l = L/(m_0 M)$ is then restricted to $-2(1+\sqrt{2}) < l < 2$.
- When they collide near the horizon, the centre-of-mass (CM) energy is given by

$$\frac{E_{\rm cm}}{2m_0} = \sqrt{\frac{1}{2}\left(\frac{2-l_1}{2-l_2}+\frac{2-l_2}{2-l_1}\right)}.$$

If we fine-tune $l_i \to 2$, $E_{\rm cm}$ can be arbitrarily high. This is a natural particle accelerator.



注釈

- specific angular momentum (**比角運動量**)
  単位質量当たりの角運動量
- **無次元角運動量**
  角運動量 L **は長さの二乗の次元なので** $l = L/(m_0 M)$ **は無次元。**
- centre-of-mass (CM) energy (**重心系エネルギー**)
  衝突のエネルギーを表す Lorentz **不変な量。**LHC **などの粒子衝突器のエネルギーはこの重心系エネルギーで測られる。正確な定義はあとで。**
- fine-tuning (**微調整**)
  パラメータ p **をある特定の値** p_c **のごく近傍に調整すること。**
  $|p - p_c| \ll p_c$ **が要求される。**

Criticisms on BSW (1)

The scenario is unphysical. Observables should be finite.

- The collision with an arbitrarily high CM energy takes arbitrarily long proper time.
- The test particle approximation will break down due to the self-force of the particles and the radiation reaction of gravitational waves. (Berti et al. 2009, Jacobson & Sotiriou 2010, Kimura et al. 2011)
- On the other hand, it is also reasonable that the CM energy will become significantly high, where the test particle approximation is valid.
- Here we assume the test particle approximation.

注釈

- **通常、ブラックホールに落ちる粒子は有限の固有時間でホライズンを横切る。**
- test particle approximation (**試験粒子近似**)
  粒子が (**自分が作り出す重力場を含まない**) **背景の時空の測地線を運動するとする近似。ブラックホールに比べて粒子が十分小さくて軽く、かつ重力波放射が無視できる場合には成り立つと考えられる。**
- self-force (**自己力**)
  ブラックホールの周りを運動する粒子が自分自身の作り出す重力場によって受ける力。
- **重力波による反作用**
  静磁場によって加速度を受けた荷電粒子は電磁波を放射して反作用を受ける。同様に静的な (**あるいは定常な**) **重力場で「加速度」を受けた粒子は重力波を放射し反作用を受ける。**

Criticisms on BSW (2)

The scenario is unrealistic. It is not realised in the Universe.

- There seems no maximally-rotating black hole in the Universe! Cf. Thorne (1974)'s bound $a_* \equiv a/M \lesssim 0.998$.
- For $a_* \approx 1$, the upper limit is estimated as

$$\frac{E_{\rm cm}}{2m_0} \sim \frac{2.41}{\sqrt[4]{1-a_*^2}}.$$

  This is at most ~ 20 for $a_* = 0.998$. (Jacobson & Sotiriou 2010).
- The fine-tuning of the angular momentum of the particle seems very difficult.

注釈

- 宇宙物理学的ブラックホールのスピンの上限
  さまざまな理由から宇宙物理学的には $a_* = 1$ は実現しないという見方がある。Thorne の上限値は、降着円盤からブラックホールに入射する光子を考慮したもの。ただし、本当に普遍的な上限値があるのか、あるとしたらどの値なのかは定説のようなものはないようである。
- LHC では p-p 衝突で CM energy=7 TeV が可能と言われている。これは $E_{\rm cm}/(2m_0) \sim 3500$ に対応する。

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Kerr black hole

- Kerr metric

$$ds^2 = -\left(1-\frac{2Mr}{\rho^2}\right)dt^2 - \frac{4Mar\sin^2\theta}{\rho^2}d\phi dt + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \left(r^2+a^2+\frac{2Mra^2\sin^2\theta}{\rho^2}\right)\sin^2\theta d\phi^2,$$

  where $\rho^2 = r^2 + a^2\cos^2\theta$ and $\Delta = r^2 - 2Mr + a^2$.
- If $0 \le |a| \le M$, there is an event horizon. Δ vanishes at $r = r_\pm = M \pm \sqrt{M^2 - a^2}$. The horizon radius is $r_H = r_+$.
- two commuting Killing vectors

$$\xi^a = \left(\frac{\partial}{\partial t}\right)^a, \quad \psi^a = \left(\frac{\partial}{\partial \phi}\right)^a.$$

- The angular velocity of the horizon

$$\Omega_H = \frac{a}{r_H^2 + a^2} = \frac{a}{2M(M+\sqrt{M^2-a^2})}.$$

注釈

- $r = r_{-}$ はブラックホールの中のホライズン。
- $|a| = M$ のときは二つのホライズンが一致する。このときブラックホールの表面重力はゼロになる。このようなブラックホールを極限的であるという。
- Killing ベクトルと対称性
  時空の対称性を表すベクトル場。対応して保存量が存在する。Kerr 時空がもつ対称性は定常軸対称とよばれる。
- ホライズンの角速度
  $\chi^a = \xi^a + \Omega_H \psi^a$ という Killing ベクトルがホライズンを生成する光的測地線の接ベクトルになっている。

Particle motion on the equatorial plane

- Four momentum $p^a = \dot{x}^a = dx^a/d\lambda$ (λ: affine parameter)
- Conserved quantities
  - Rest mass $m^2 = -p_a p^a$
  - Energy $E = -p_t = -\xi^a p_a$
  - Angular momentum $L = p_\phi = \psi^a p_a$,
- $\dot{x}^a$ can be given explicitly in terms of r, m, E and L. In particular,

$$\frac{1}{2}\dot{r}^2 + V(r) = 0,$$

where the effective potential $V(r)$ is given by

$$V(r) = -\frac{m^2 M}{r} + \frac{L^2 - a^2(E^2 - m^2)}{2r^2} - \frac{M(L - aE)^2}{r^3} - \frac{E^2 - m^2}{2}.$$

- 'Forward in time' condition: to have $\dot{t} > 0$ near the horizon,

$$L \leq \Omega_H^{-1} E \equiv L_c.$$

注釈

- massive particle (proton, electron, neutron, ん neutron star, black hole) も massless particle (photon) も両方念頭に置く。
- 粒子の保存エネルギー E は静止エネルギーと運動エネルギーと位置エネルギーからなる。
- massive particle の場合、ここでの affine parameter λ は固有時間 τ そのものではなく $d\tau = m d\lambda$ の関係にある。
- ブラックホールに近づくためには角運動量が上限値より小さくないといけない。photon の場合 impact parameter b が $b = L/E \leq \Omega_H^{-1}$ と制限される。

Centre-of-mass (CM) energy

- Suppose particles 1 and 2 are at the same spacetime point.
- The sum of the two momenta

$$p^a_{\rm tot} = p^a_1 + p^a_2.$$

- Centre-of-mass energy

$$E^2_{\rm cm} = -p^a_{\rm tot} p_{{\rm tot}a} = m^2_1 + m^2_2 - 2g_{ab} p^a_1 p^b_2.$$

- This is a scalar. This is coordinate-independent and in principle observable.

注釈

- 重心系エネルギーは座標変換に対して普遍な量であり、衝突のエネルギーを局所的に評価する物理的な量である。
- 重心系エネルギーと粒子の保存エネルギーは別のもの。重心系エネルギーは二粒子が同一時空点にある時にのみ定義される。
- 重心系エネルギーは保存エネルギーと直接の関係はないので、どこかにある保存エネルギーから「取り出される」必要はない。

Arbitrarily high CM energy

- It can be proven that the CM energy is bounded except in the near-horizon limit.
- For massive particles of the same rest mass m_0, in the near-horizon limit $r \to r_H$, we obtain

$$\frac{E_{\rm cm}}{2m_0} = \sqrt{1 + \frac{4M^2 m_0^2[(E_1 - \Omega_H L_1) - (E_2 - \Omega_H L_2)]^2 + (E_1 L_2 - E_2 L_1)^2}{16M^2 m_0^2 (E_1 - \Omega_H L_1)(E_2 - \Omega_H L_2)}}.$$

- If we fine-tune $L_i \to \Omega_H^{-1} E_i \equiv L_{ci}$ for either particle, $E_{\rm cm}$ can be arbitrarily high.
- This includes the BSW case, i.e. $E_1/m_0 = E_2/m_0 = 1$ and $\Omega_H = (2M)^{-1}$, as a special case.

注釈

- Near-horizon limit （地平線近傍極限)
  ここでは二粒子が地平線の外側で衝突することを考え、その重心系エネルギーを計算する。その後、衝突点を地平線に近づける極限をとって、重心系エネルギーがどういう極限値に近づくかを見る。
- なぜ発散が起こるのか？
  回転するブラックホールの重力場によって粒子が加速されたと言ってよい。より深遠な理由あるいは初等的な説明があるのかどうかよくわからない。
- Penrose 過程ではない！
  Penrose 過程ではエネルギーが負の粒子が重要な役割を果たす。今回の場合は負のエネルギーの粒子は必要ない。

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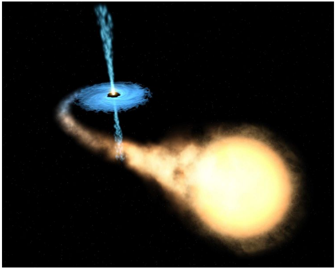
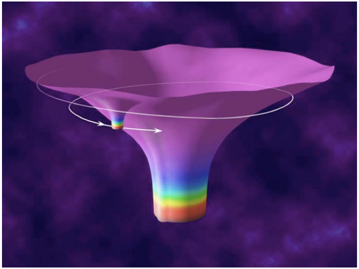
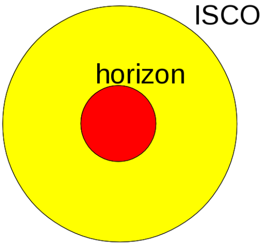
Astrophysical significance of the ISCO





- The inner edge of the standard accretion disk is given by the ISCO.
- Compact objects inspiral around a supermassive black hole and begin to plunge into the horizon at the ISCO.
- The counterpart of an ISCO for general geodesic particles is called a *last stable orbit (LSO)*.

注釈



- **安定円軌道はある半径の外側には存在するが内側には存在しない。したがってこの半径はもっとも内側の安定円軌道に対応する。**
  Innermost Stable Circular Orbit=ISCO

ISCO particle

- The circular orbit is given by $V(r) = V'(r) = 0$. Then, we find $E = E(r)$ and $L = L(r)$.
- The ISCO radius $r_{\rm ISCO}$ is determined by $dE(r)/dr = dL(r)/dr = 0$. (Bardeen, Press & Teukolsky 1972)

$$\begin{aligned} \frac{r_{\rm ISCO}}{M} &= 3 + Z_2 - [(3 - Z_1)(3 + Z_1 + 2Z_2)]^{1/2}, \\ Z_1 &= 1 + (1 - a_*^2)^{1/3}[(1 - a_*)^{1/3} + (1 + a_*)^{1/3}], \\ Z_2 &= (3a_*^2 + Z_1^2)^{1/2}. \end{aligned}$$

- $r_{\rm ISCO}/r_H$ decreases from 3 to 1 as a_* is increased from 0 to 1.
- The fine-tuning is naturally realised. $E/m \to 1/\sqrt{3}$, $L/(mM) \to 2/\sqrt{3}$, $\Omega_H \to (2M)^{-1}$ and hence $L \to \Omega_H^{-1} E \equiv L_c$ as $a_* \to 1$.

注釈

- ISCO がもっともエネルギーの低い安定円軌道になっている。
- 実は ISCO には順方向回転と逆方向回転があるが、いま重要なのは順方向回転の ISCO。

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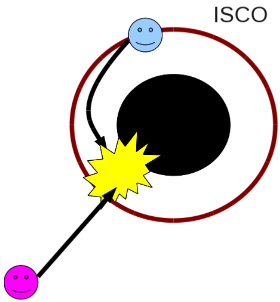
Near-horizon collision of an ISCO particle

- Substitute E_1 and L_1 for the ISCO particle into the formula.



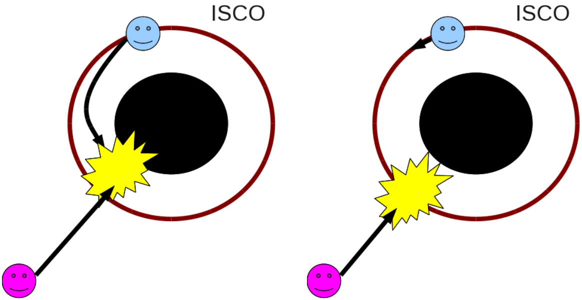
- For $a_* \approx 1$,

$$\frac{E_{\rm cm}}{2m_0} \approx \frac{1}{2^{1/2}3^{1/4}} \frac{\sqrt{2e_2 - l_2}}{\sqrt[4]{1-a_*^2}}, \quad \text{where } e_2 = \frac{E_2}{m_0} \text{ and } l_2 = \frac{L_2}{m_0 M}.$$

- $E_{\rm cm}$ diverges as $(1-a_*^2)^{-1/4}$ for $a_* \to 1$.

On-ISCO collision of an ISCO particle

- Since $r_{\rm ISCO} \to r_H$ as $a_* \to 1$, we don't need to take the near-horizon limit? Let's consider the collision on the ISCO.

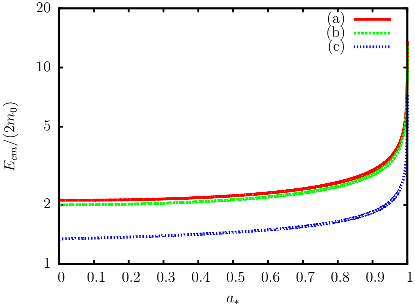


Near-horizon collision On-ISCO collision

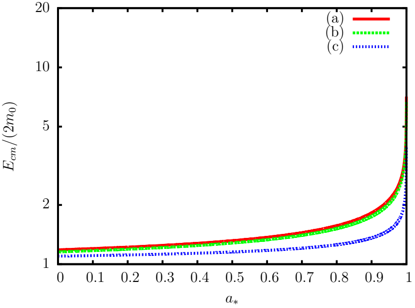
- The different diverging behaviour for $a_* \to 1$.

$$\frac{E_{\rm cm}}{2m_0} \approx \frac{1}{2^{1/6}3^{1/4}} \frac{\sqrt{2e_2 - l_2}}{\sqrt[6]{1-a_*^2}} \quad \text{for} \quad a_* \approx 1.$$

CM energy for general black hole spins



Near-horizon collision

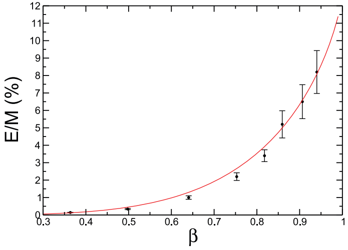


On-ISCO collision

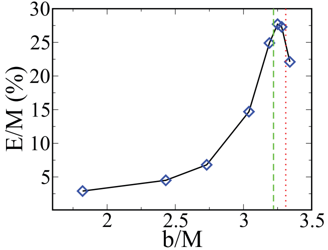
For $a_* = 0.998$, $\gamma = E_{\rm cm}/(2m_0) \simeq 3.86 - 6.95$ for the near-horizon collision and $2.43 - 4.11$ for the on-ISCO collision.

Significance of the high-velocity collision

- $\gamma = E_{\rm cm}/(2m_0) \sim 4-7$ is not so high for the particle accelerator.
- However, the collision with $\sim$ 10 GeV occurs near the inner edge of the accretion disk, which might be observable.
- The high-velocity collision of compact objects can occur around a supermassive black hole. The high-velocity collision of compact obejets is currently under investigation by numerical relativity.



Sperhake et al. 2008



Sperhake et al. 2009

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General geodesic particle

- The conserved quantities are m, E, L and the Carter constant Q.
- The tragectories in r and θ are determined by (Carter 1968)

$$\begin{aligned} \rho^2 \dot{r} &= \sigma_r \sqrt{R}, \\ \rho^2 \dot{\theta} &= \sigma_\theta \sqrt{\Theta}, \end{aligned}$$

where $\sigma_r = \pm 1$, $\sigma_\theta = \pm 1$,

$$\begin{aligned} R &= R(r) = P^2 - \Delta[m^2 r^2 + (L - aE)^2 + Q], \\ P &= P(r) = (r^2 + a^2)E - aL, \\ \Theta &= \Theta(\theta) = Q - \cos^2\theta \left[a^2(m^2 - E^2) + \frac{L^2}{\sin^2\theta} \right]. \end{aligned}$$

- 'Forward in time' condition: to have $\dot{t} > 0$ near the horizon,

$$L \leq \Omega_H^{-1} E \equiv L_c.$$

CM energy of two general geodesic particles

- The CM energy for the two-particle collision is bounded except in the near-horizon limit, in which it is given by

$$\begin{aligned} E_{\rm cm}^2 &= m_1^2 + m_2^2 + \frac{1}{r_H^2 + a^2\cos^2\theta}\left[(m_1^2 r_H^2 + \mathcal{K}_1)\frac{E_2 - \Omega_H L_2}{E_1 - \Omega_H L_1}\right. \\ & +(m_2^2 r_H^2 + \mathcal{K}_2)\frac{E_1 - \Omega_H L_1}{E_2 - \Omega_H L_2} \\ & \left.-\frac{2(L_1 - a\sin^2\theta E_1)(L_2 - a\sin^2\theta E_2)}{\sin^2\theta} - 2\sigma_{1\theta}\sqrt{\Theta_1}\sigma_{2\theta}\sqrt{\Theta_2}\right], \end{aligned}$$

  where $\mathcal{K}_i \equiv Q_i + (L_i - aE_i)^2$. This includes the equatorial result as a special case.
- If we fine-tune $L_i \to \Omega_H^{-1} E_i$ for either particle, the CM energy can be arbitrarily high.

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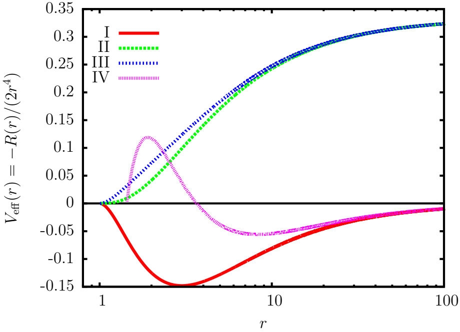
Classification of critical particles

- The effective potential V is defined by $\frac{1}{2}\dot{r}^2 + \frac{r^4}{\rho^4}V(r) = 0$.



Class	$V(r)$ at $r = r_H$	BH spin	Scenario
I	$V = V' = 0,\ V'' < 0$	$a_* = 1$	Direct collision
II	$V = V' = V'' = 0$	$a_* = 1$	LSO (ISCO) collision
III	$V = V' = 0,\ V'' < 0$	$a_* = 1$	Multiple scattering
IV	$V = 0,\ V' < 0$	$0 < a_* < 1$	Multiple scattering

Multiple-scattering scenario

- Grib & Pavlov (2010) proposed a possibility of an arbitrarily high CM energy even for a non maximally rotating black hole.

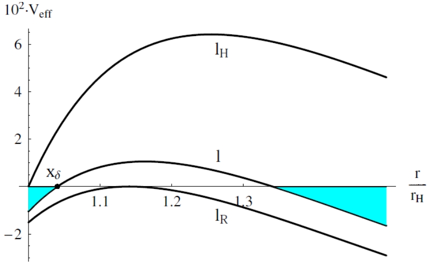


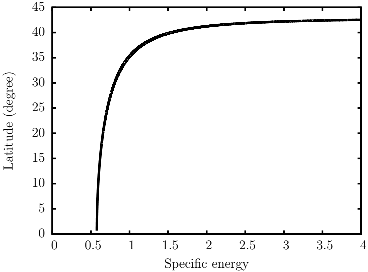
Figure 1: The effective potential for $A = 0.95$ and $l_R \approx 2.45$, $l = 2.5$, $l_H \approx 2.76$. Allowed zones for $l = 2.5$ are shown by the green color.

- A particle with $L = L_c - \delta$ for sufficiently small $\delta(> 0)$ cannot approach the horizon from well outside by the geodesic motion due to the potential barrier.
- However, a particle might be put near the horizon 'initially' through multiple scattering with other particles beforehand.

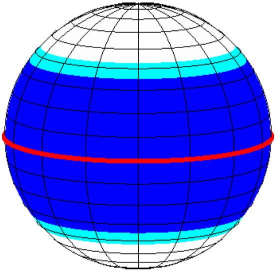
High-velocity collision belt

- We do not take the multiple scattering scenario into account here. Therefore, we assume $V = V' = 0$ and $V'' \leq 0$ at $r = r_H$.
- Then, the black hole must be maximally rotating. For Q to exist,

$$\sin\theta \geq \sqrt{\frac{-(4E^2 - m^2) + \sqrt{12E^4 - 4E^2m^2 + m^4}}{m^2 - E^2}}.$$



Highest latitude



The belt lies between $\pm 43°$.

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Summary

- An arbitrarily high CM energy occurs for the near-horizon collision under the test particle approximation, with the fine-tuning of the angular momentum and in the maximal rotation of the black hole.

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- An arbitrarily high CM energy occurs for the near-horizon collision under the test particle approximation, with the fine-tuning of the angular momentum and in the maximal rotation of the black hole.
- The fine-tuning of the angular momentum is naturally realised by an ISCO particle around a rapidly rotating black hole.

Summary

- An arbitrarily high CM energy occurs for the near-horizon collision under the test particle approximation, with the fine-tuning of the angular momentum and in the maximal rotation of the black hole.
- The fine-tuning of the angular momentum is naturally realised by an ISCO particle around a rapidly rotating black hole.
- If the maximal rotation limit can be taken, the CM energy for the collision of an ISCO particle can be arbitrarily high. The high-velocity collision around a rapidly rotating black hole can be significant in astrophysics.

Summary

- An arbitrarily high CM energy occurs for the near-horizon collision under the test particle approximation, with the fine-tuning of the angular momentum and in the maximal rotation of the black hole.
- The fine-tuning of the angular momentum is naturally realised by an ISCO particle around a rapidly rotating black hole.
- If the maximal rotation limit can be taken, the CM energy for the collision of an ISCO particle can be arbitrarily high. The high-velocity collision around a rapidly rotating black hole can be significant in astrophysics.
- These features can be extended to general geodesic particles. There is a high-velocity collision belt centered at the equator on the maximally rotating black hole.

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- On the other hand, the physical quantity is still diverging. This is not yet astrophysics. Bounding the CM energy is very important.
- Observational effects? That is the question! Be patient! Remember Blandford-Znajek effect appeared in 1977, which was 8 years after the discovery of Penrose-process in 1969. We are working in this direction.